Random Sources in Secure Computation

Geoffroy Couteau, Adi Rosén

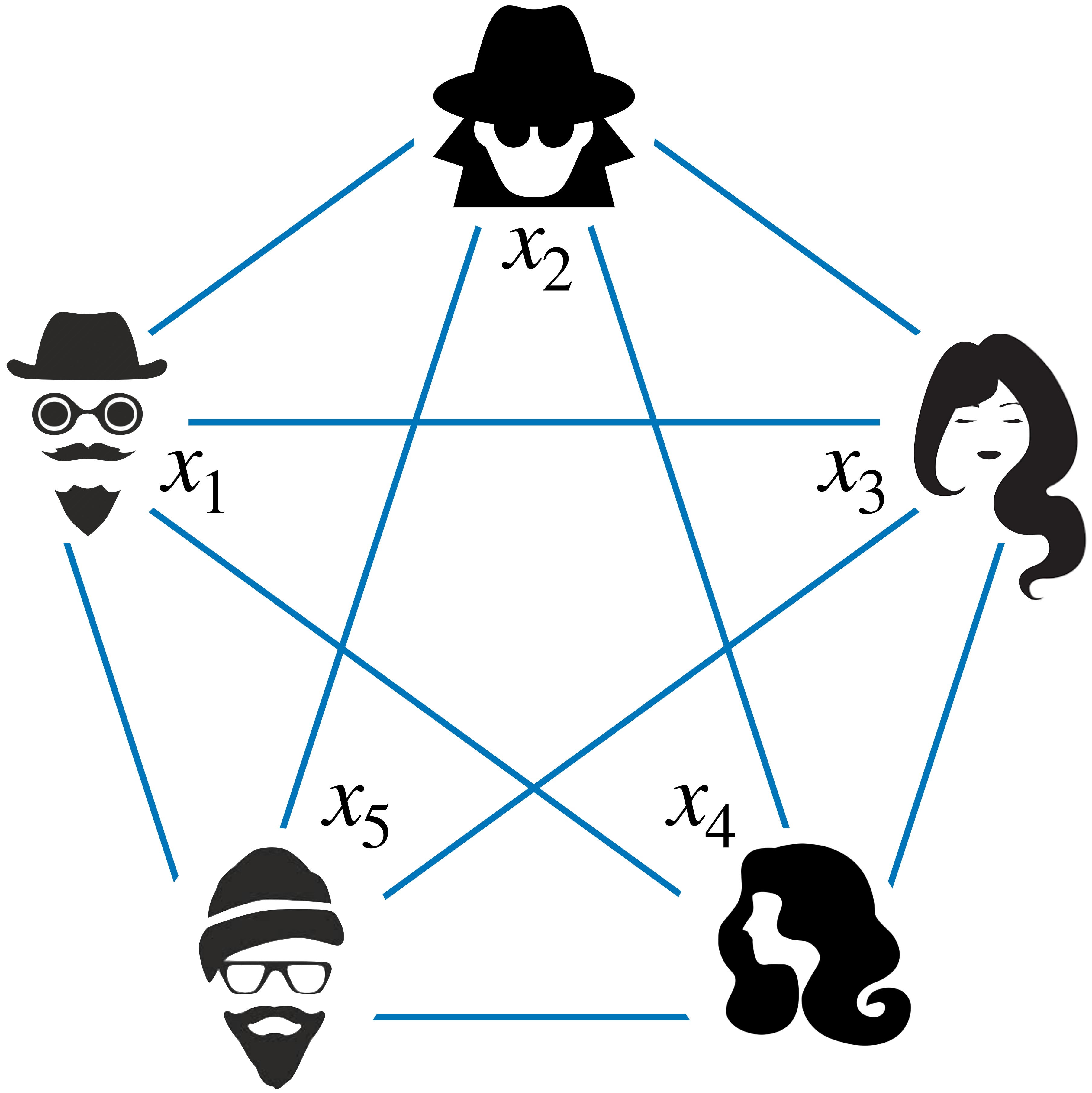
The Model

Information-Theoretic Secure Computation

- n parties with inputs $(x_1, \cdots, x_n)$
- The adversary corrupts at most t parties
- **Goal:** computing $f(x_1, \cdots, x_n)$ without revealing more

Background

This Work



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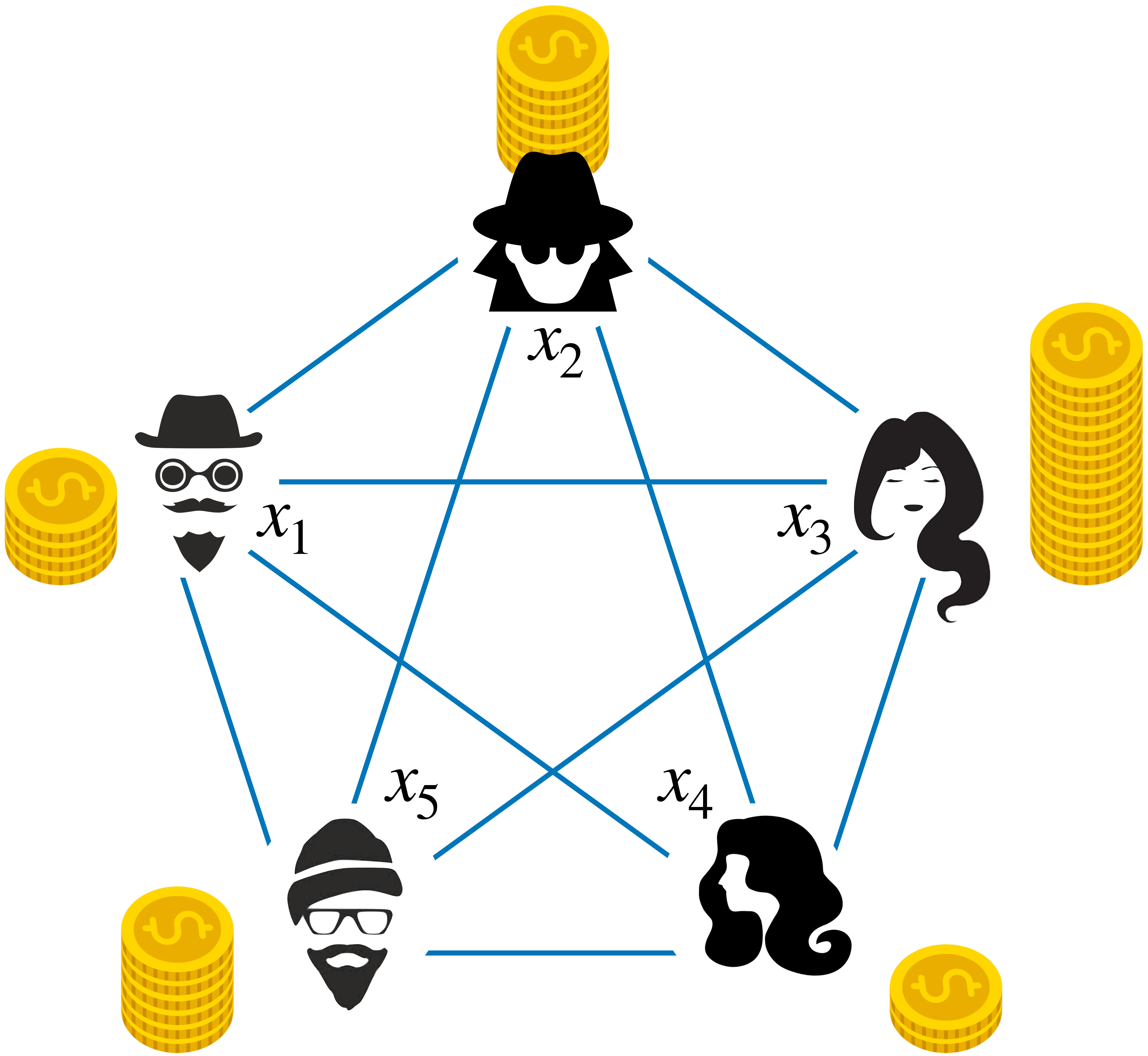

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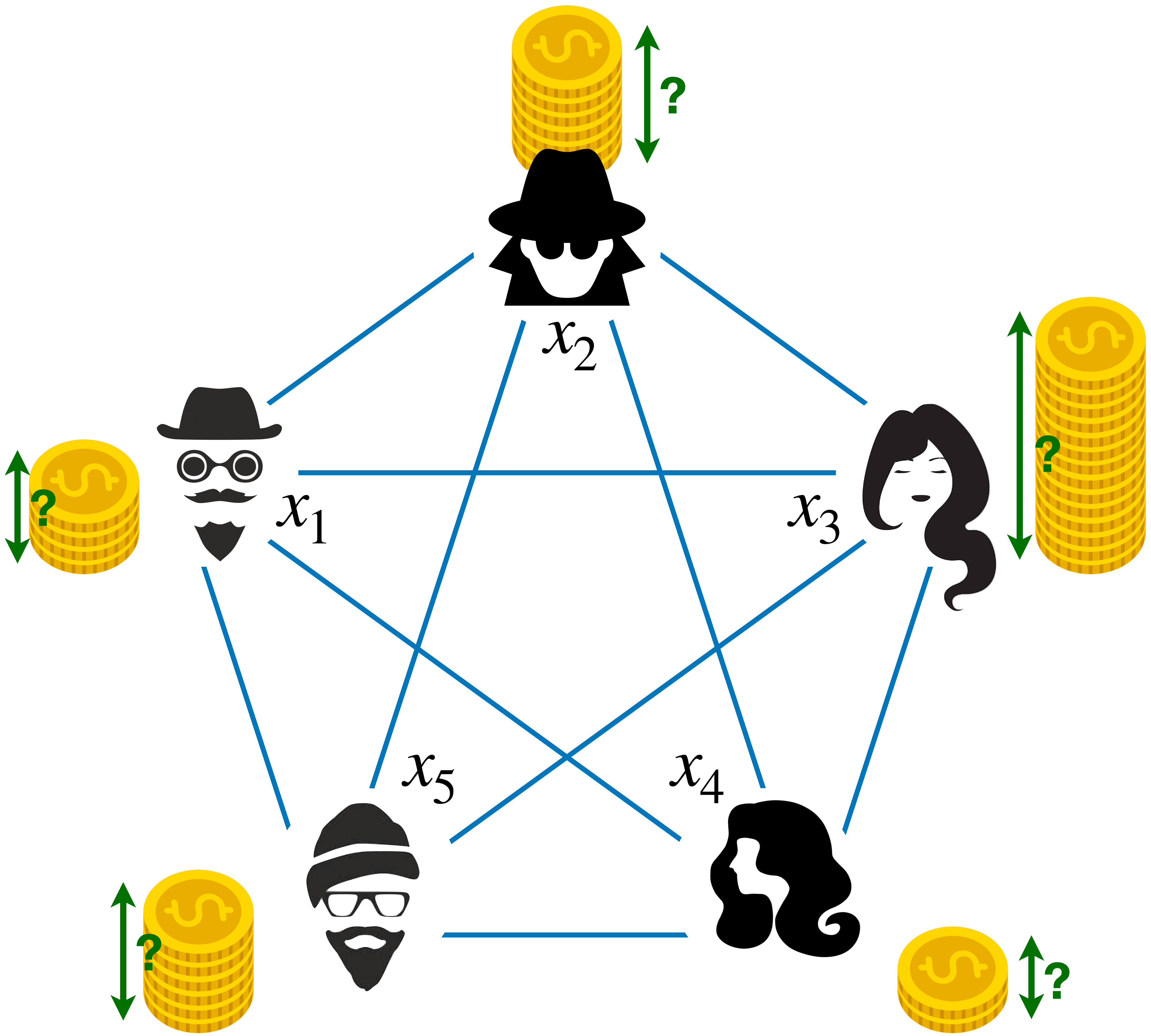

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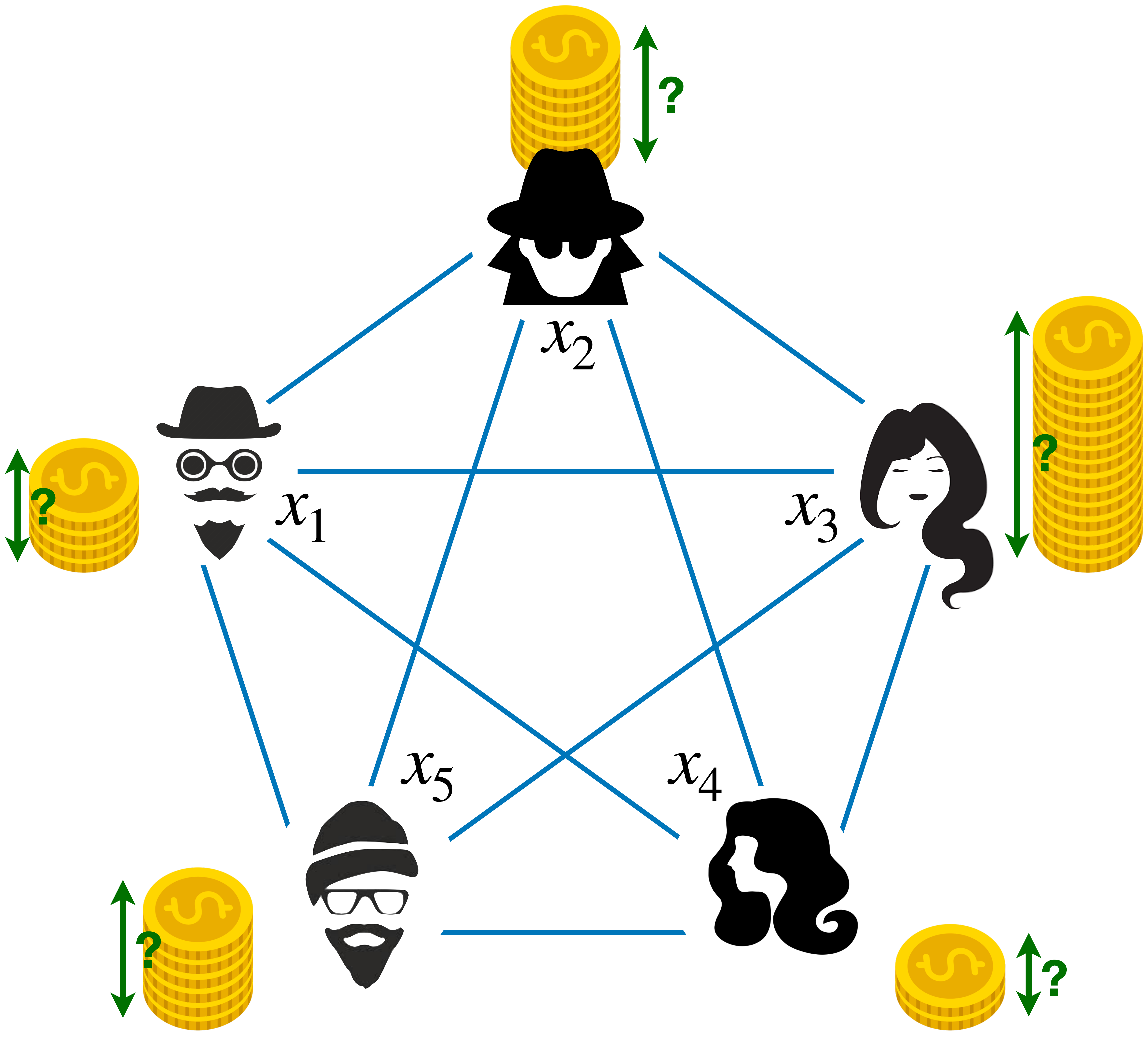

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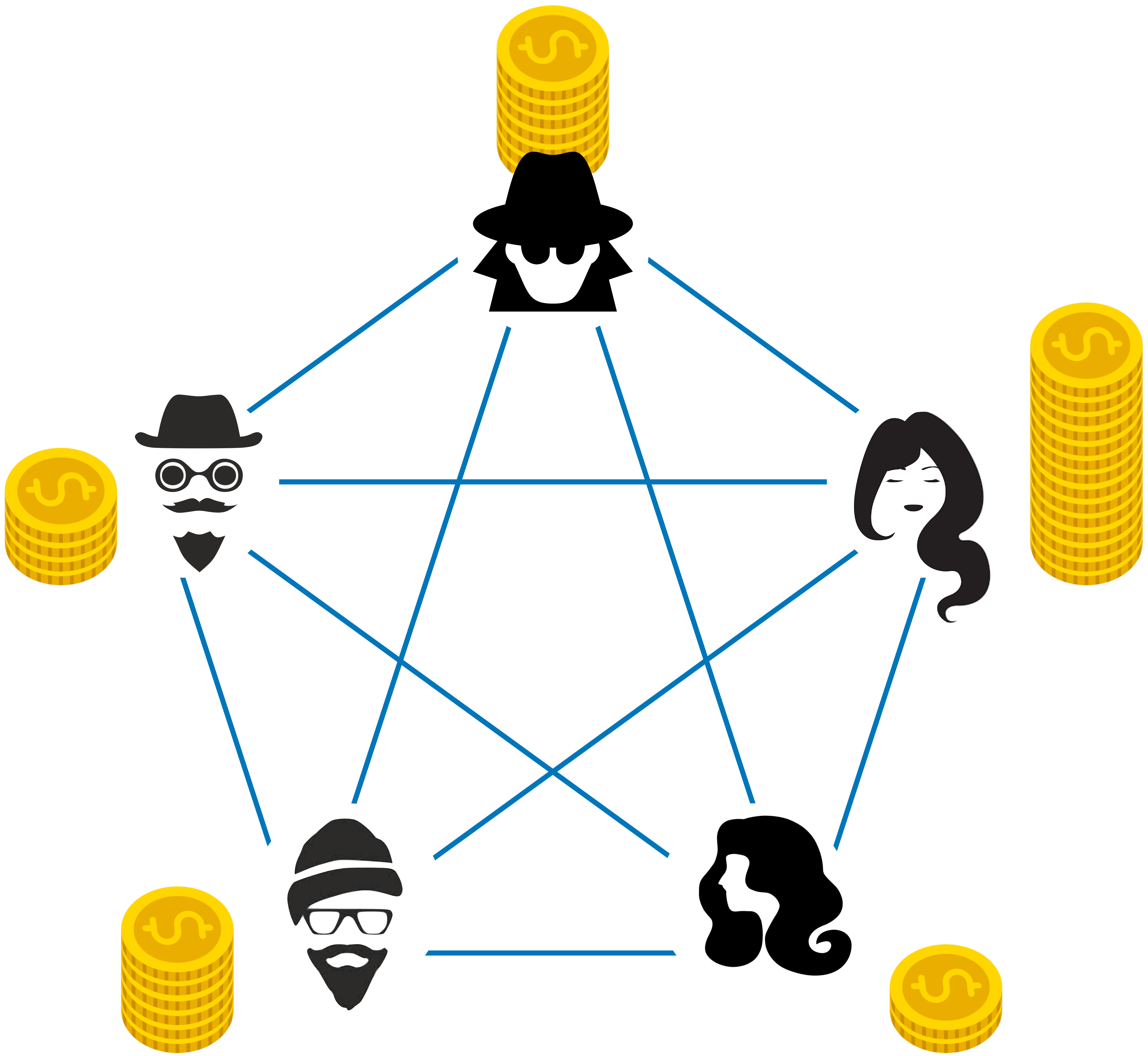

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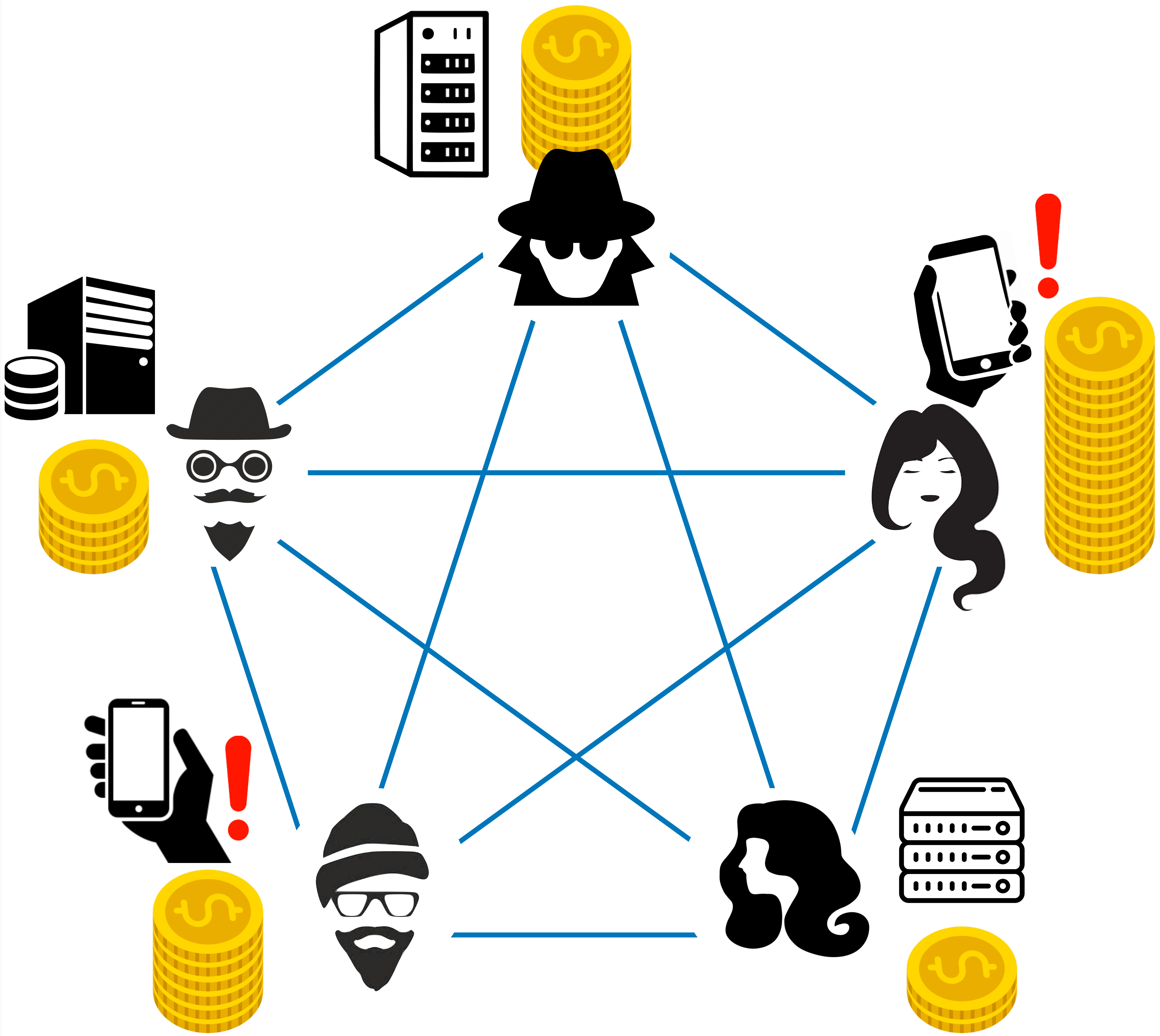

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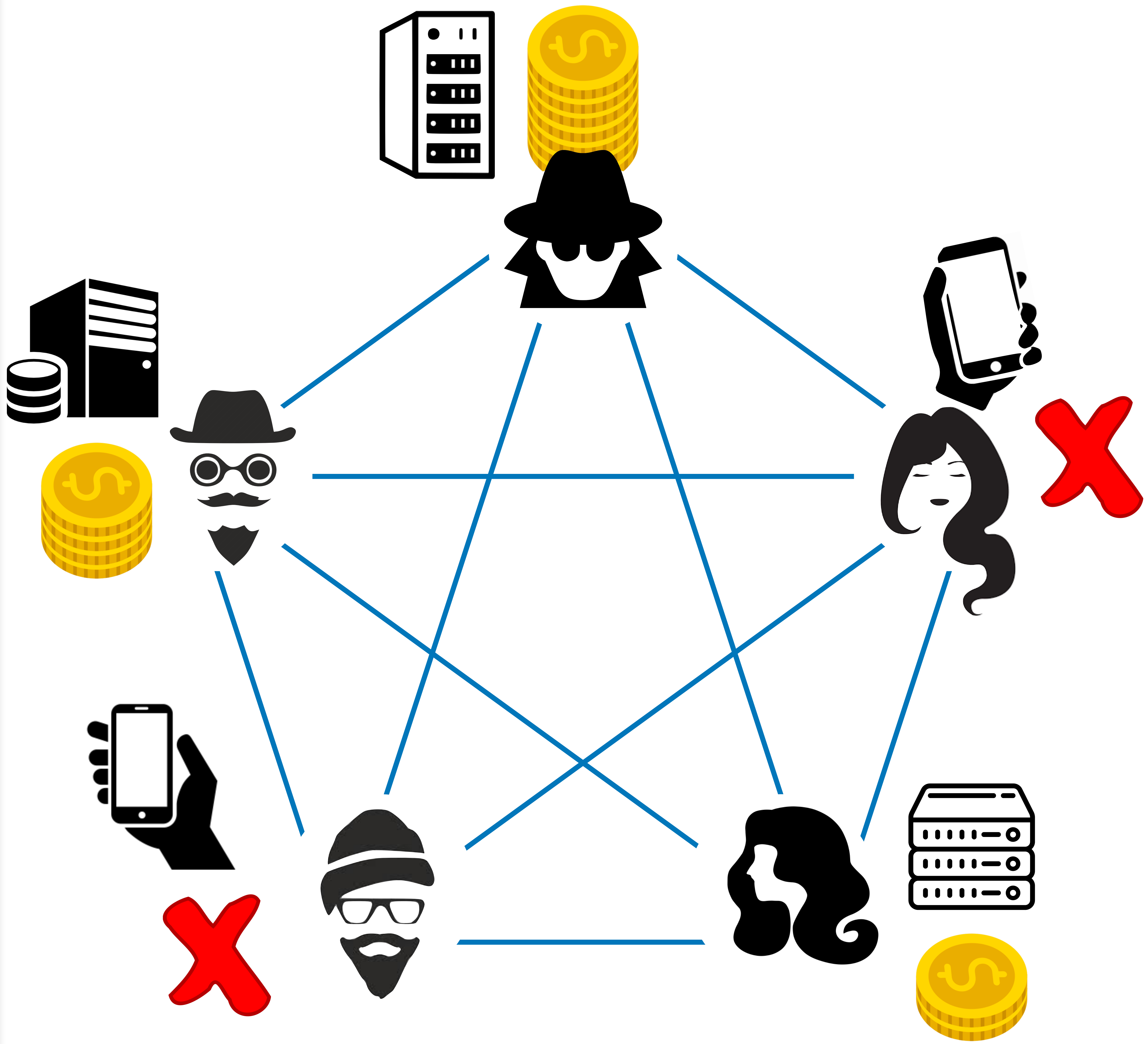

How many parties with [coin] to compute $f(x_1, \cdots, x_n)$?

First Result: Random Source Complexity of t-Private Computation

Kushilevitz & Mansour, PODC'96 show that t parties must toss coins for t-private XOR

First Result: upper and lower bounds on the number of sources

This work: there is a randomized functionality $\mathscr{F}$ that cannot be t-privately comp. with t sources

	Deterministic Functionalities	Randomized Functionalities
Lower Bound	t	$t+1$
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This work: It *suffices* that t parties toss coins to t-privately compute all deterministic functionalities

This work: $t+1$ sources suffice to t-privately compute any randomized functionality

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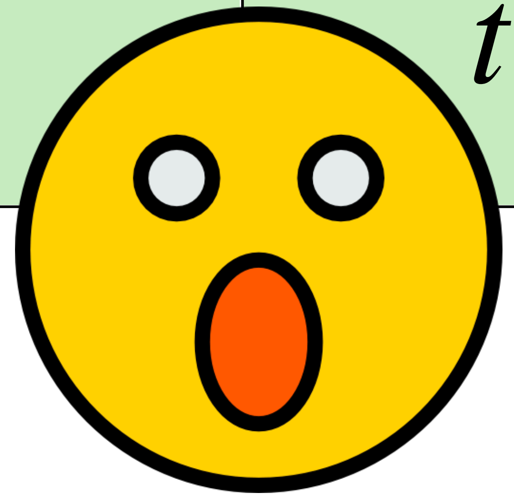

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Second Result: Randomness vs Random Sources, and 1-Private AND

Second Result: the randomness complexity of 1-private AND

- **Question.** What is the tradeoff between the number of sources and the randomness complexity? Do we need much more randomness to use a minimal number of sources?

  Proving tight bounds on randomness is notoriously very hard. Towards making progress, as in previous works, we focus on a natural functionality: the n-party AND.

  - **Best known protocol for 1-private, n-party AND:** 8 bits, 2 sources (KOPRTV, TCC'19)
  - **Question.** Can we match this bound with a single source?
  - **Our result.** Surprisingly, we manage to improve *both* the randomness complexity and the number of sources: we describe a protocol using only 6 bits and a single source.

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Starting point (warmup)

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Most of the presentation

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Previous work

Easy - see paper!

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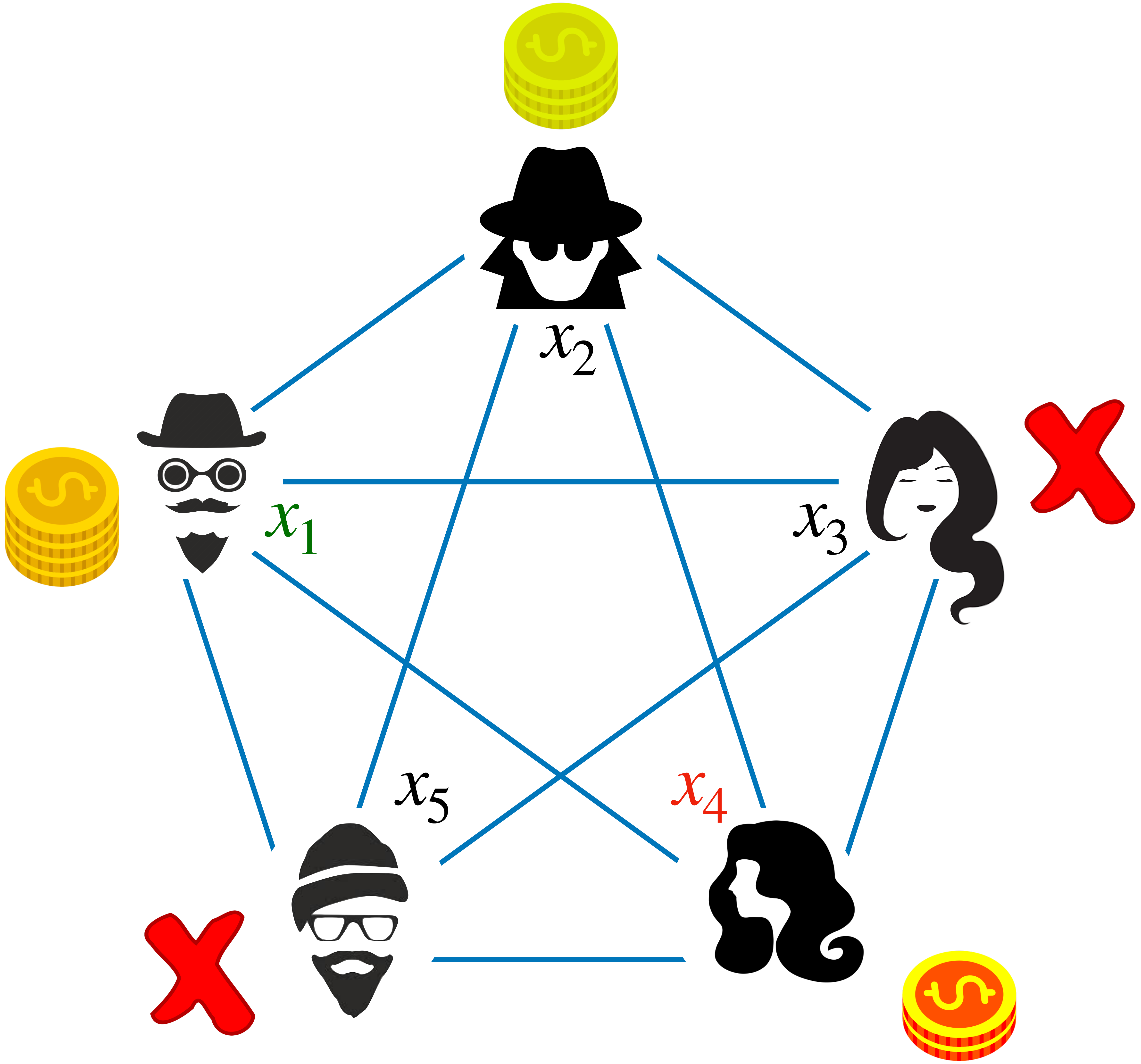
Warmup: $(t+1)$ sources, randomized functionalities

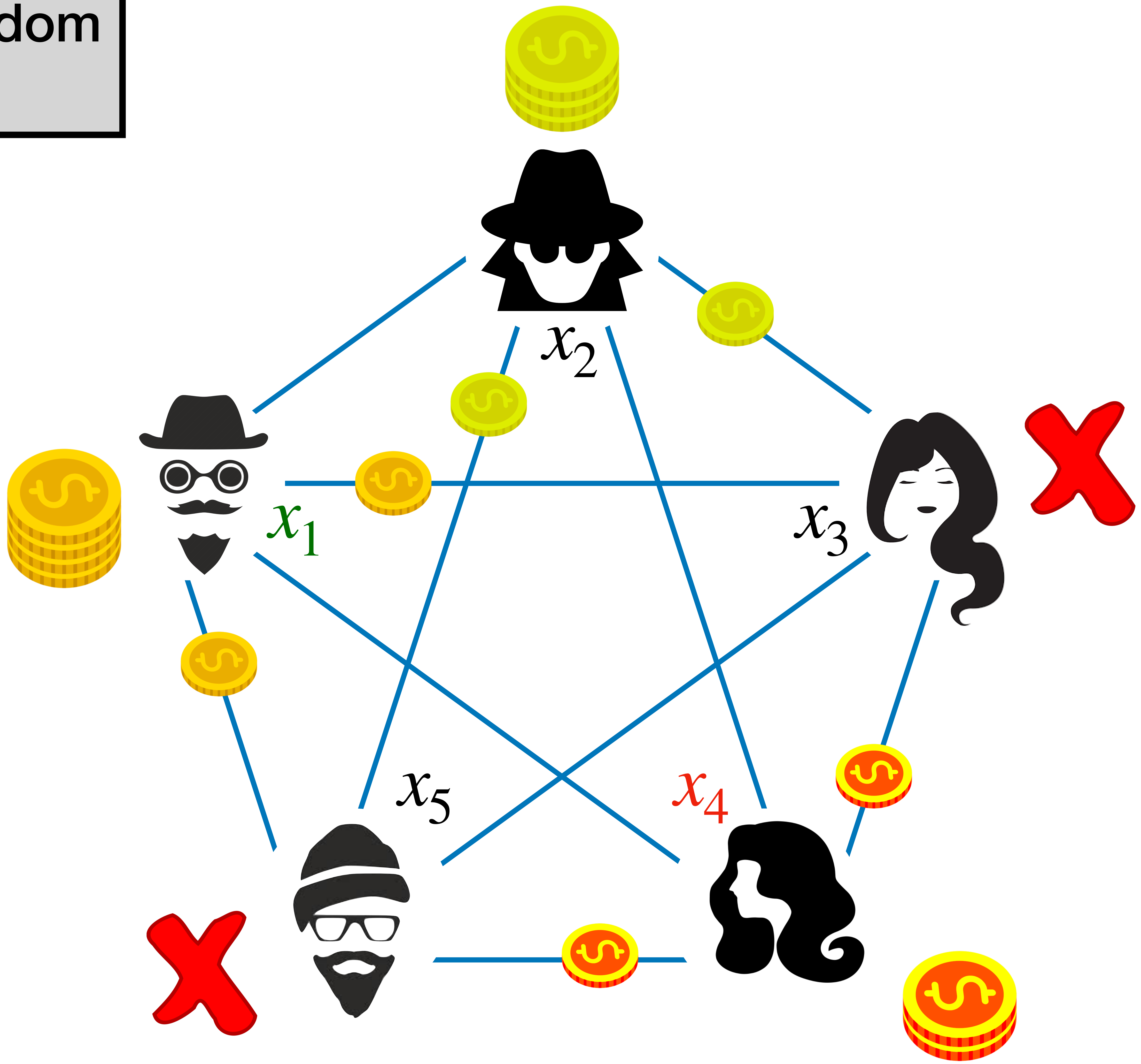
$$F(x_1, x_2, x_3, x_4, x_5; r)$$



Warmup: $(t+1)$ sources, randomized functionalities

Each source sends a random tape to each player

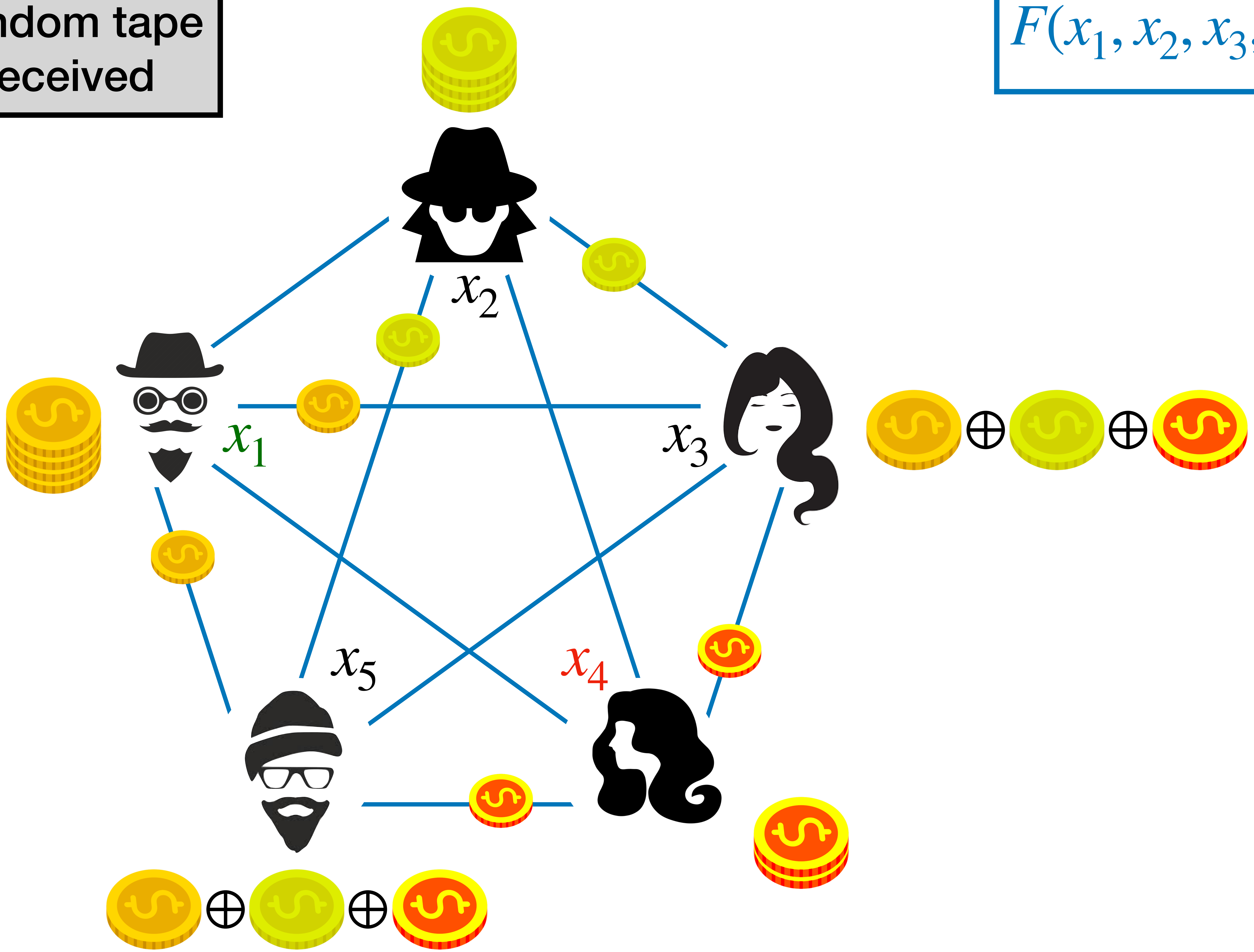
$F(x_1, x_2, x_3, x_4, x_5; r)$



Warmup: $(t+1)$ sources, randomized functionalities

Each player sets their random tape to the XOR of the tapes received

$F(x_1, x_2, x_3, x_4, x_5; r)$



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All parties run BGW with these tapes to compute $F(x_1, x_2, x_3, x_4, x_4; r)$

BGW (Ben-Or, Goldwasser, Wigderson 1988): information-theoretic secure computation for any $t < n/2$. Any other IT protocol would work.

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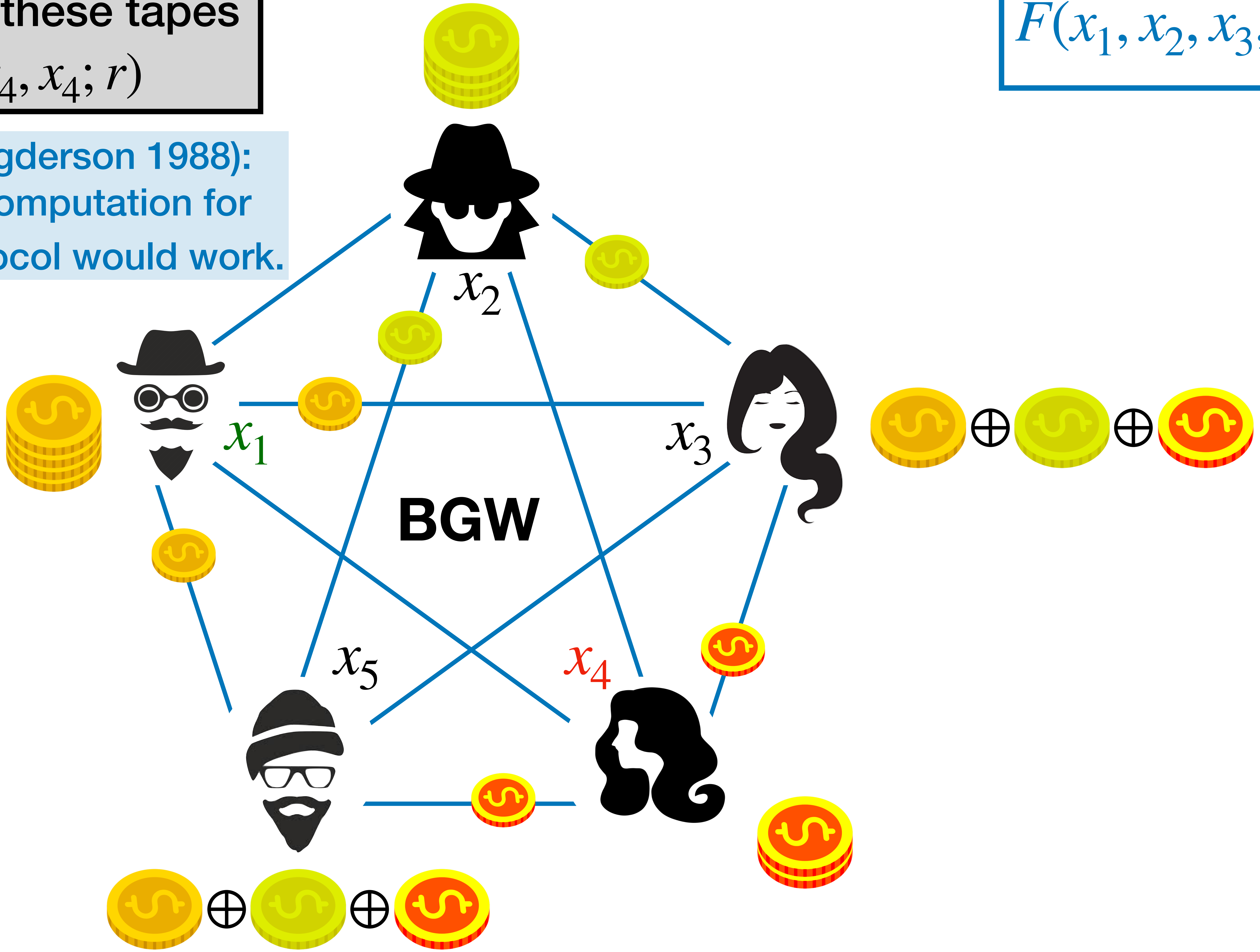

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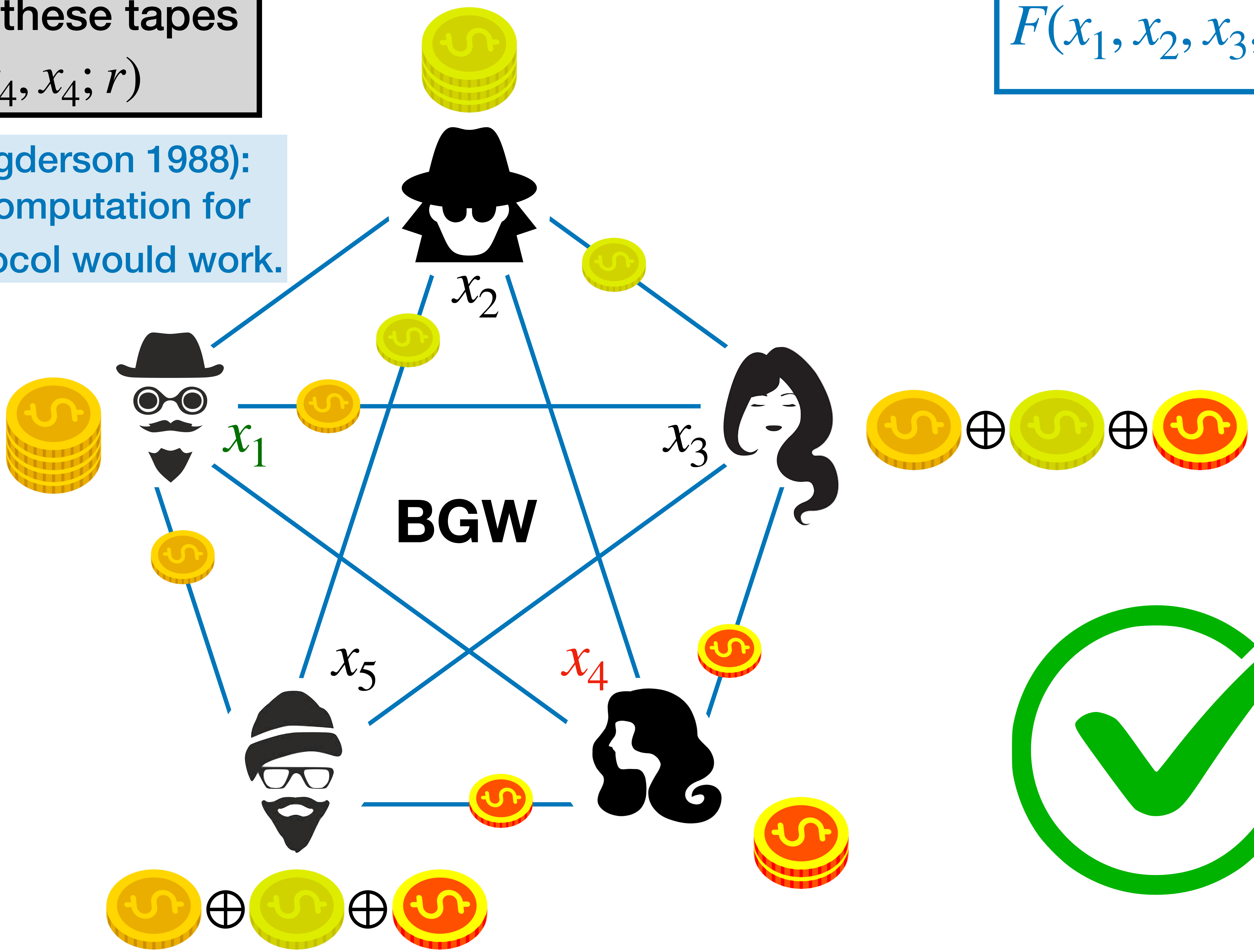

Core Result: t sources, deterministic functionalities

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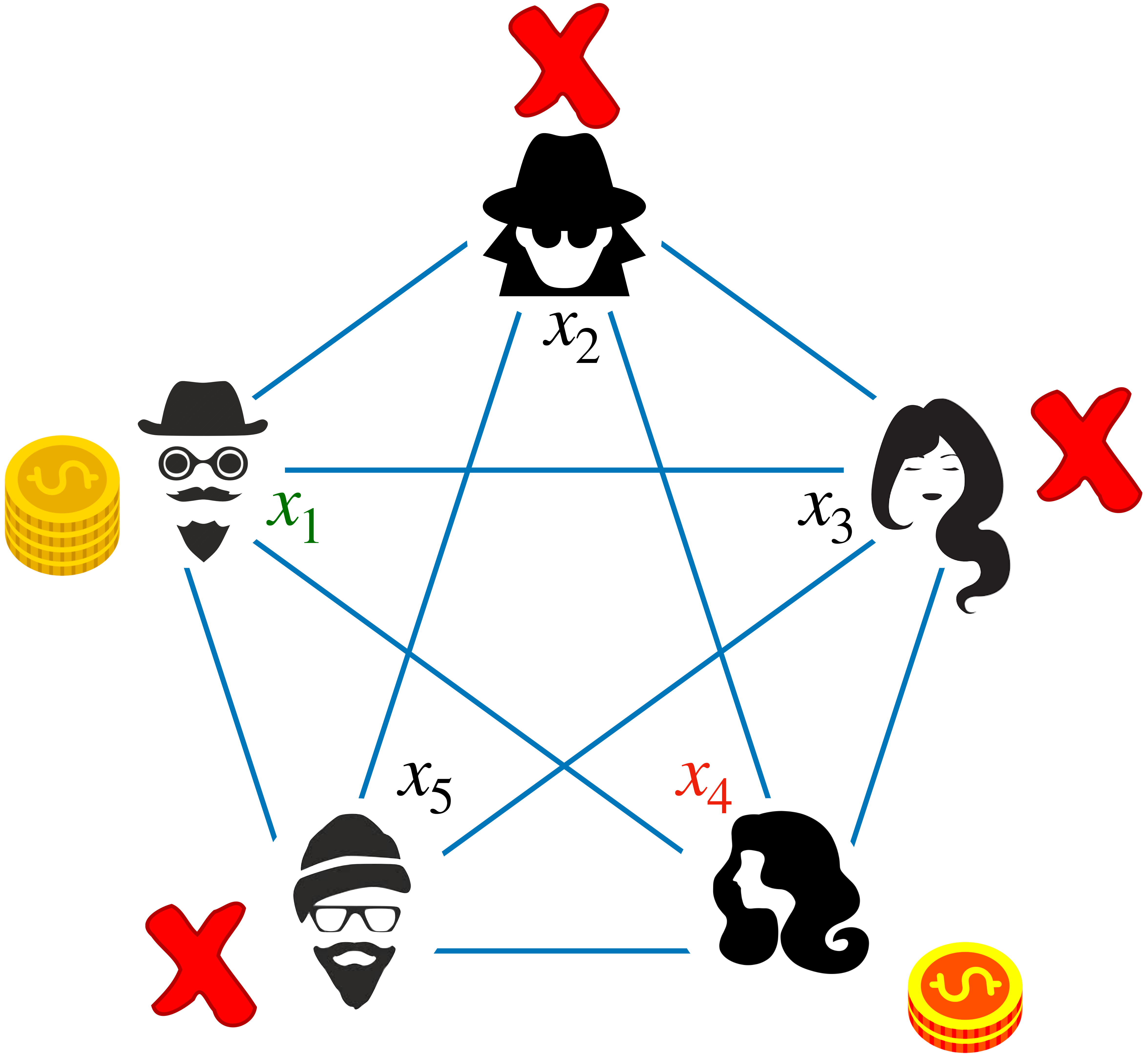

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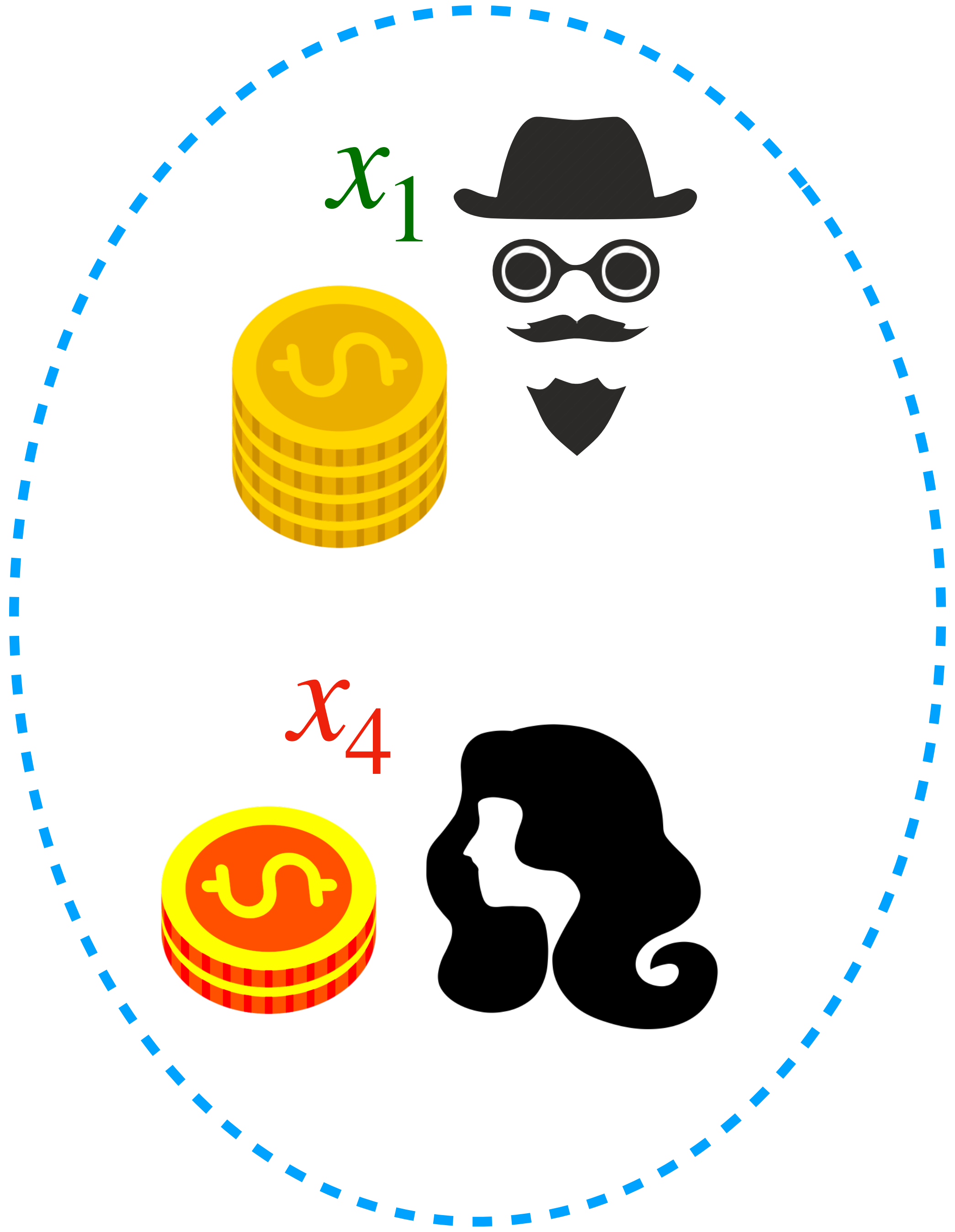
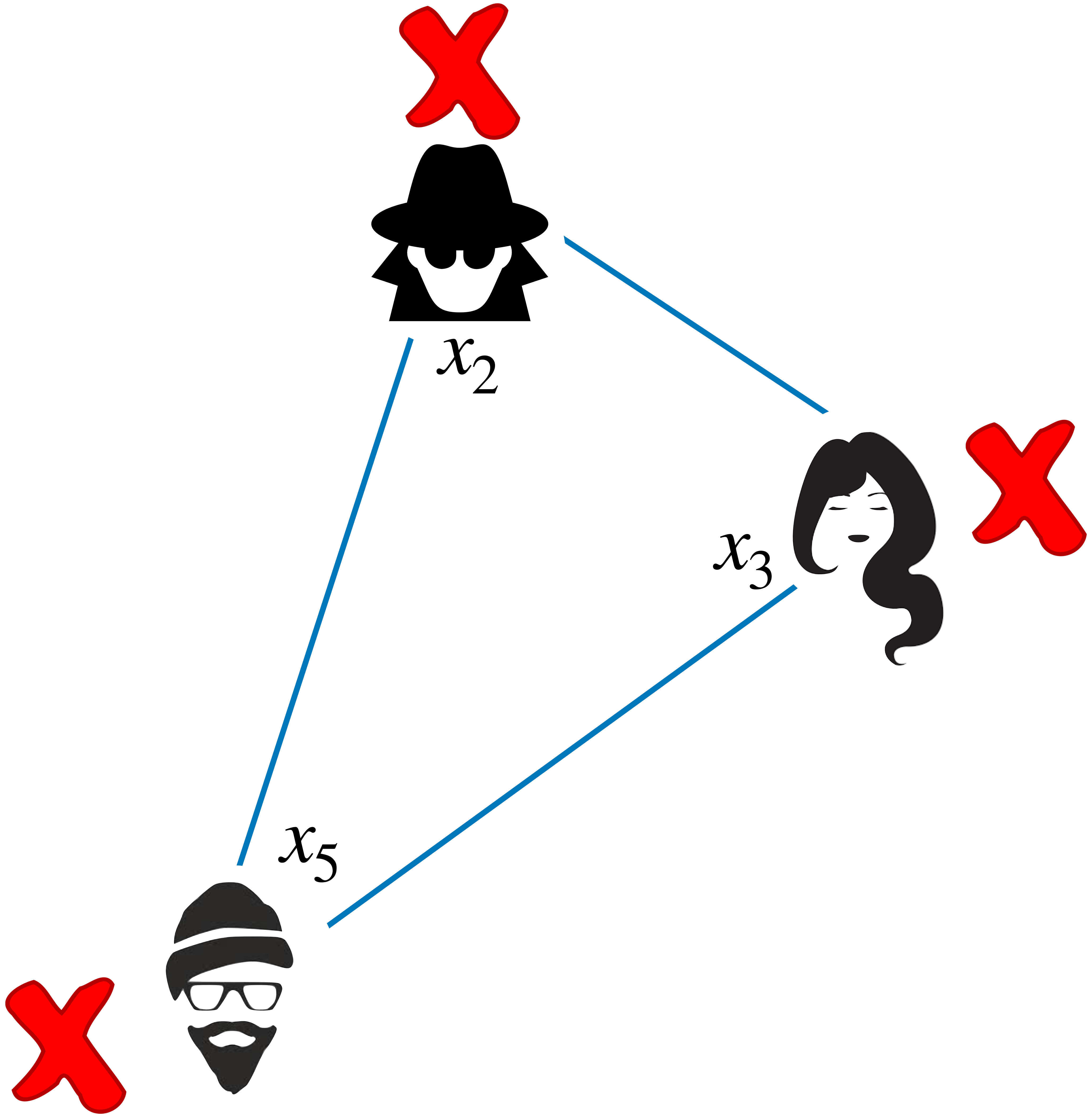
Isolating the sources

$F(x_1, x_2, x_3, x_4, x_5)$





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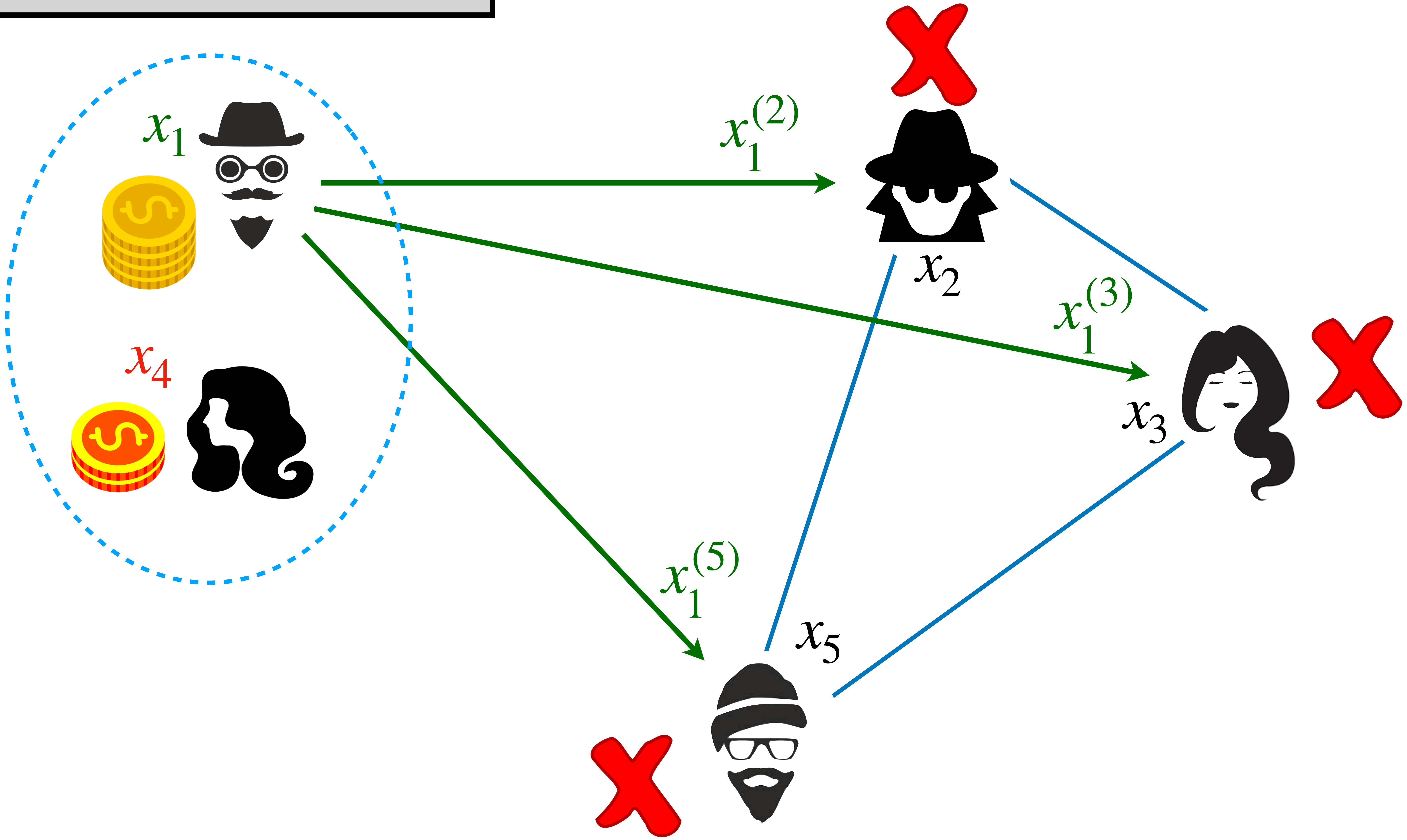
Input Sharing

$F(x_1, x_2, x_3, x_4, x_5)$



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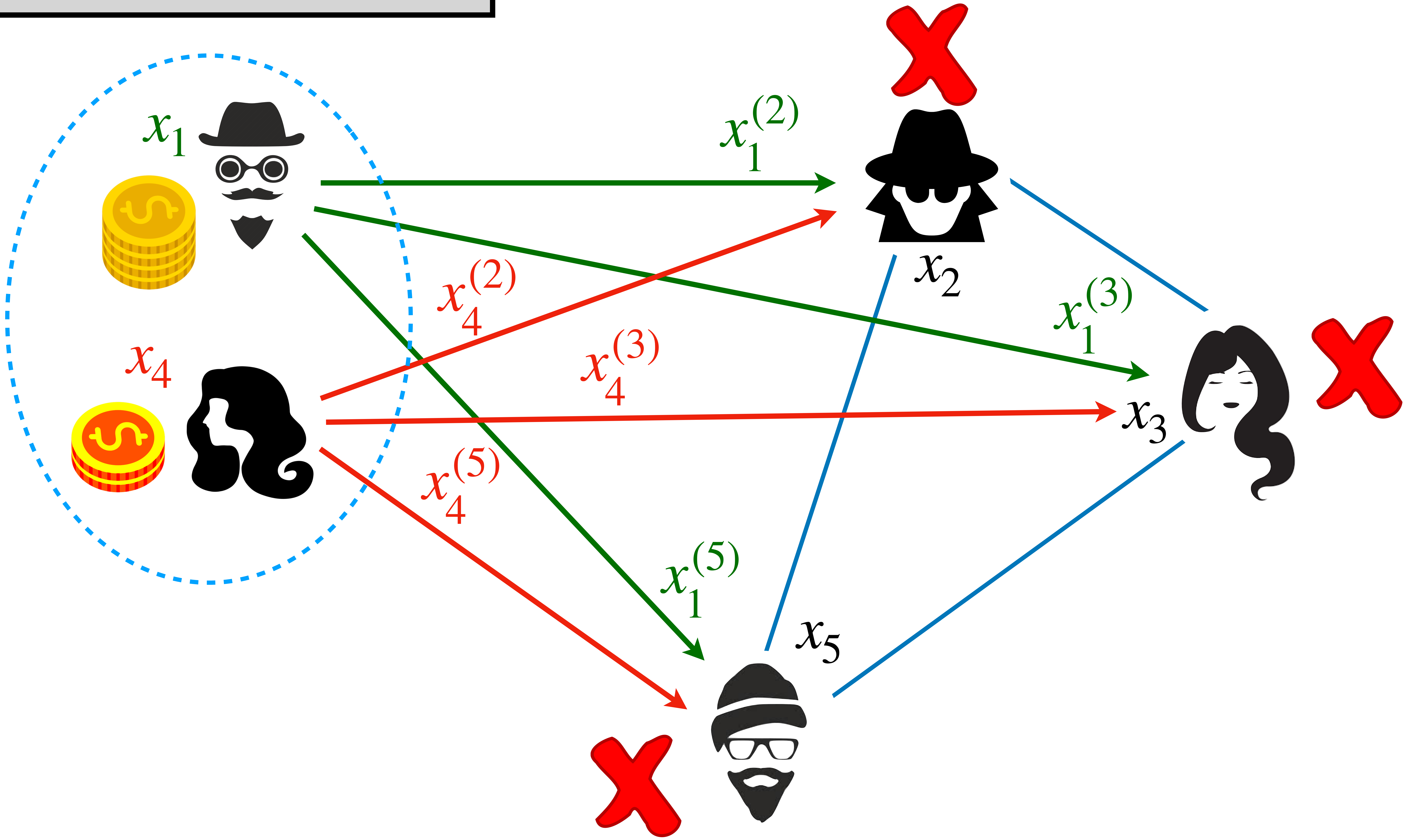

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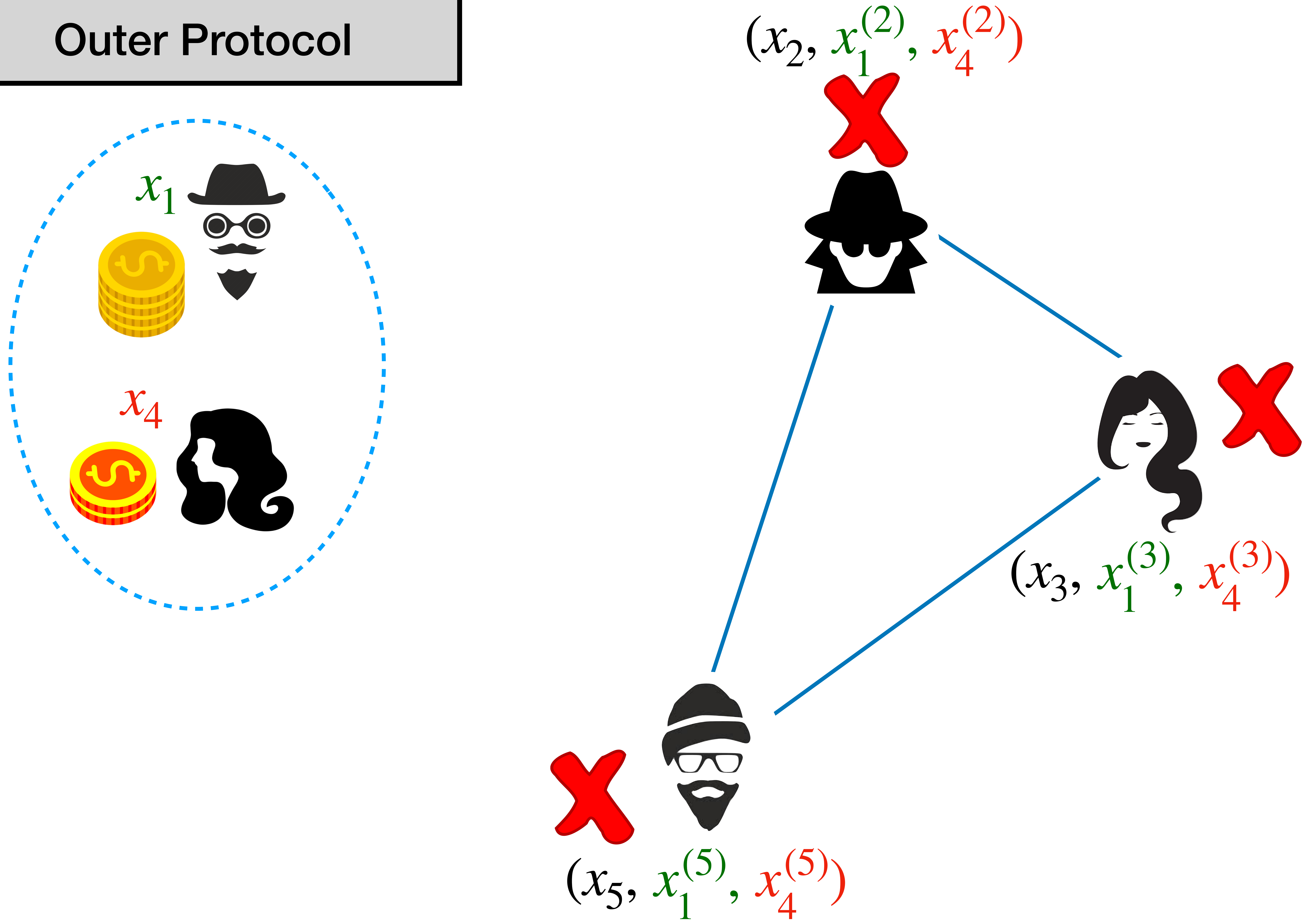
Outer Protocol



$$F'(\text{shares}, x_2, x_3, x_5)$$
$$= F(x_1, x_2, x_3, x_4, x_5)$$

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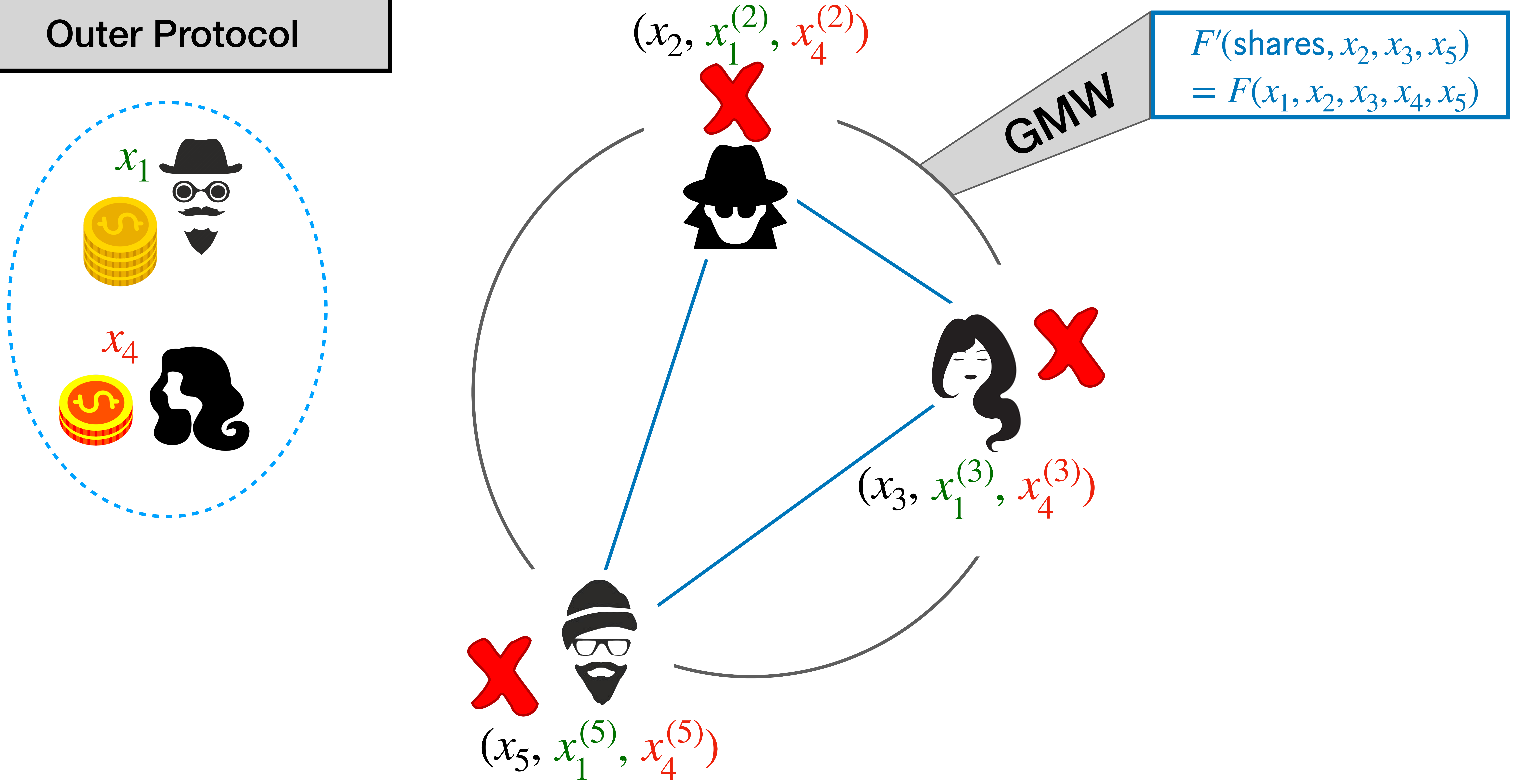

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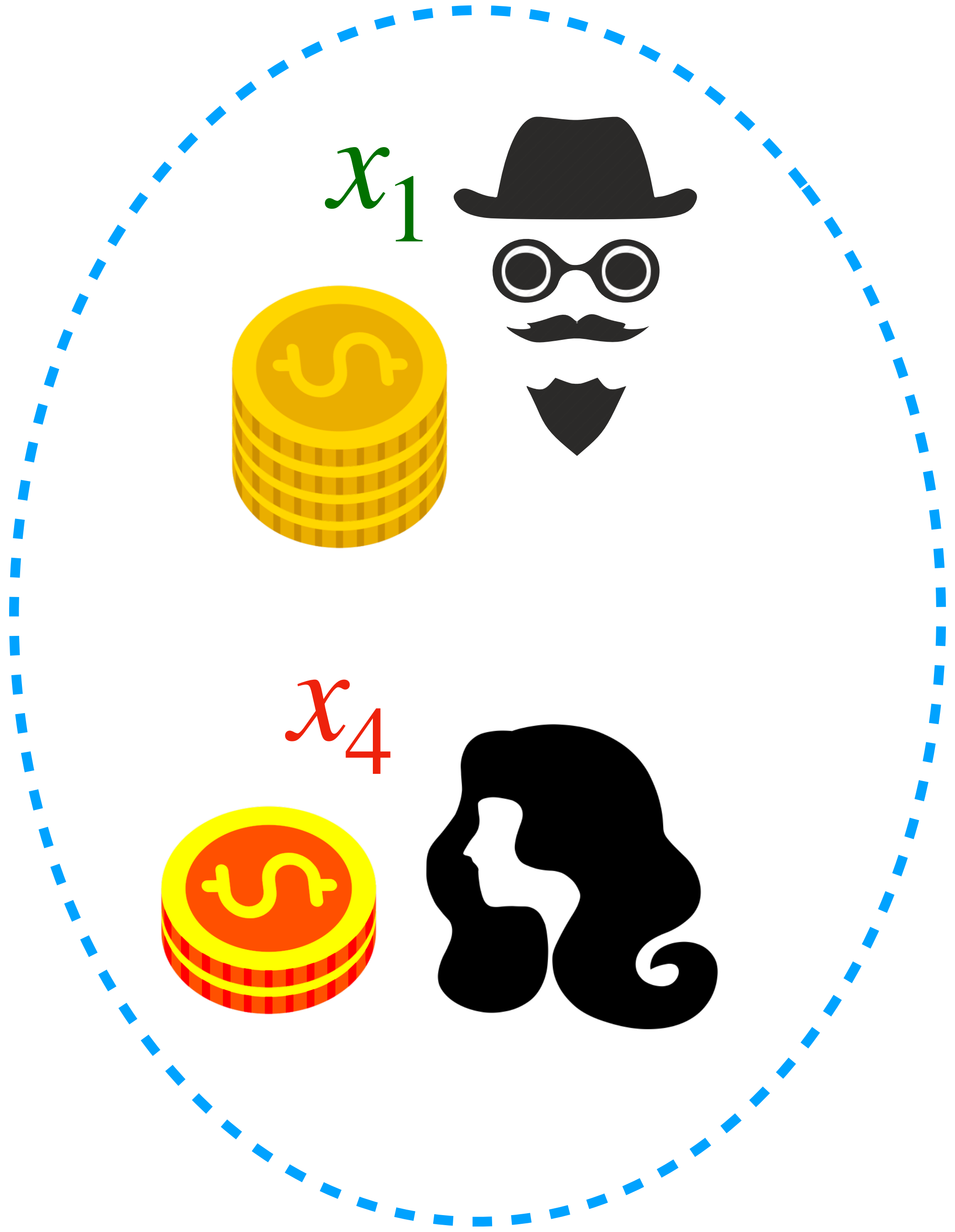
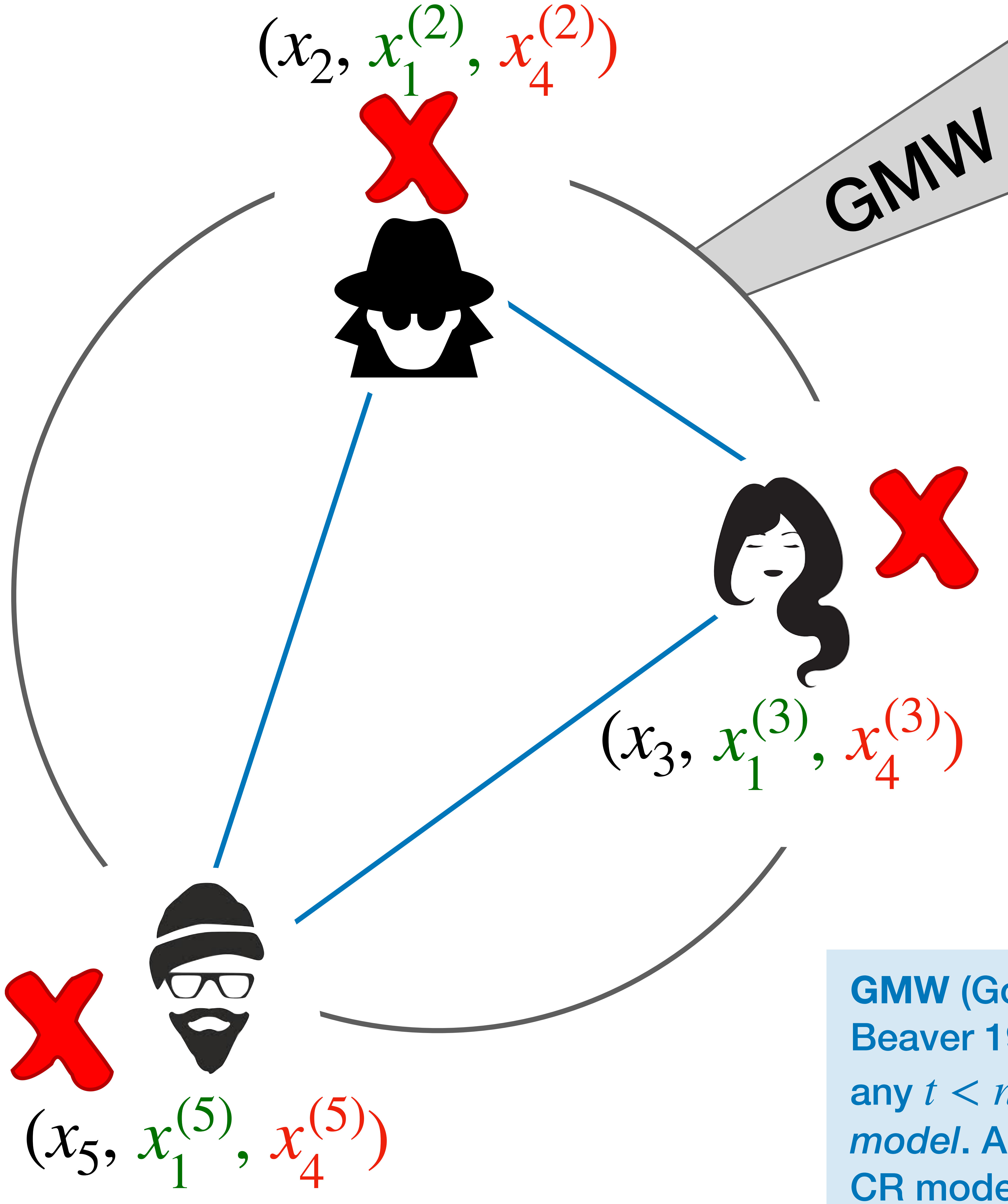




$F'(\text{shares}, x_2, x_3, x_5)$
$= F(x_1, x_2, x_3, x_4, x_5)$

GMW computes the circuit gate-by-gate on shares of the input wires.

XOR gates are local.

AND gates can be handled by *consuming a Beaver triple*

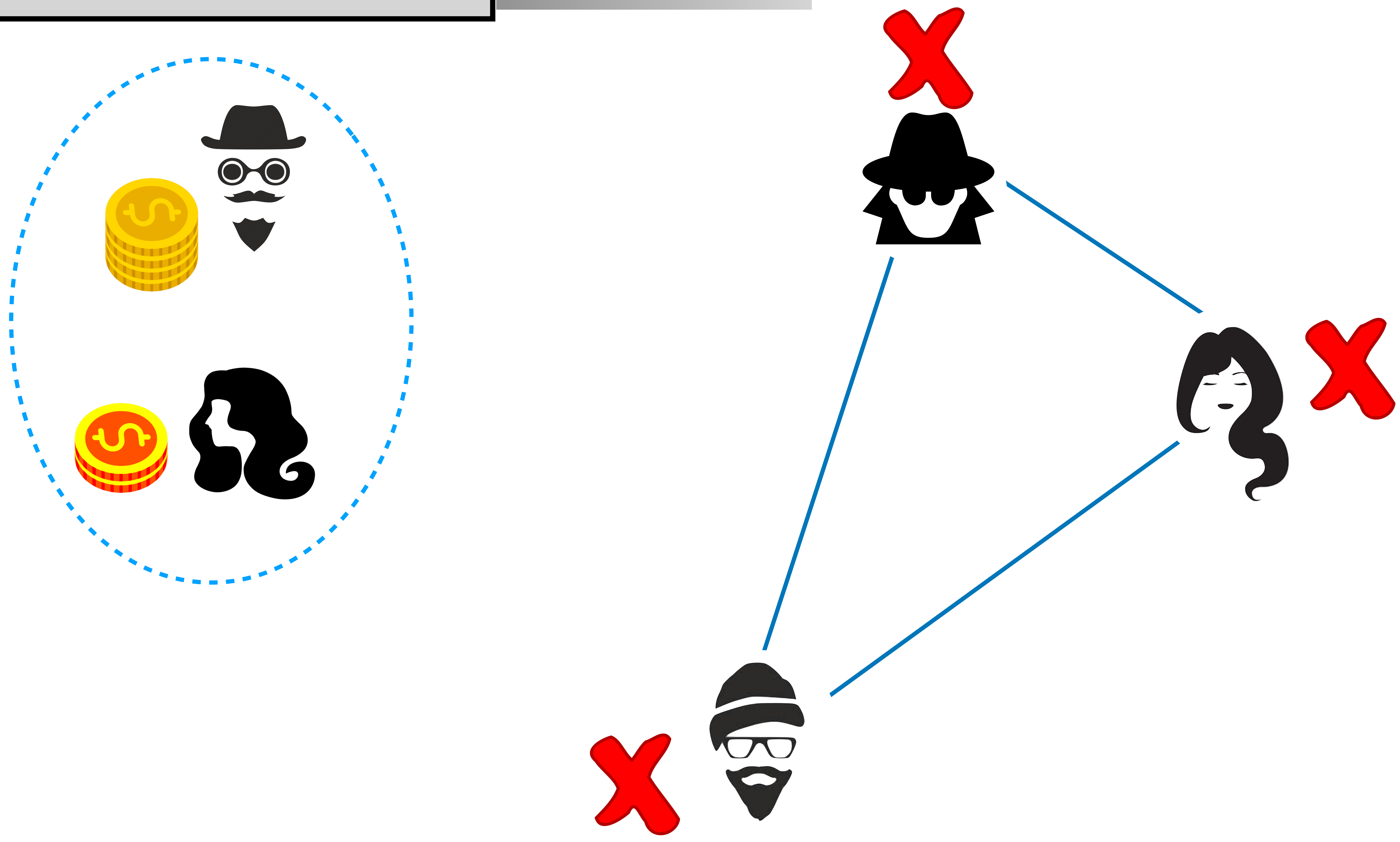
Use an *Inner Protocol*

GMW (Goldreich, Micali, Wigderson 1987 + Beaver 1995): IT secure computation for any $t < n - 1$ in the *correlated randomness model*. Any all-but-one IT protocol in the CR model would work.

Core Result: t sources, deterministic functionalities

Inner Protocol

Tape sharing



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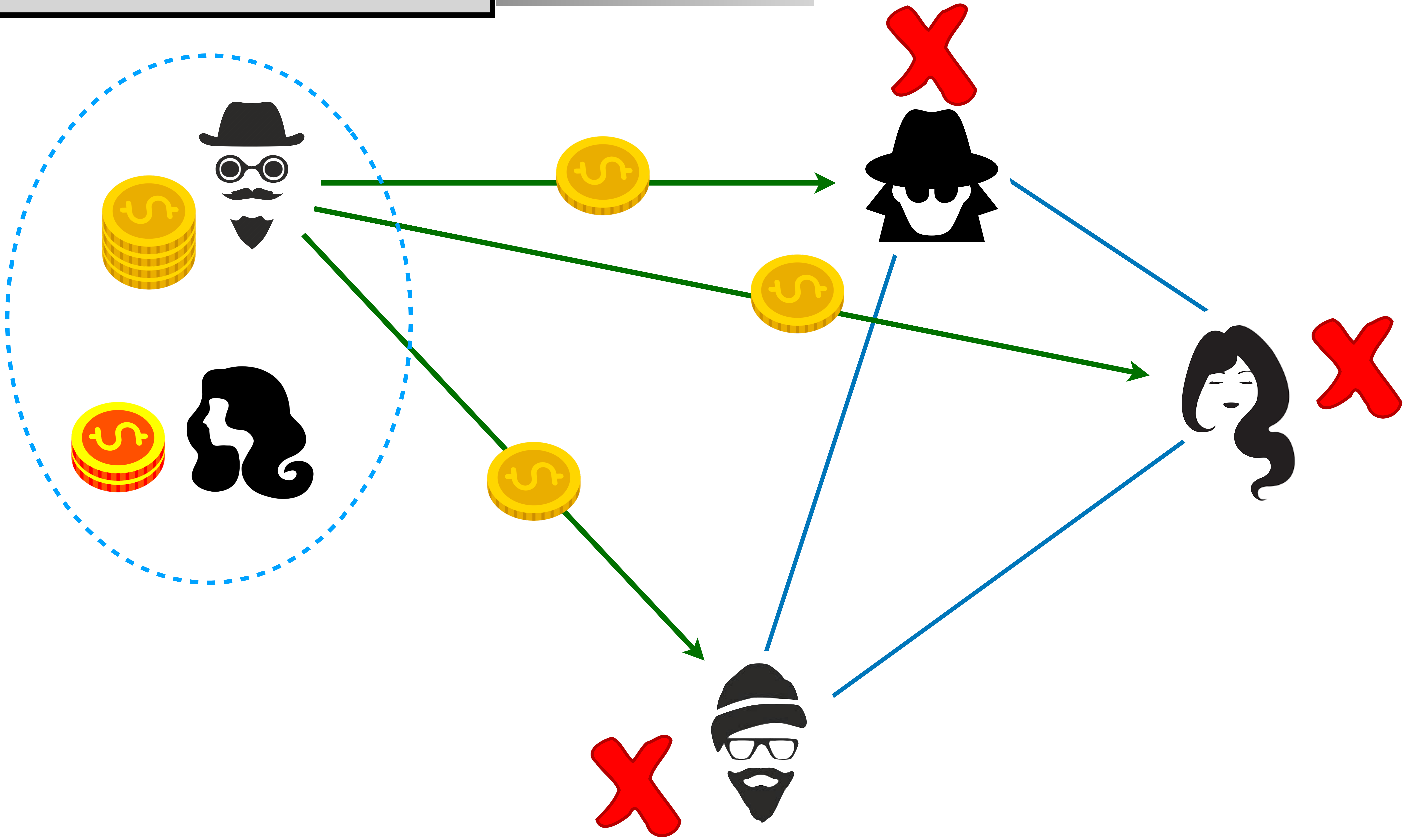

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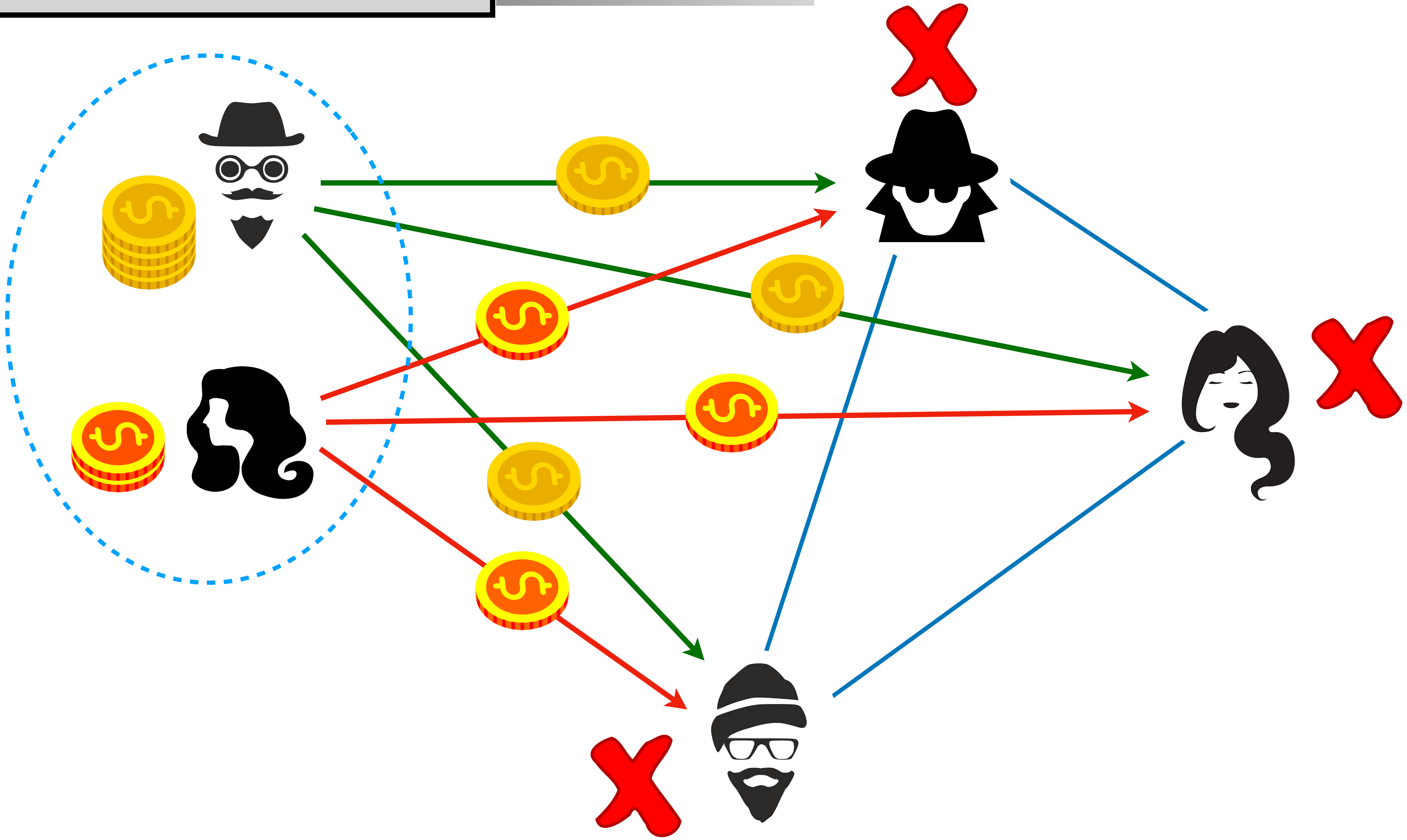

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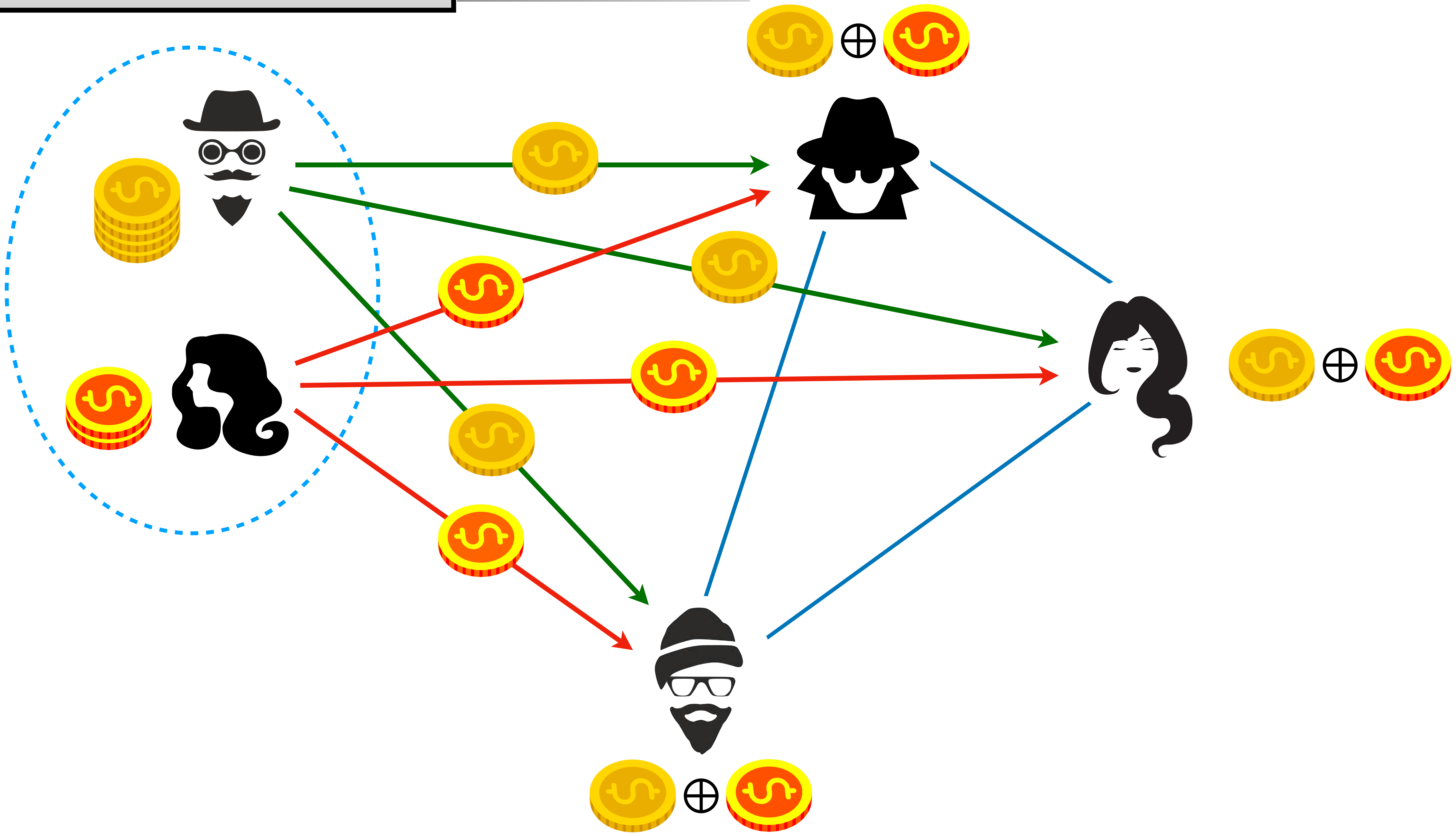

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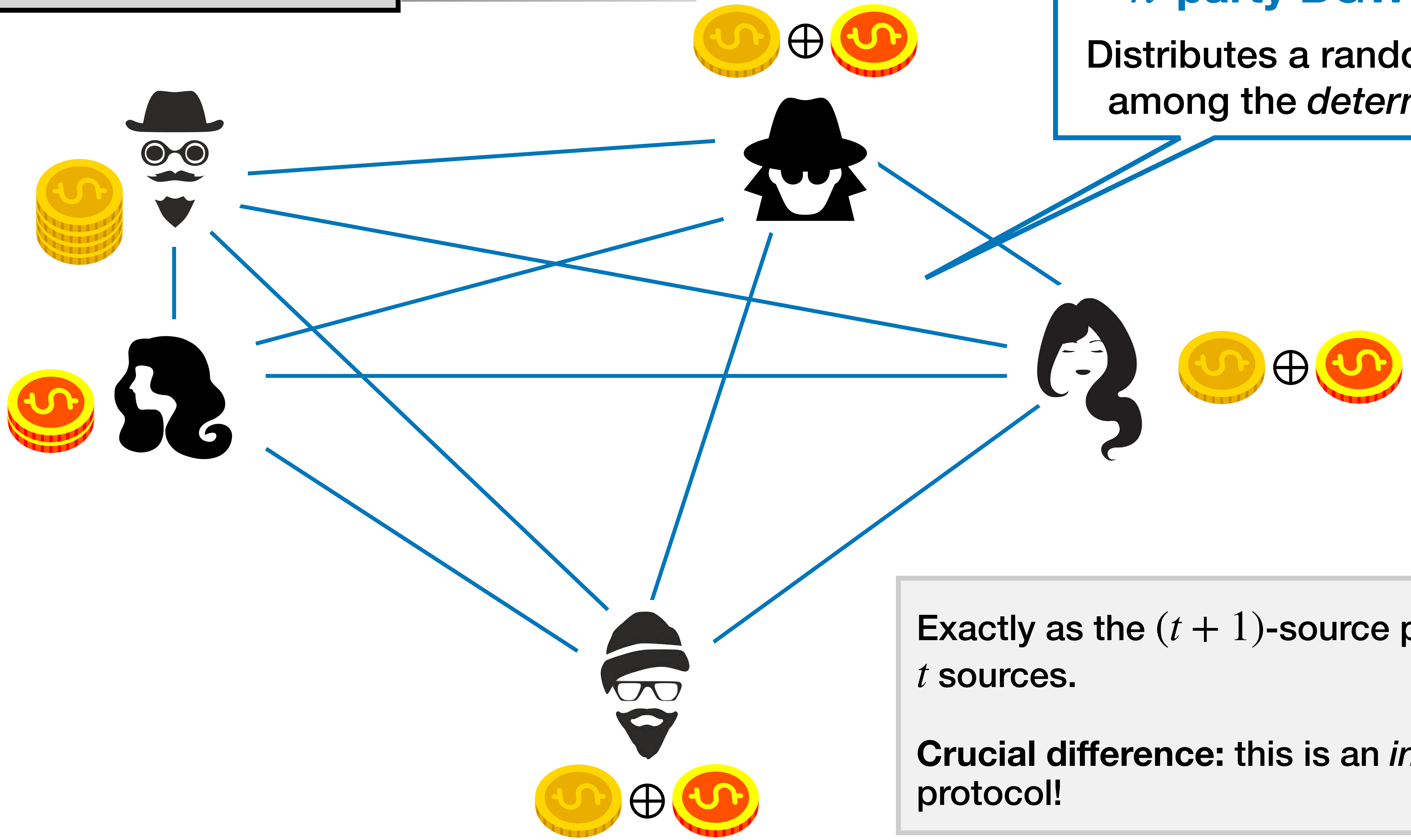
Inner Protocol n-party BGW



n-party BGW for $\mathscr{F}_{\text{Beaver}}$

Distributes a random Beaver triple among the *deterministic* parties

Exactly as the $(t+1)$-source protocol, but with t sources.

Crucial difference: this is an *input-independent* protocol!

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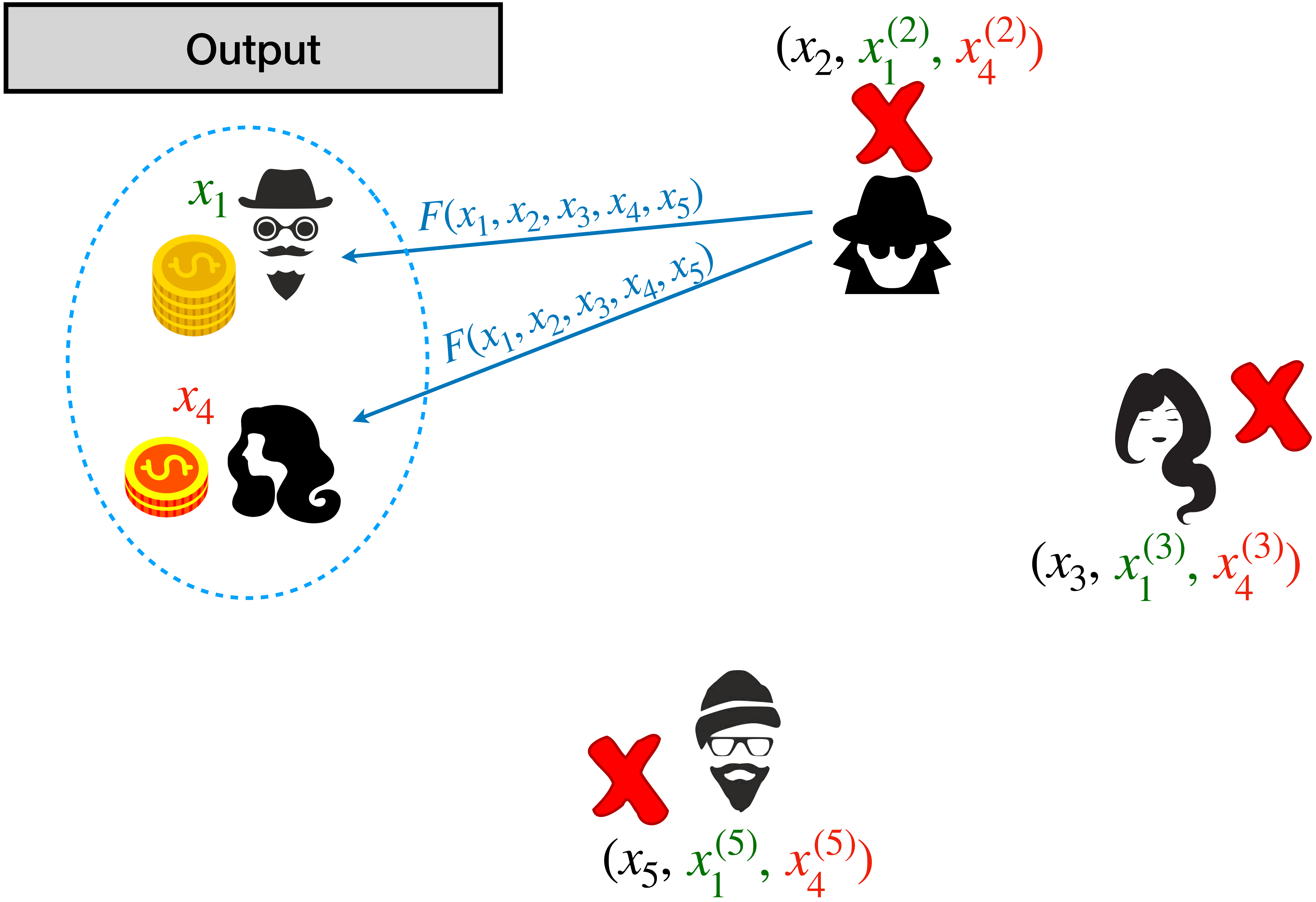

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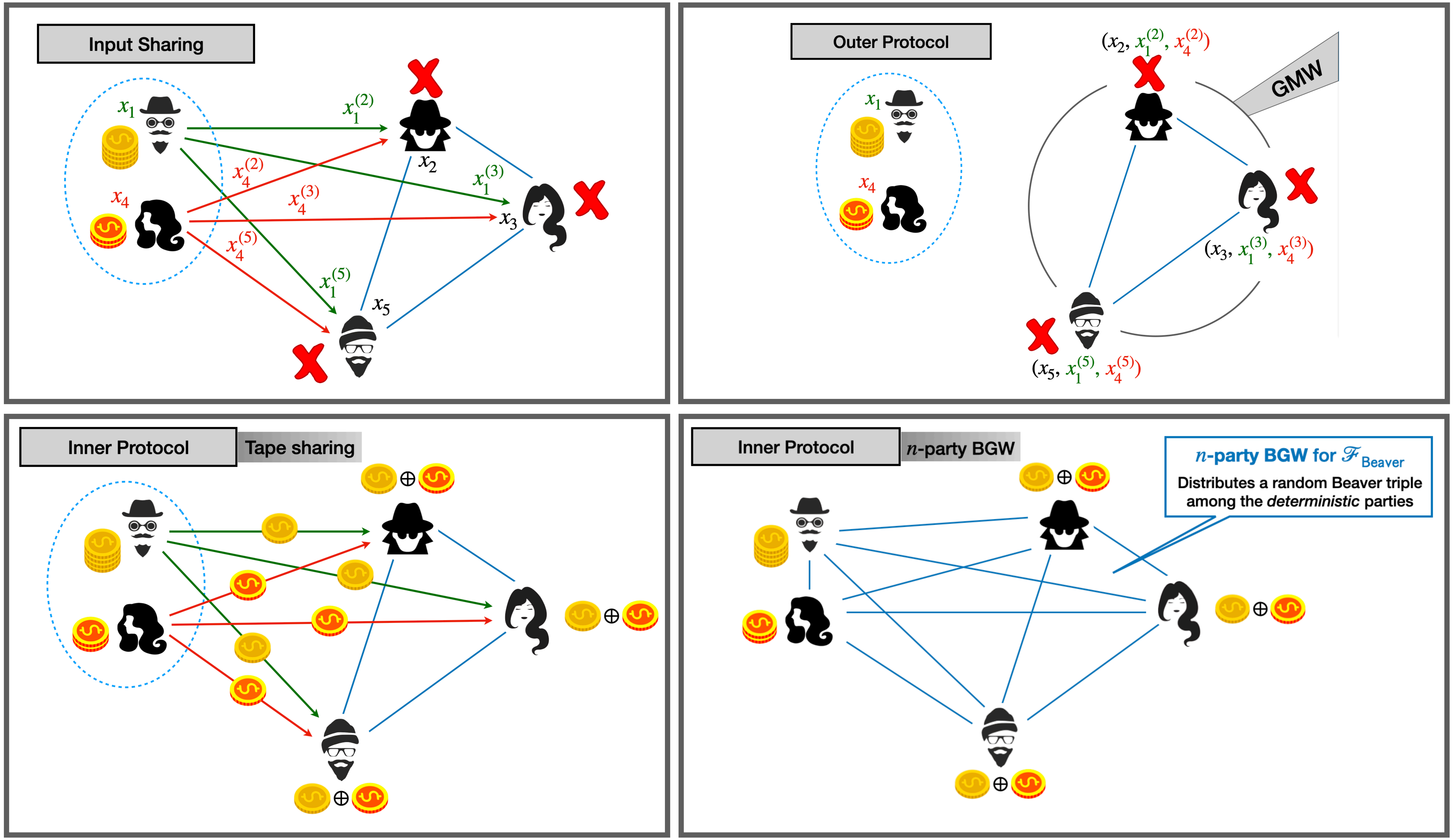
Summary of the protocol



Security

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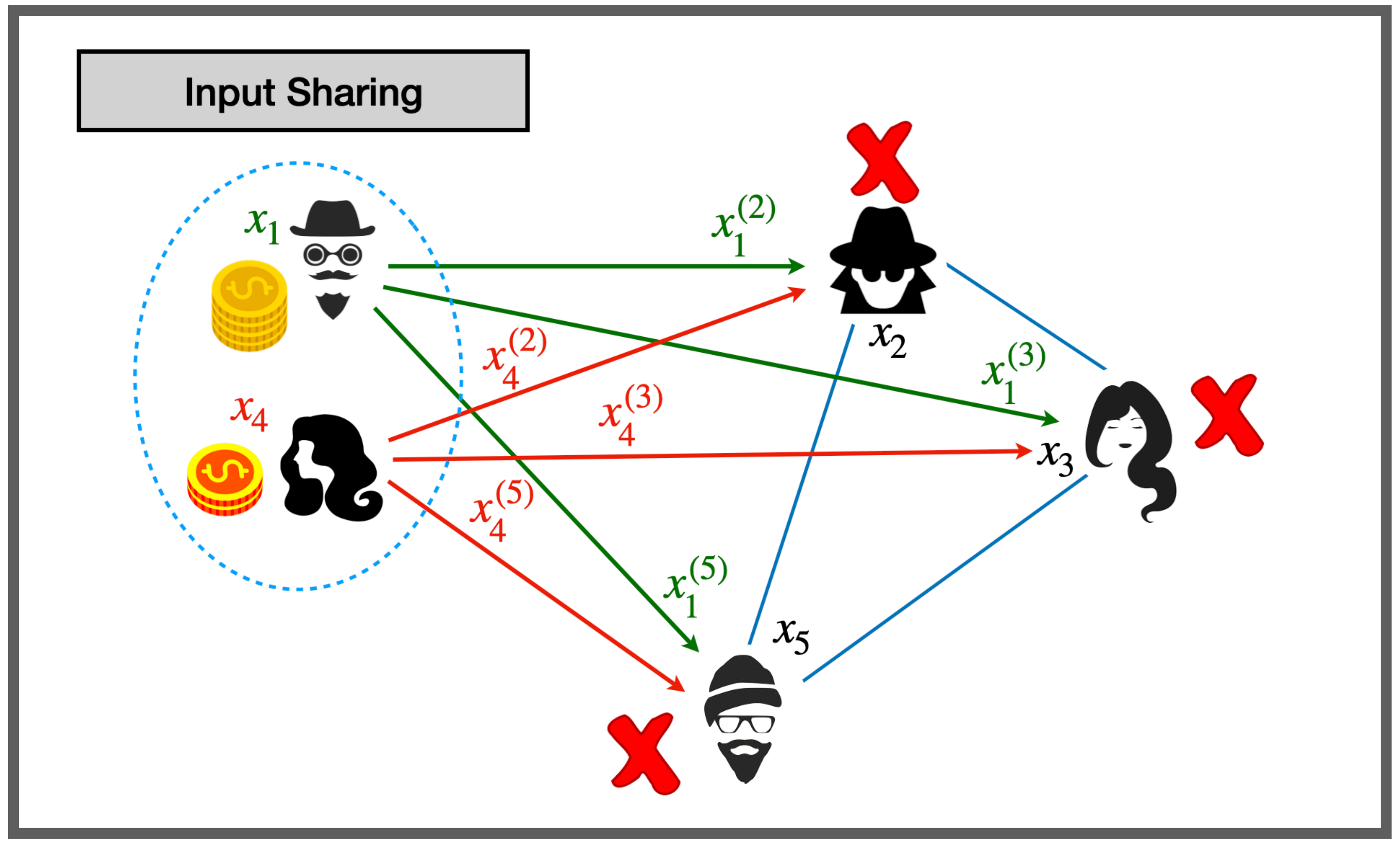

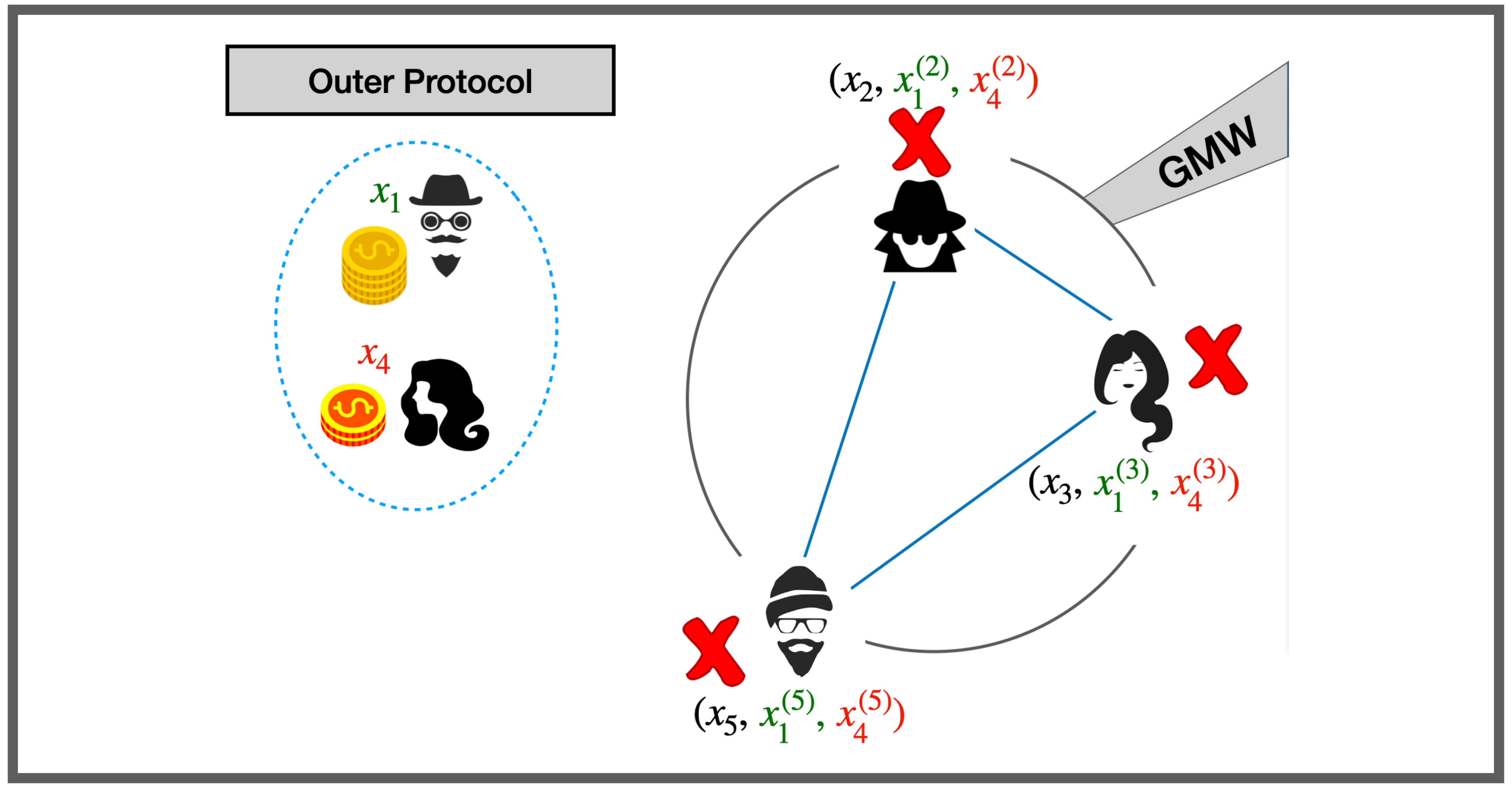

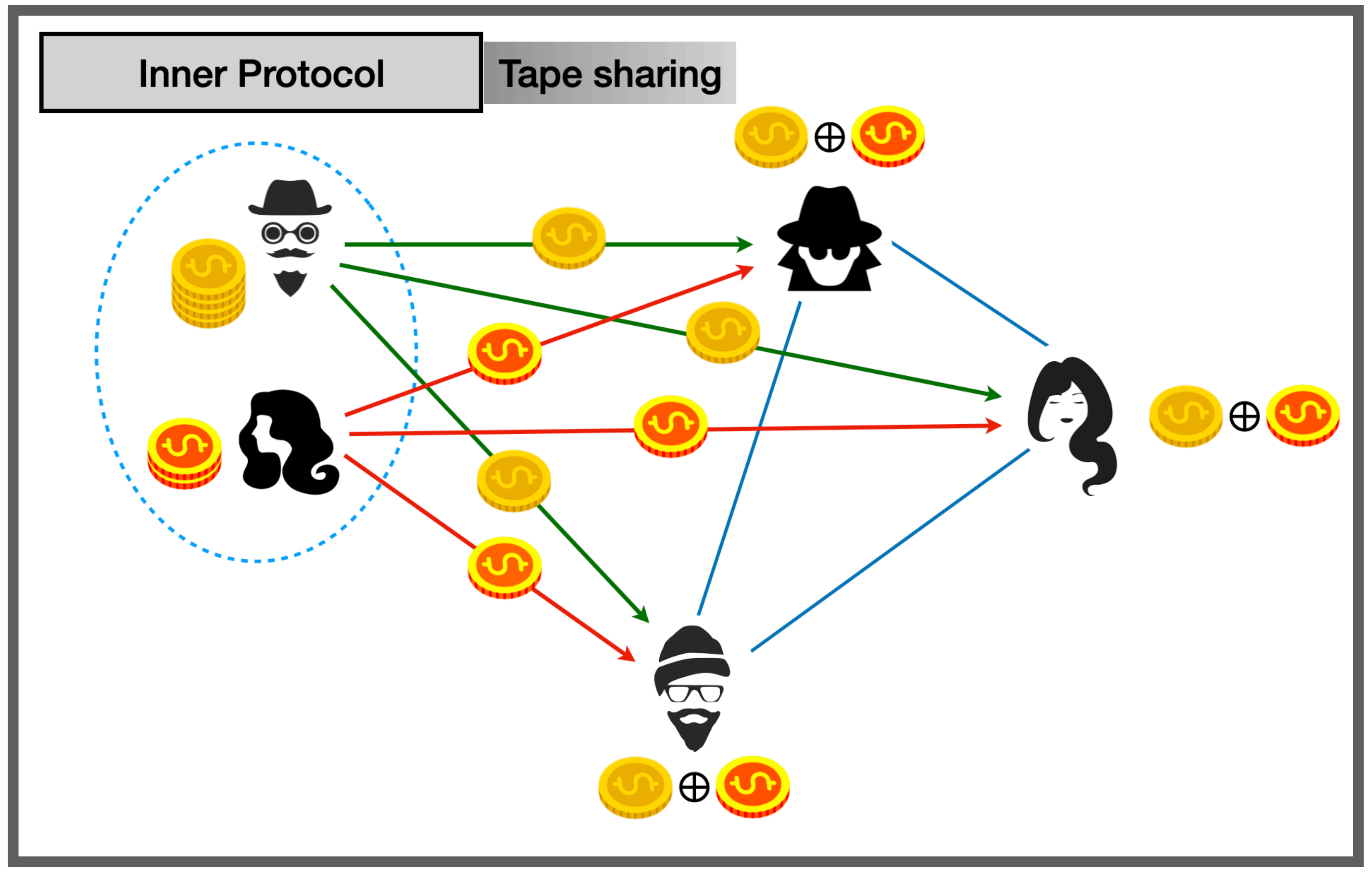

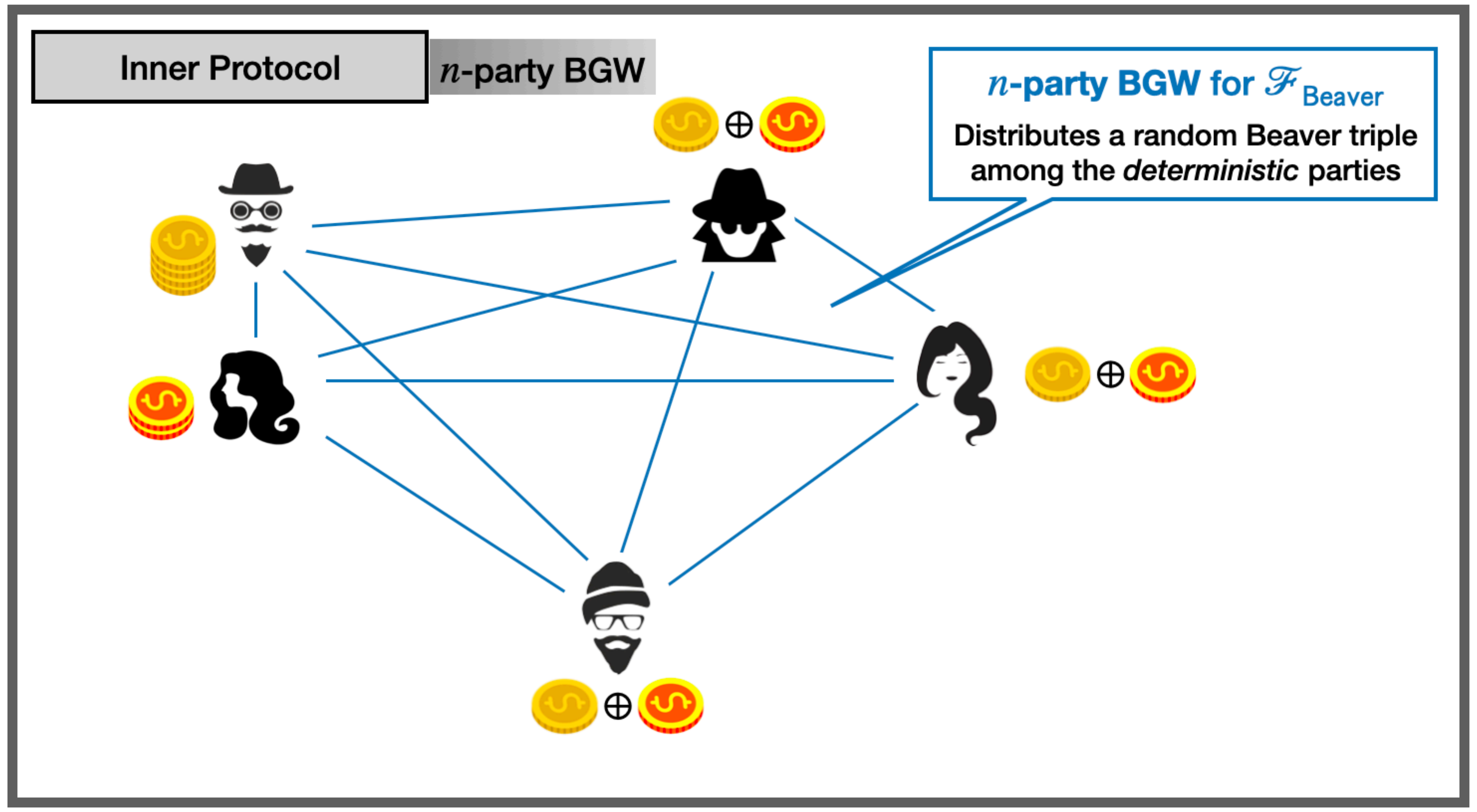

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Two cases

(1) there is one honest source

(2) $\mathscr{A}$ corrupts only the sources

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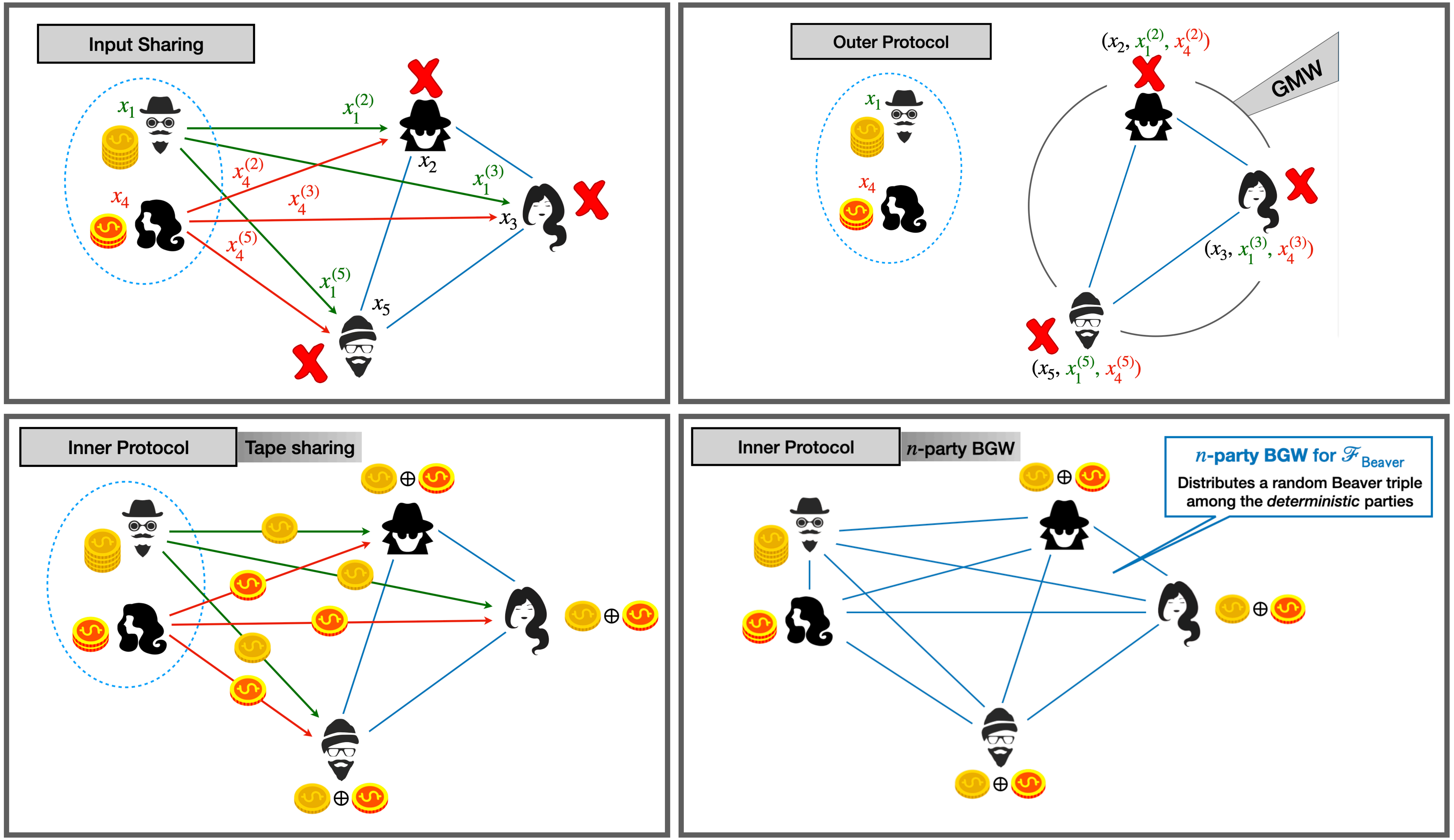

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① Two observations

(a) $\mathscr{A}$ can't see the shared tapes

$\Longrightarrow$ BGW is secure

$\Longrightarrow$ The Beaver triples are trusted

(b) $t < n/2 \implies t < n - t$

$\Longrightarrow$ there is one honest player

$\Longrightarrow$ GMW is secure ✓

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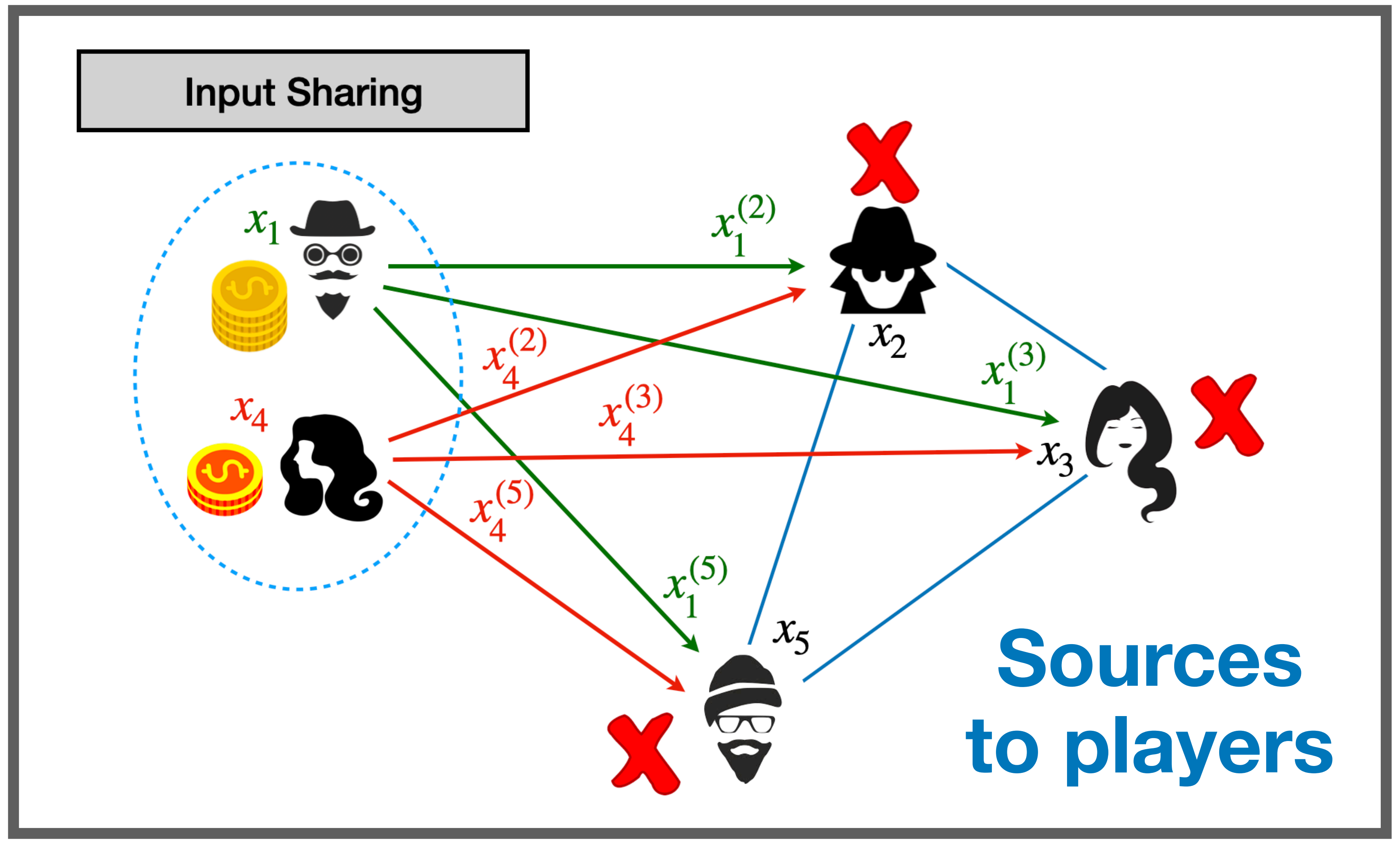

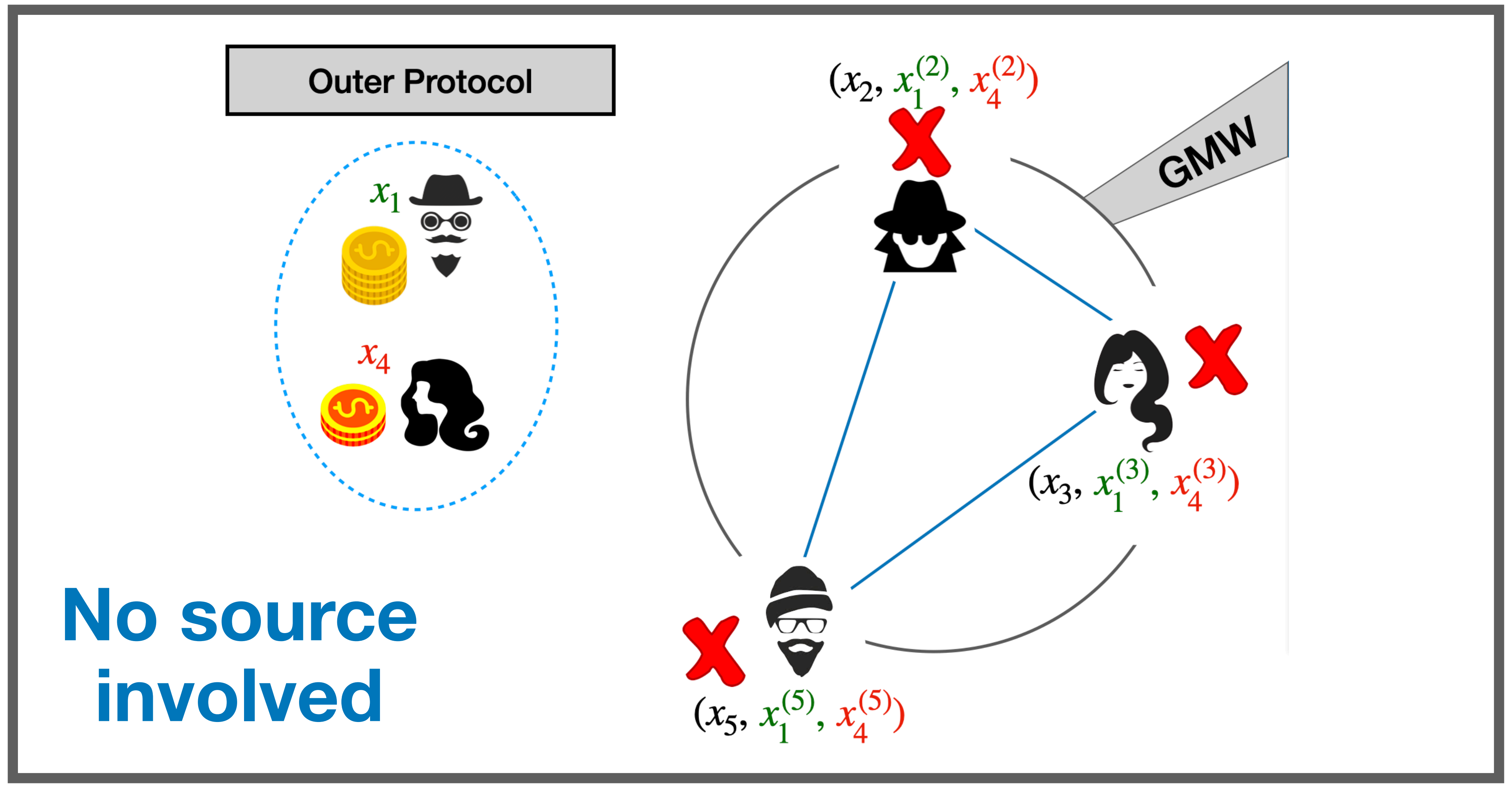

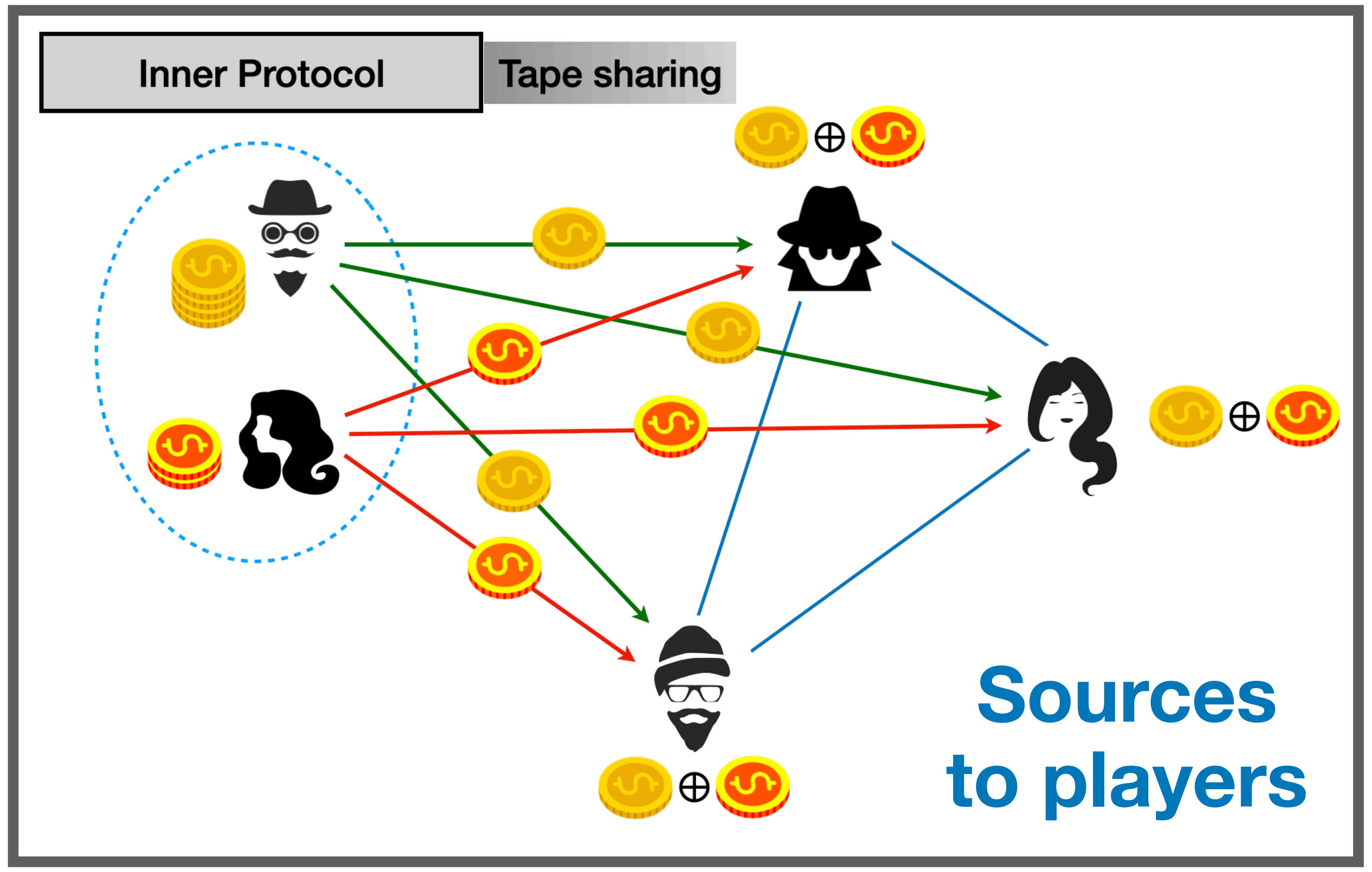

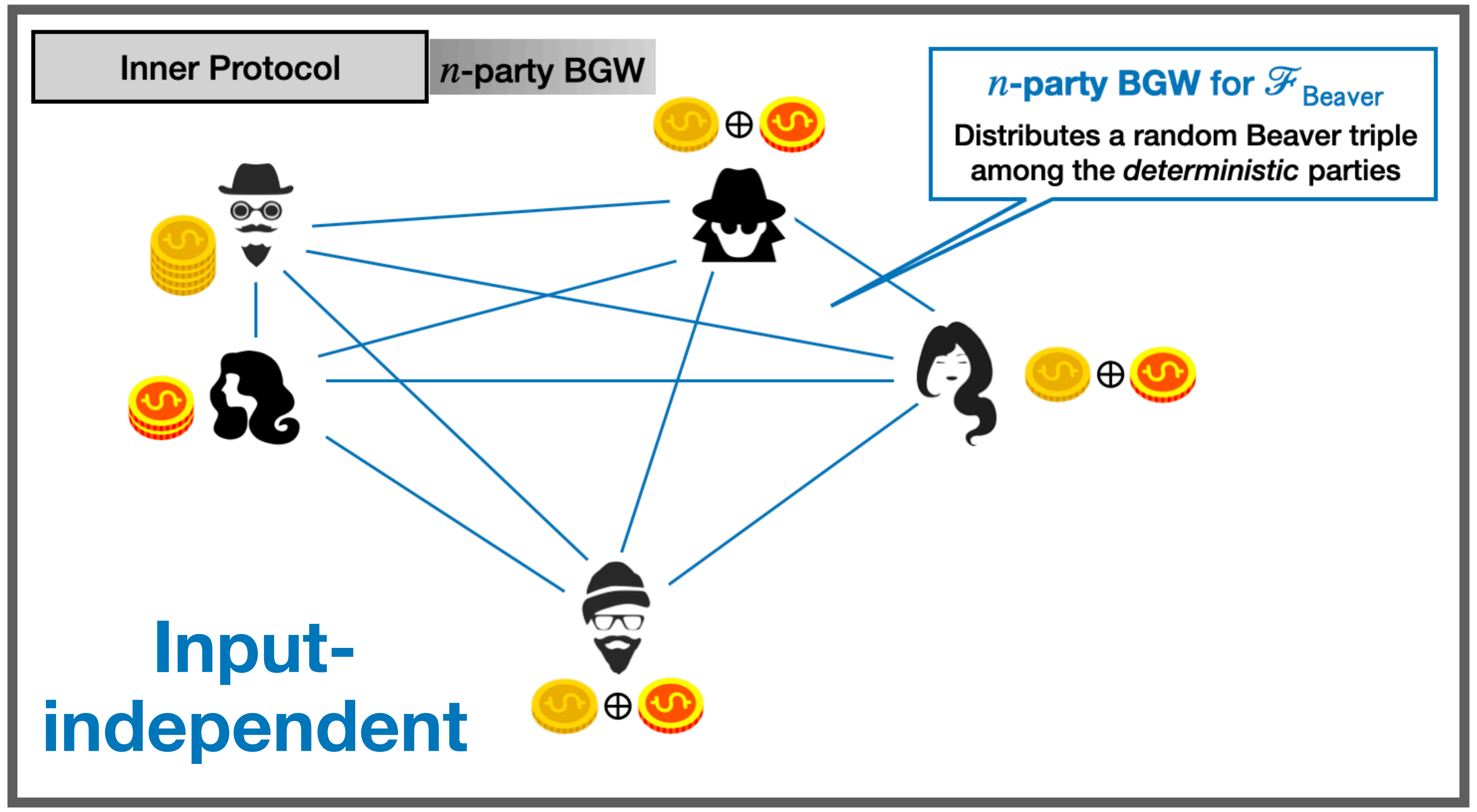

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2 One observation

no source ever sees an input-dependent message, beyond the output! ✓

Second Result: 1-Private n-Party AND with 6 Bits and 1 Source

Motivation & Previous Work

- AND is a basic building block of MPC, together with XOR (for which we already have tight — trivial — bounds)
- The randomness complexity of 1-private n-party AND has been studied in previous works
- Most recent result [TCC:KOPRTV'19]: AND can be computed using 8 bits (and two sources)

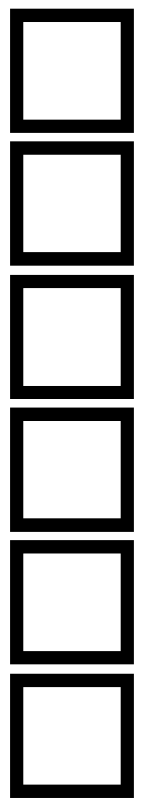
Setting

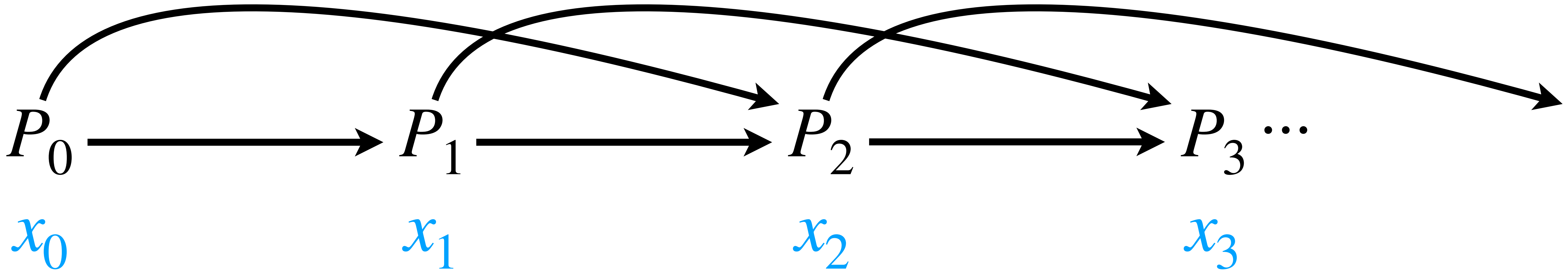
- n parties $(P_0, \cdots, P_{n-1})$ with respective inputs $(x_0, \cdots, x_{n-1})$
- Output: $\wedge_{i=0}^{n-1} x_i$
- At most one corrupted party (= no collusion)

Second Result: 1-Private n-Party AND with 6 Bits and 1 Source

Main Phase

Budget of random bits





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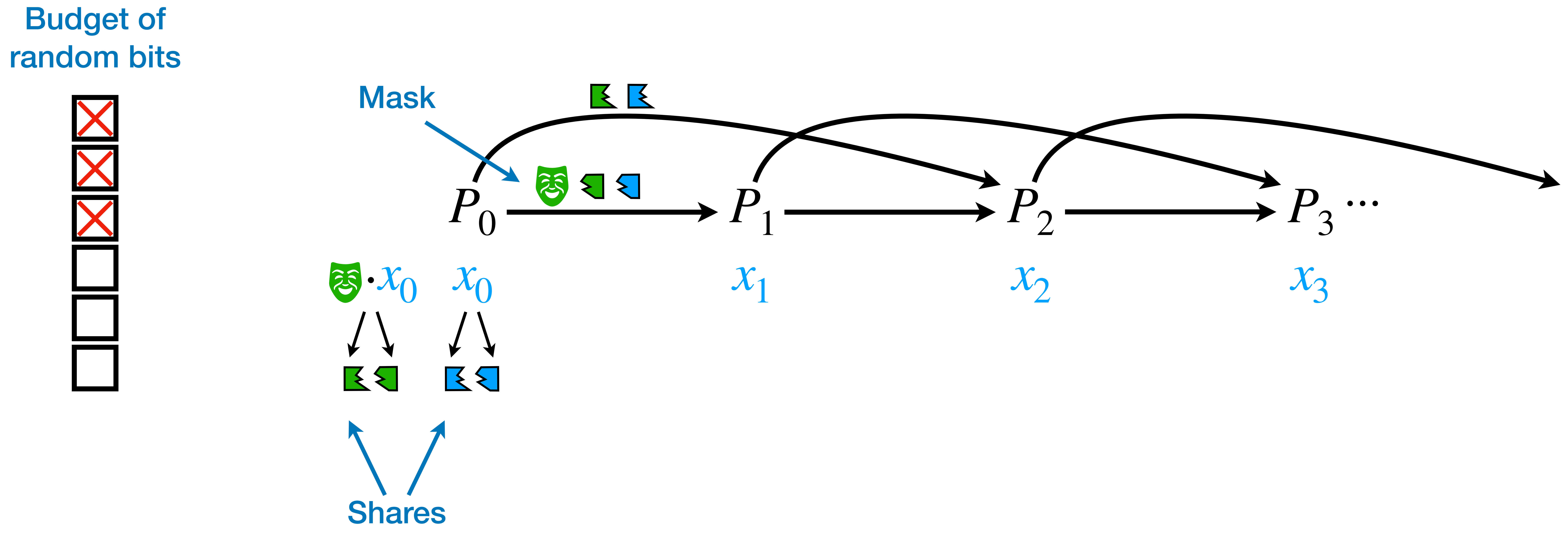

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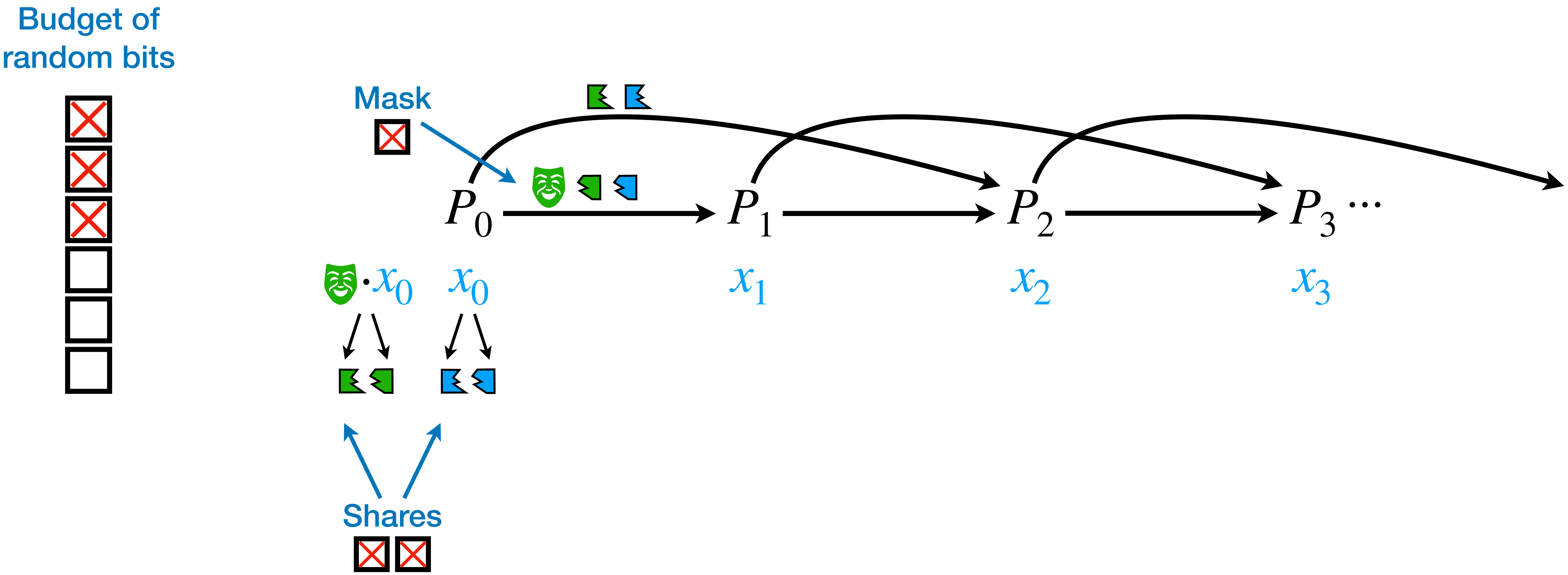

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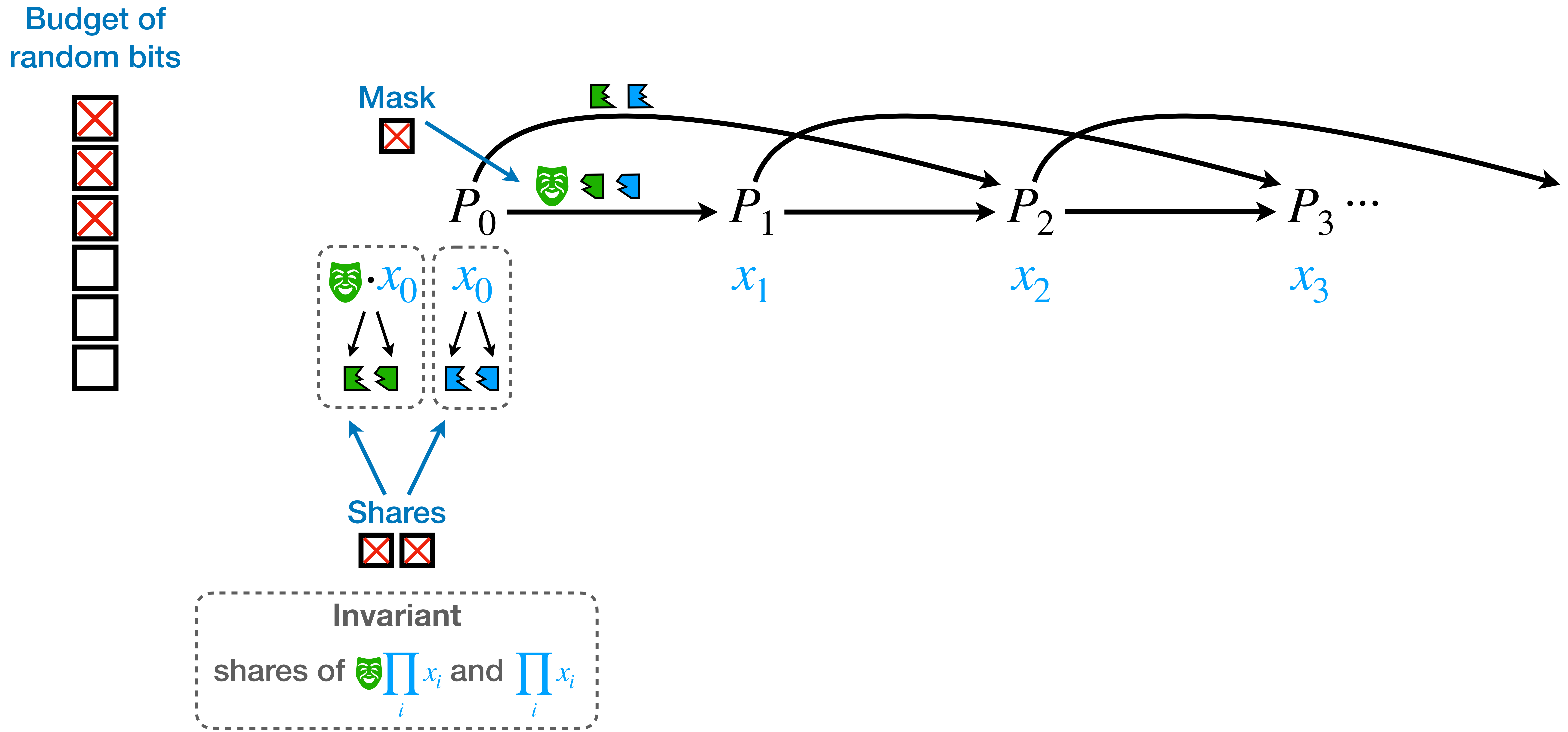

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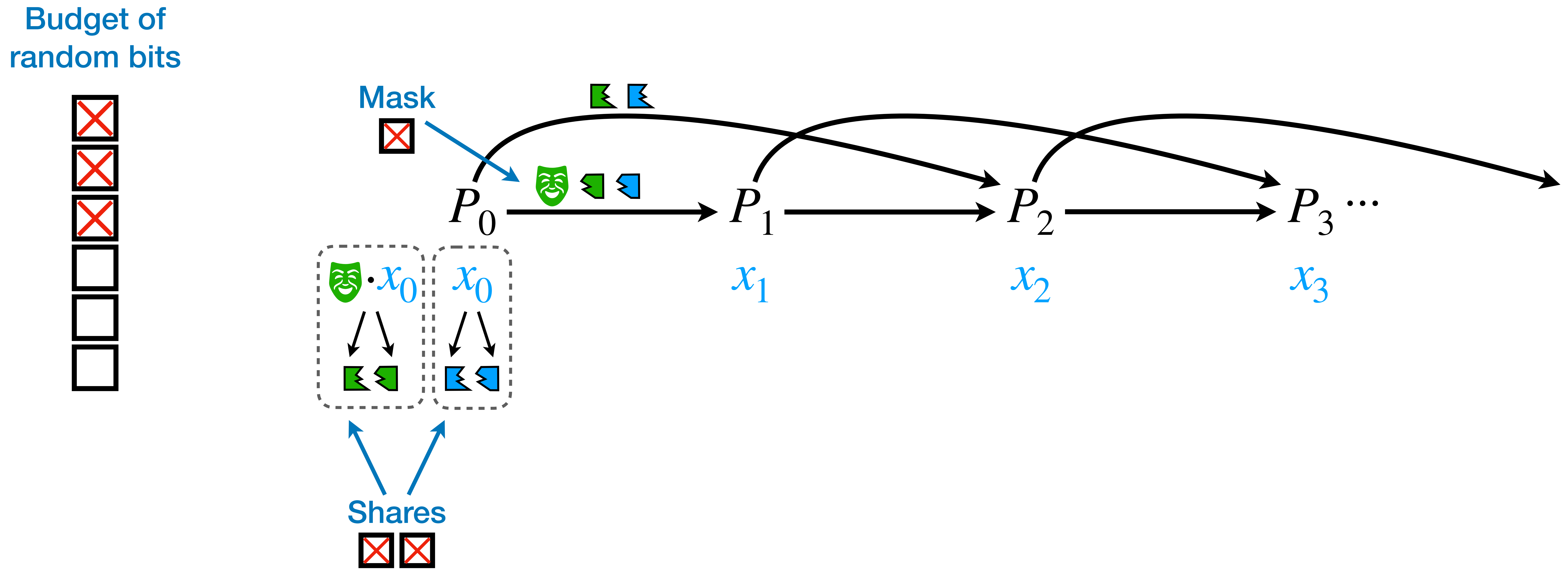


Invariant

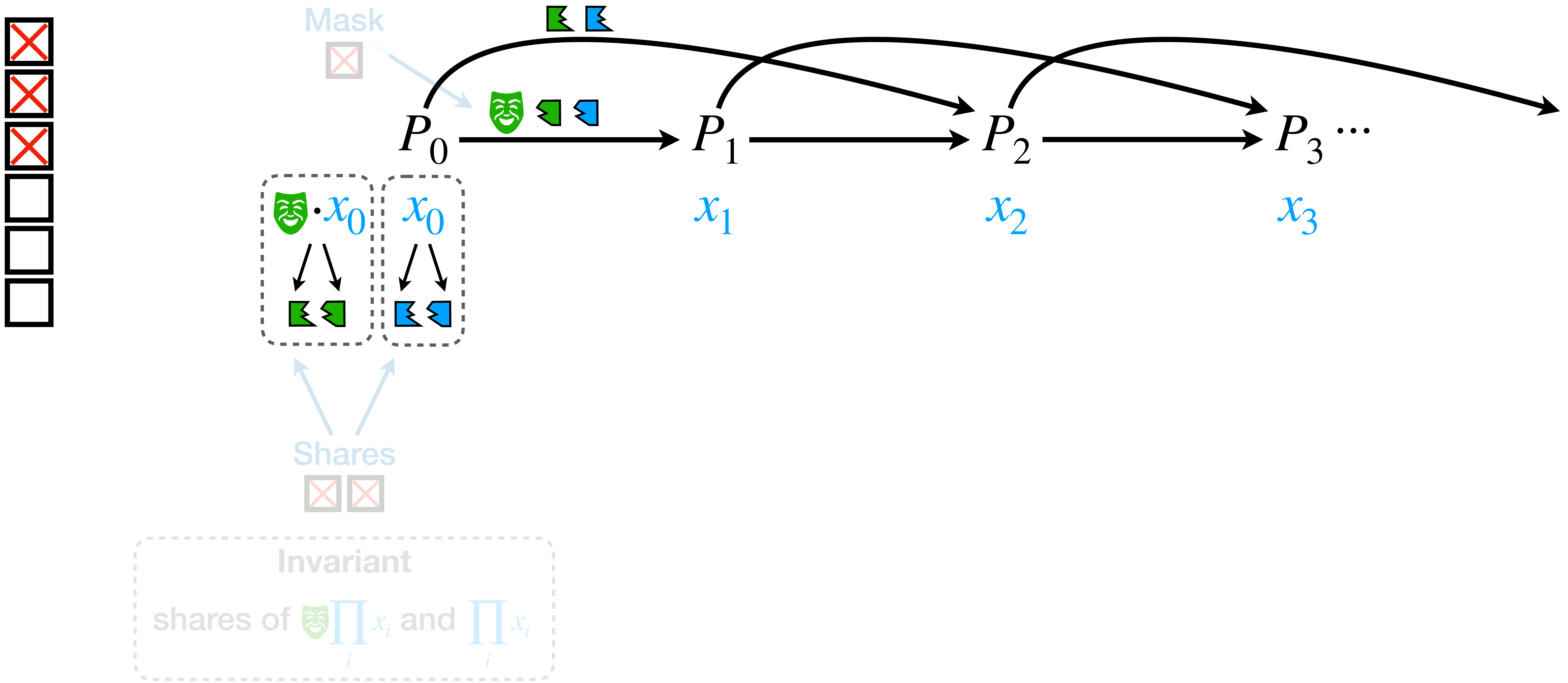
shares of $\prod_i x_i$ and $\prod_i x_i$

We *propagate* the invariant: throughout the computation, P_{i-1} and P_i will hold shares of these products.

Second Result: 1-Private n-Party AND with 6 Bits and 1 Source

Main Phase

Budget of random bits



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Main Phase

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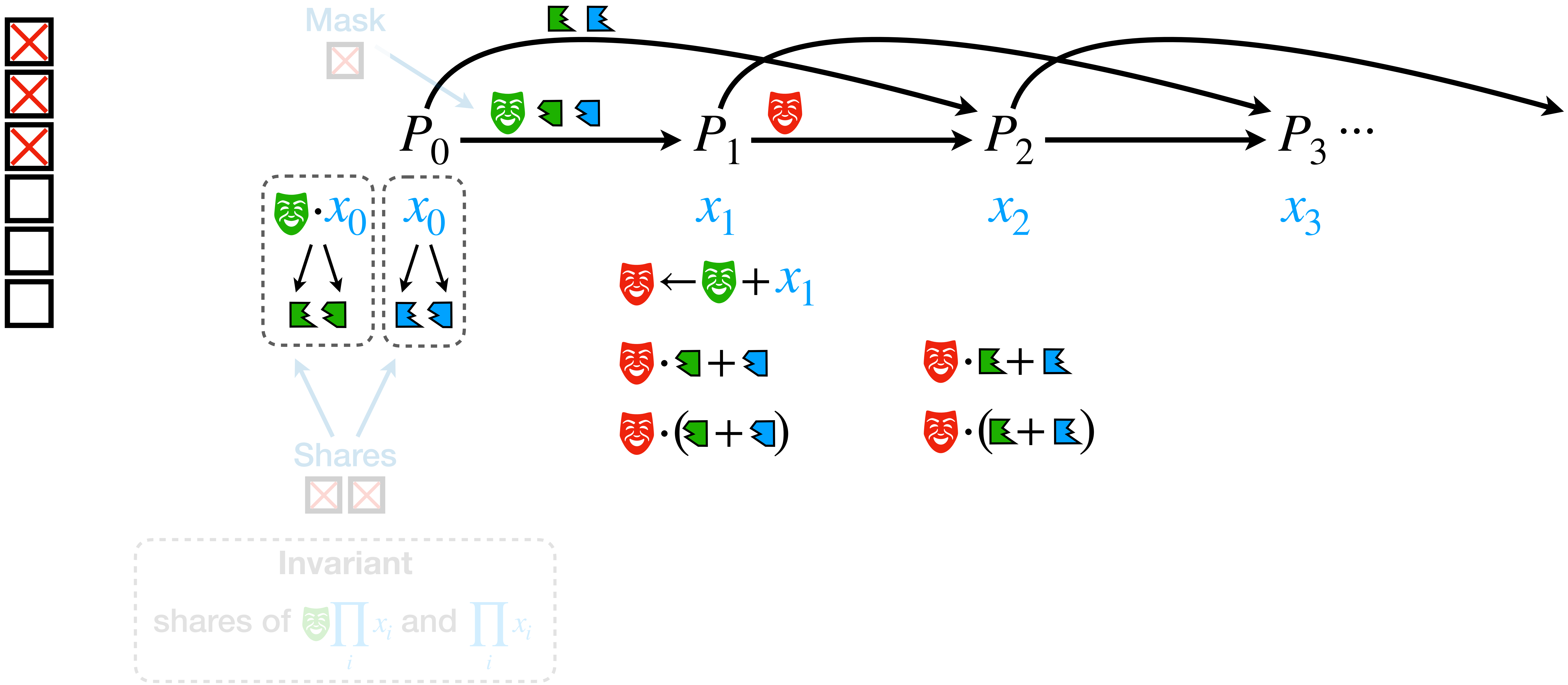

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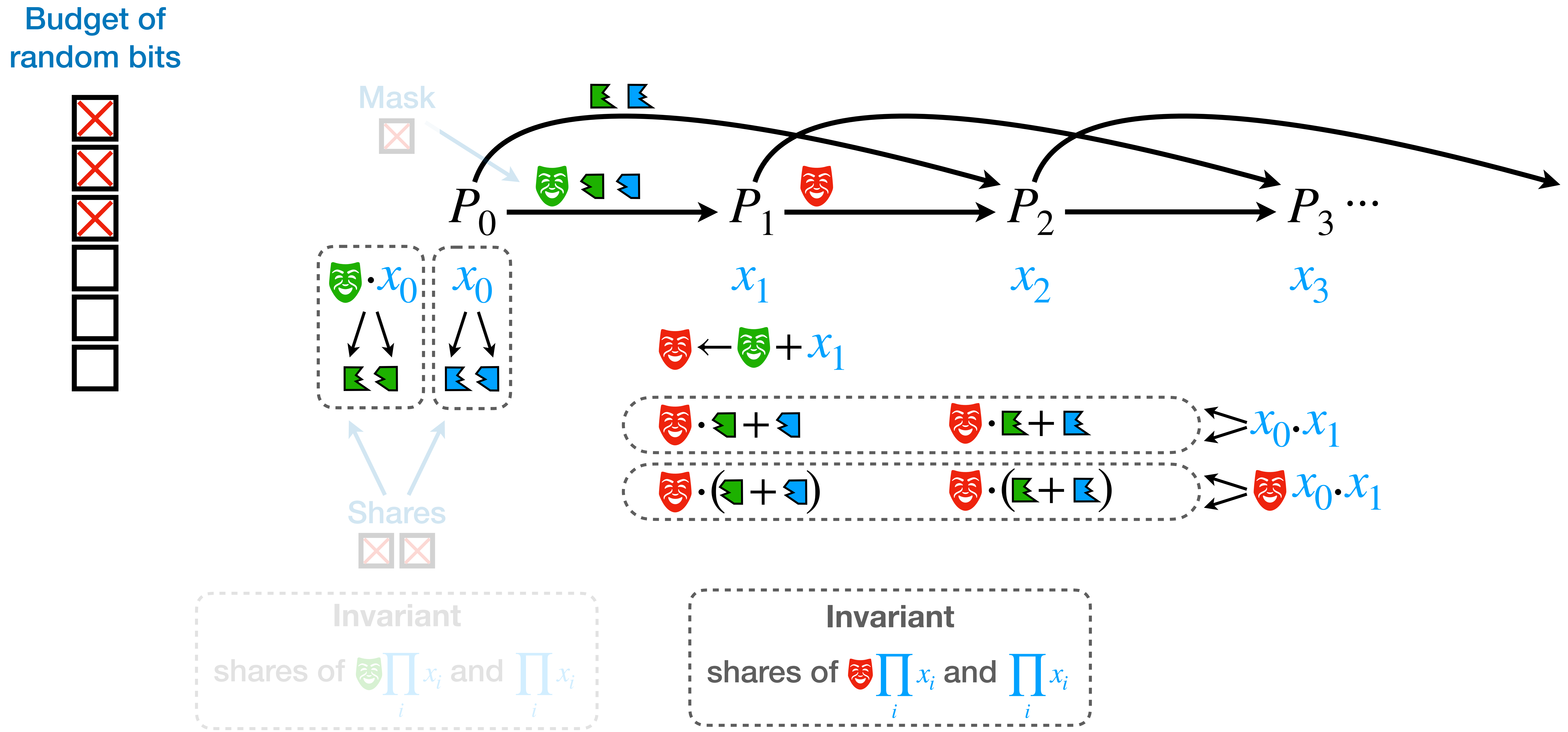

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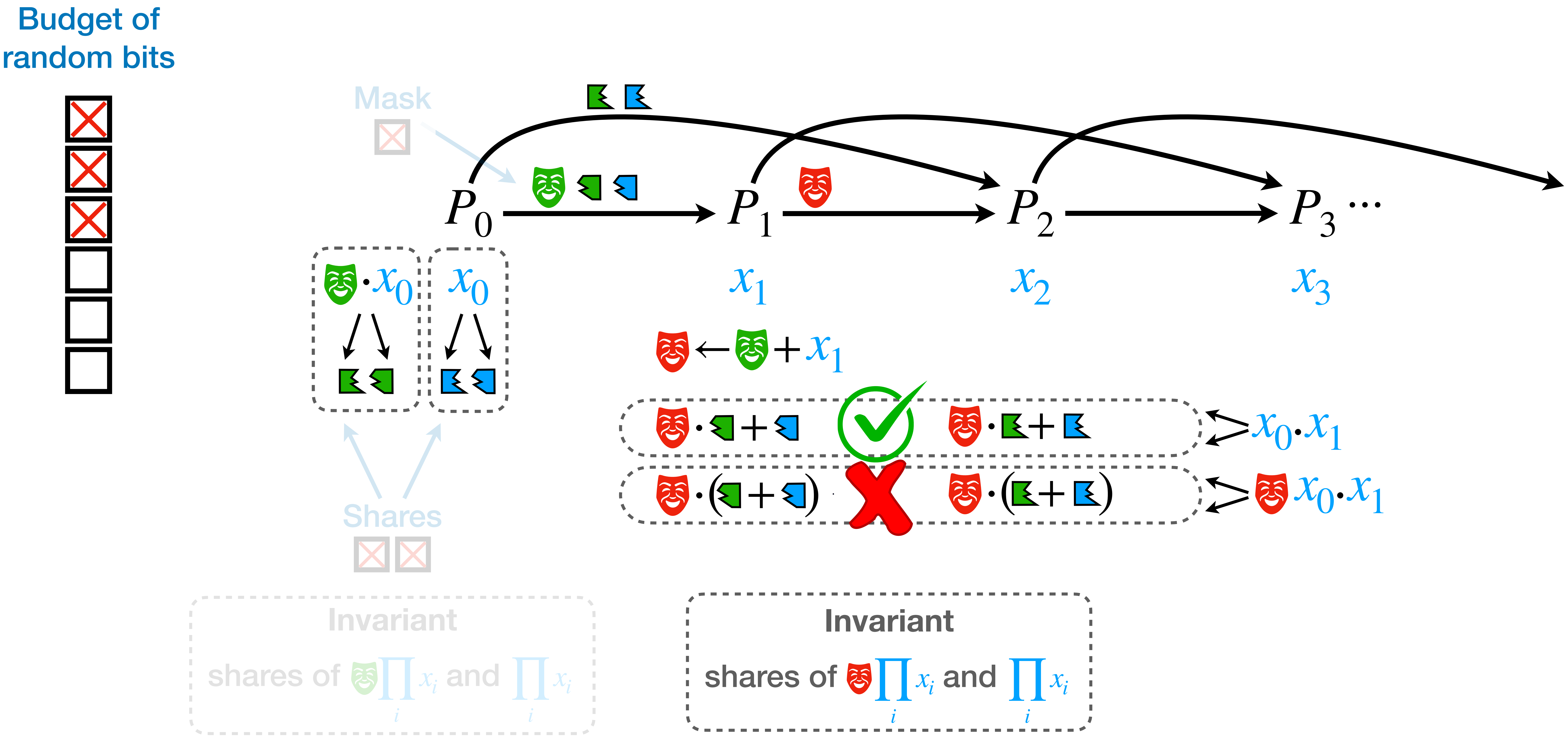
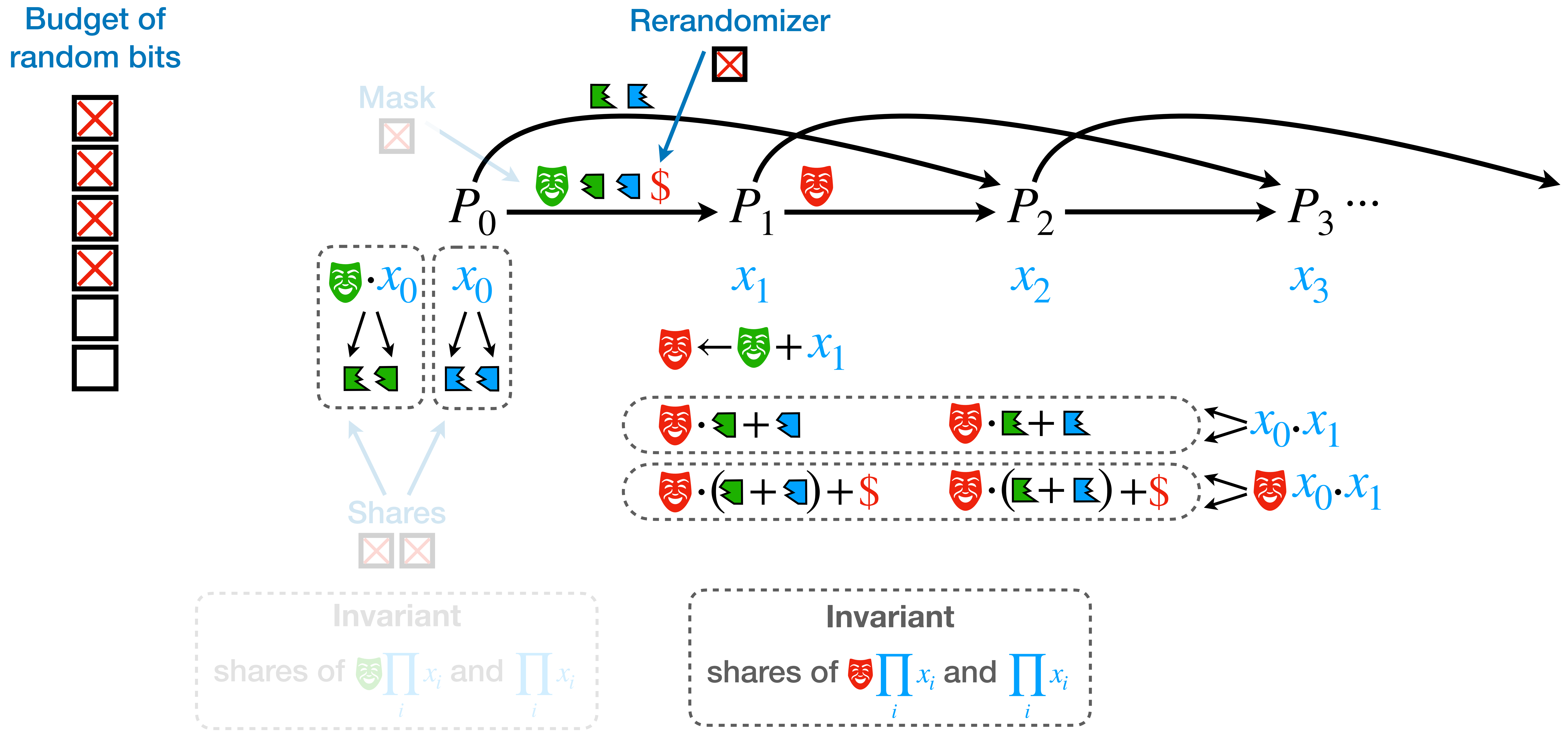


Shares of $x_0 \cdot x_1$ are not uniform! (Biased toward 0)

Second Result: 1-Private n-Party AND with 6 Bits and 1 Source

Main Phase



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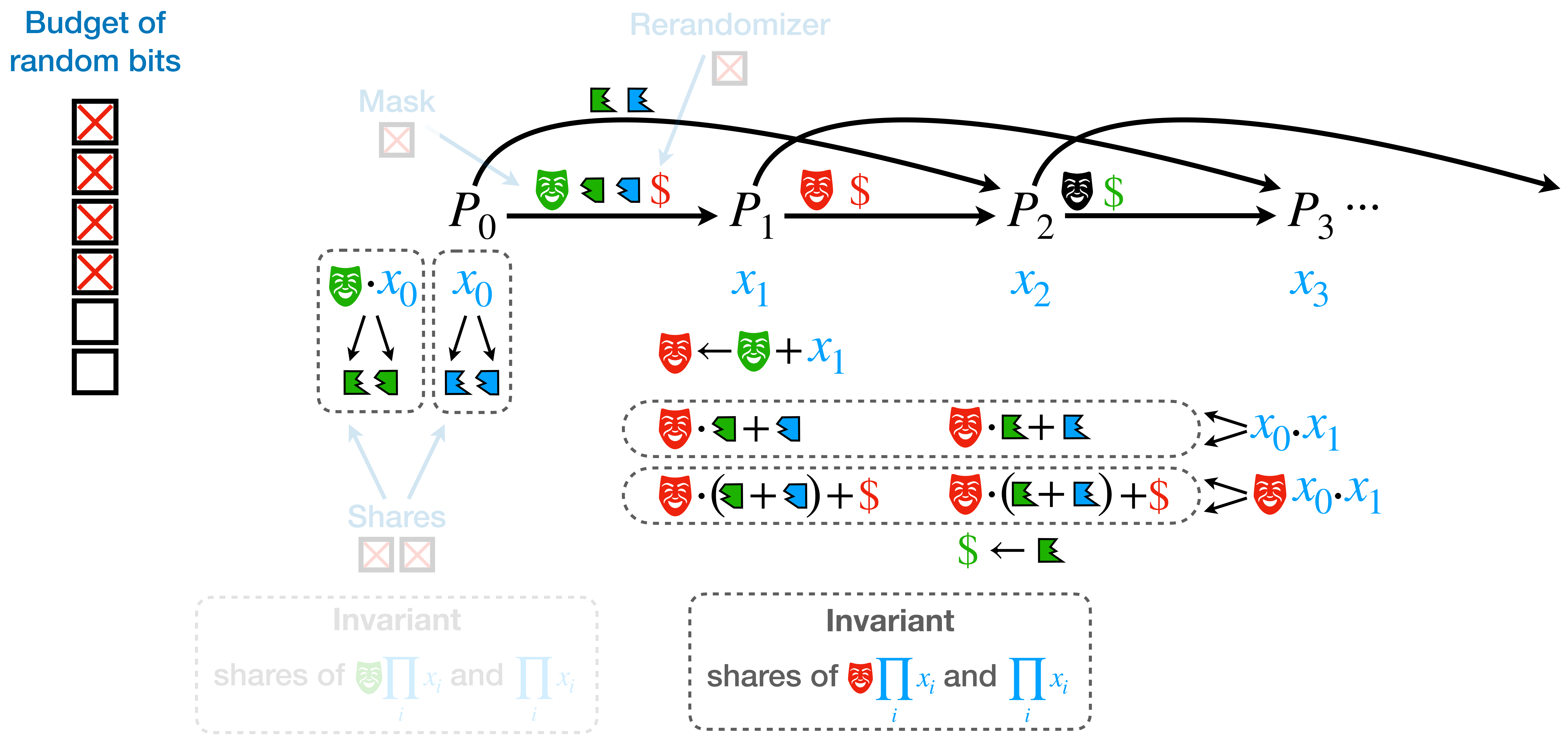

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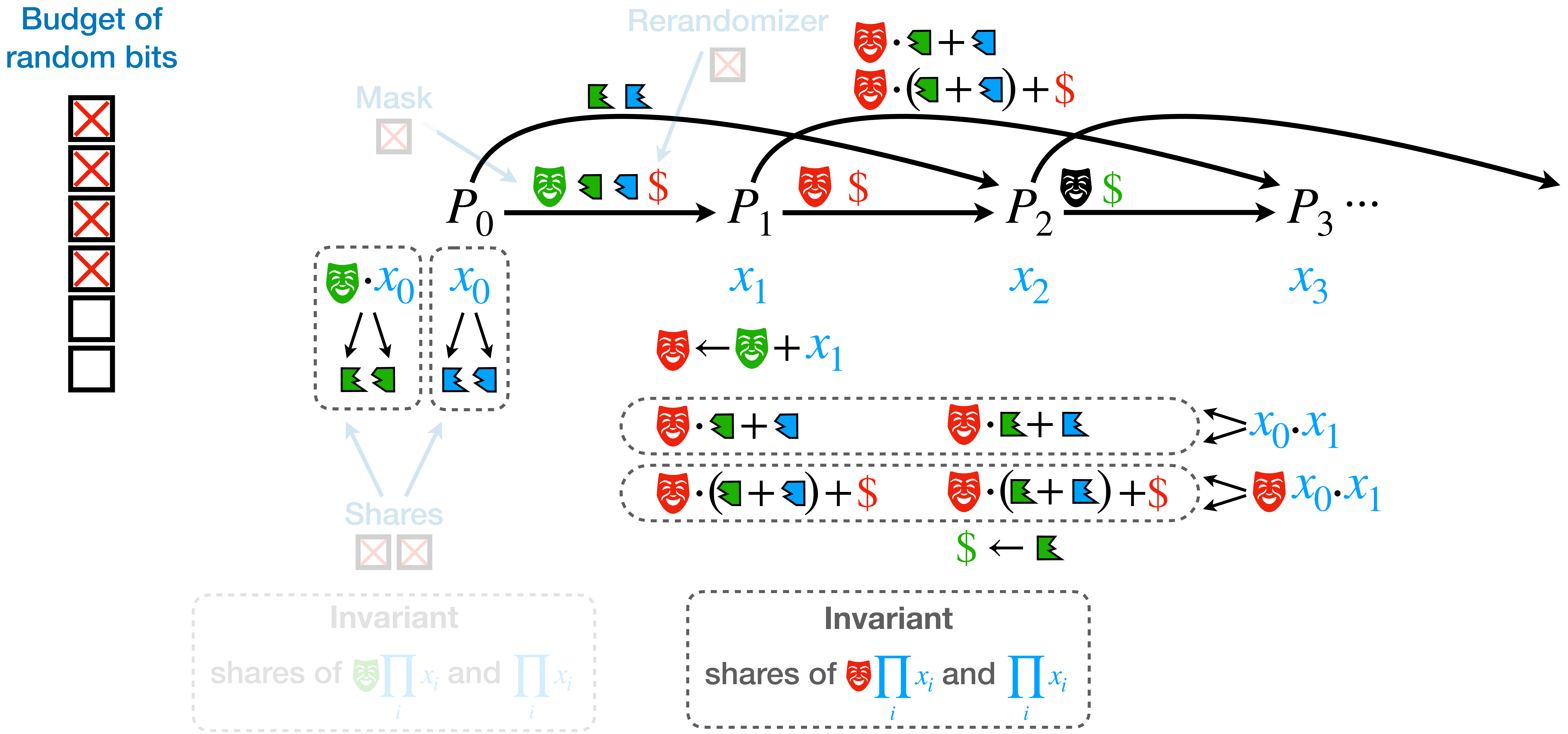

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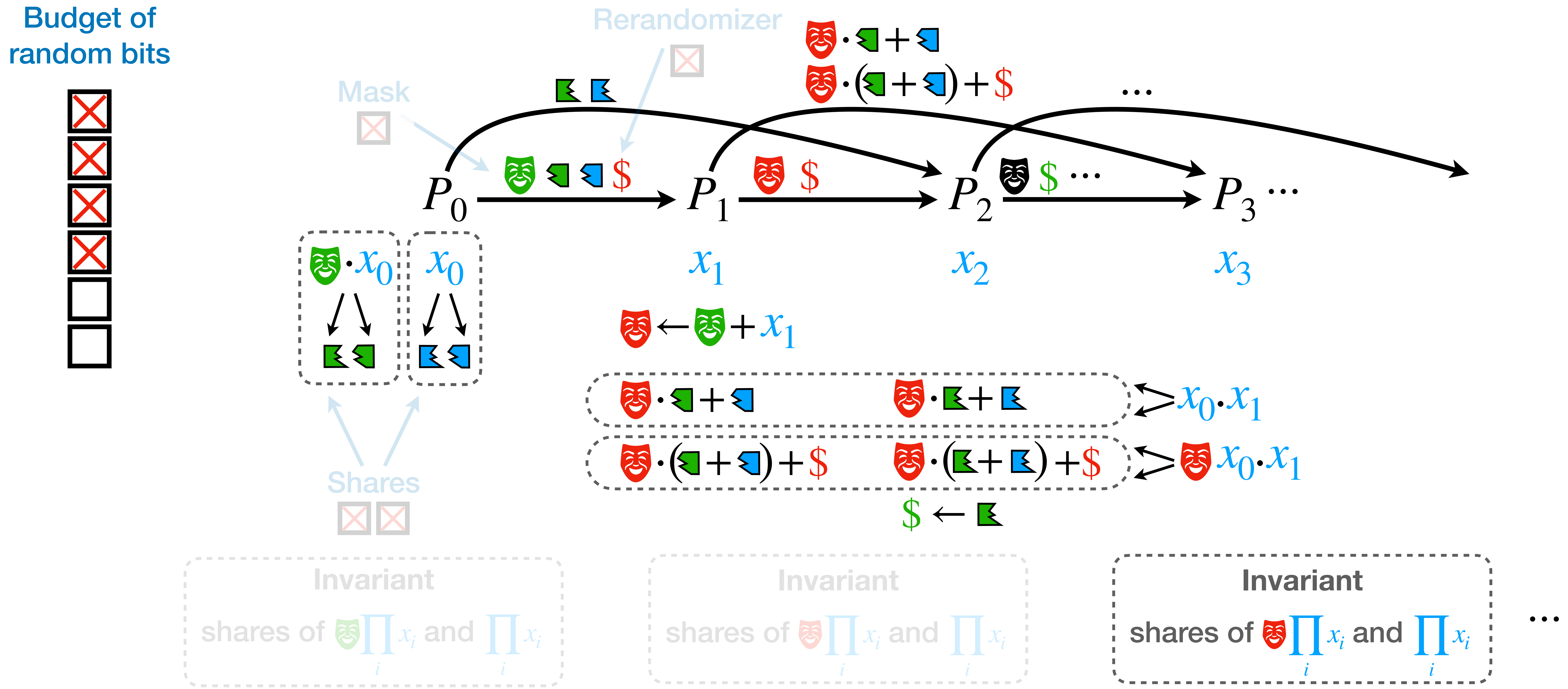

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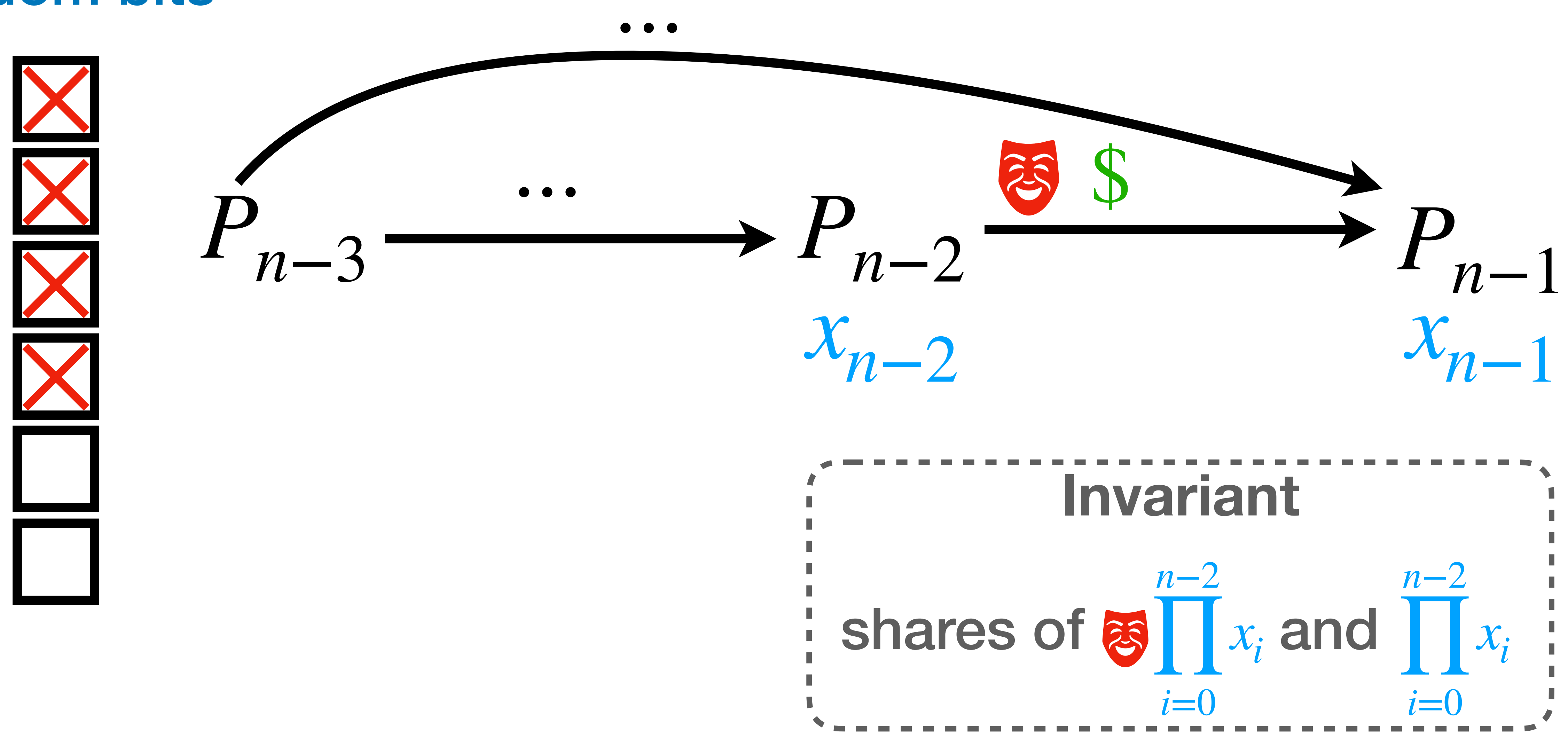
Output Phase

Budget of random bits



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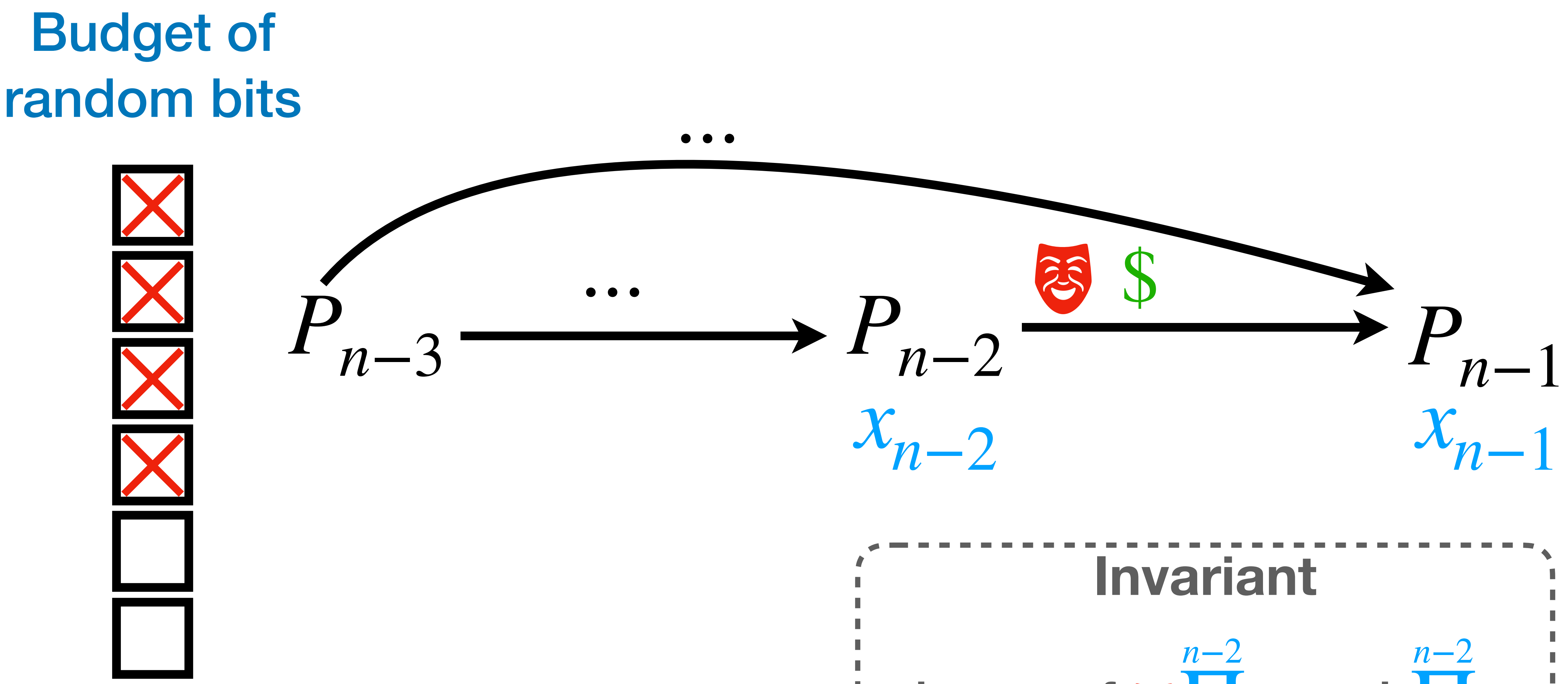


Invariant

shares of $\prod_{i=0}^{n-2} x_i$ and $\prod_{i=0}^{n-2} x_i$

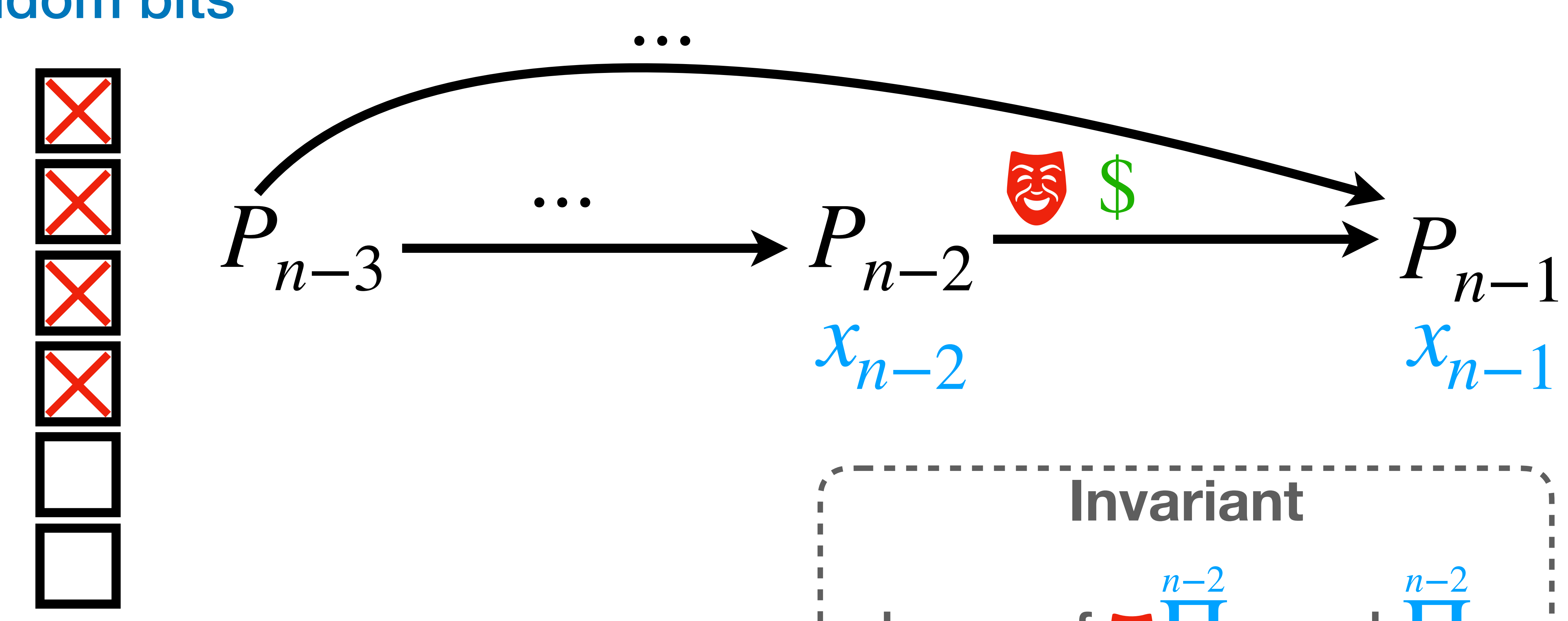
Output

$$\begin{cases} 0 & \text{if } x_{n-1} = 0 \\ \prod_{i=0}^{n-1} x_i & \text{if } x_{n-1} = 1 \end{cases}$$

Second Result: 1-Private n-Party AND with 6 Bits and 1 Source

Output Phase

Budget of random bits



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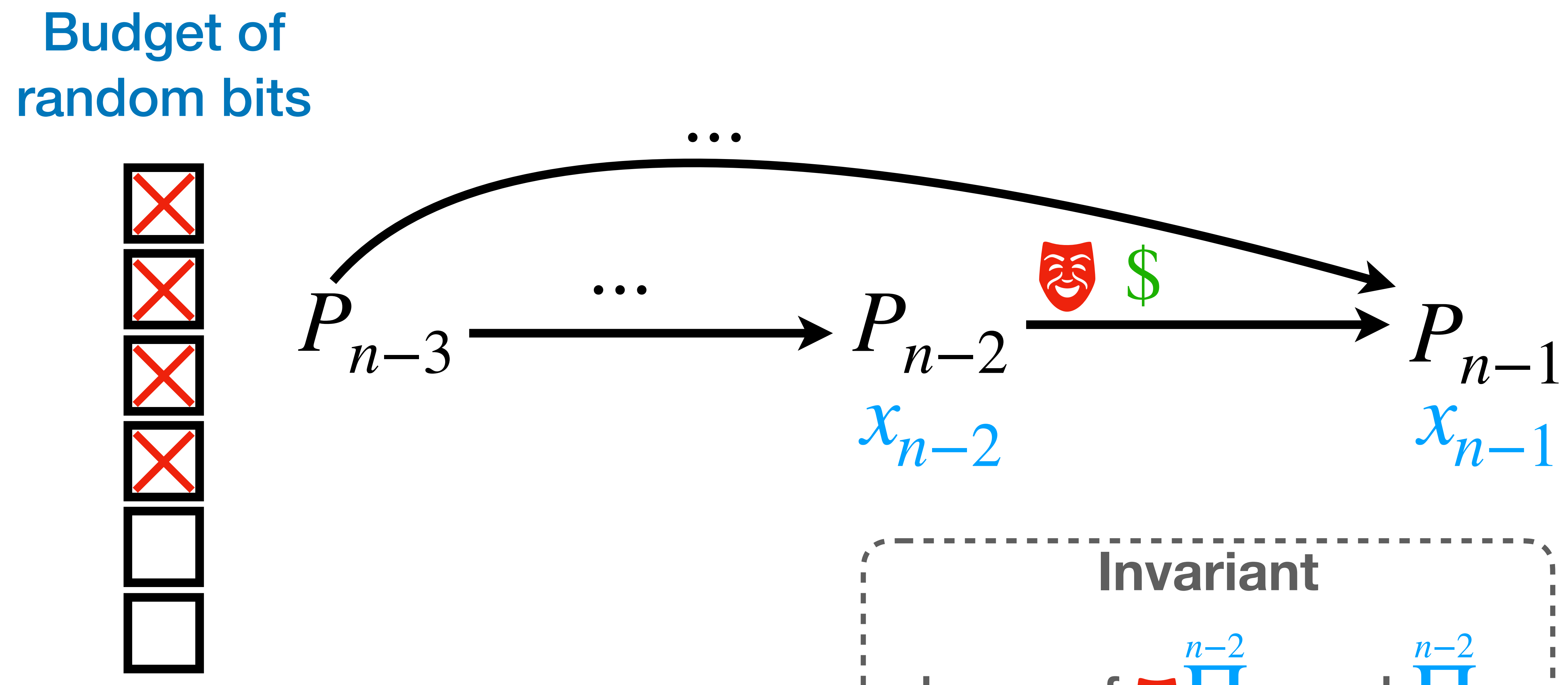
Idea 1

Use an oblivious transfer with x_{n-1} as selection bit, and sender inputs 0 and P_{n-2}'s share of $\prod_{i=0}^{n-1} x_i \implies$ uses 3 random bits!

Second Result: 1-Private n-Party AND with 6 Bits and 1 Source

Output Phase



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P_{n-2} and P_{n-1} don't need the rerandomization bit $\$ \Longrightarrow$ can reuse it for the OT!

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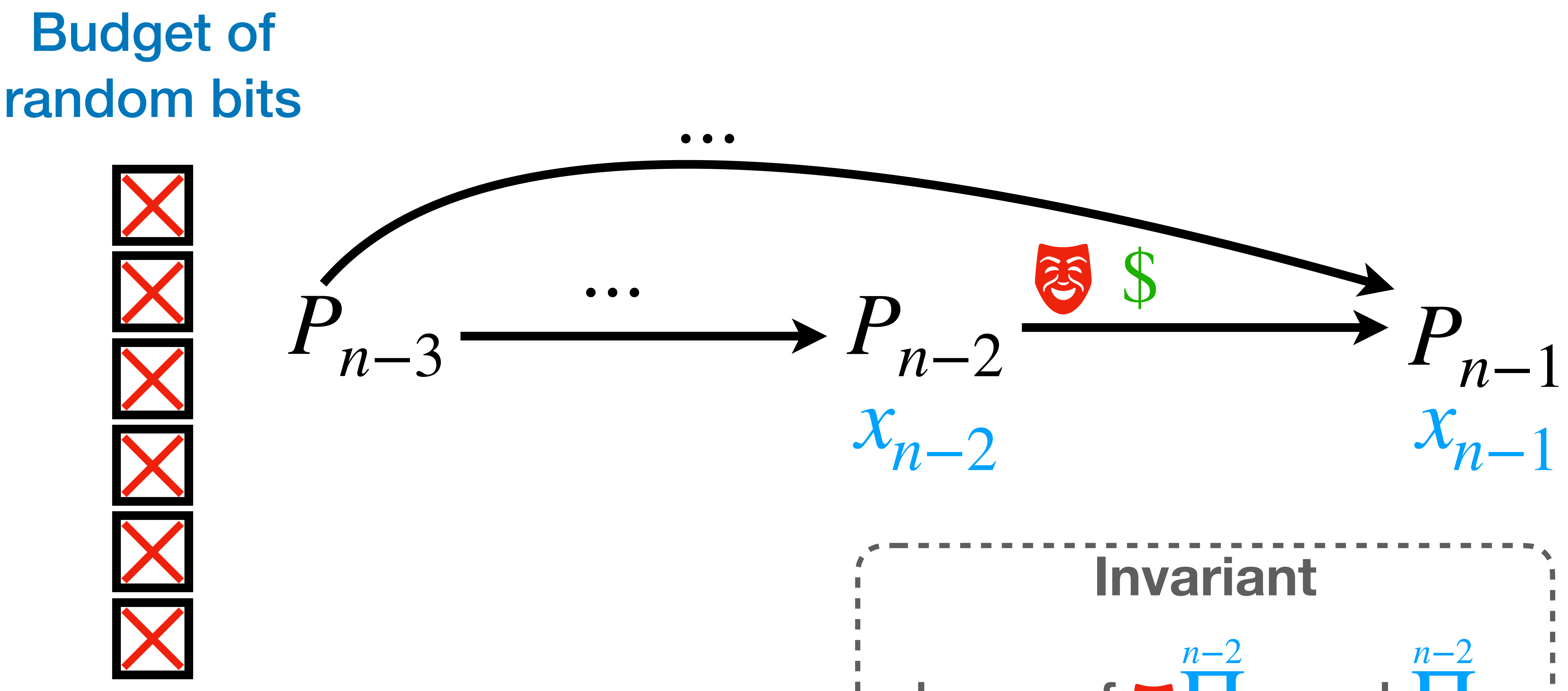
Output Phase



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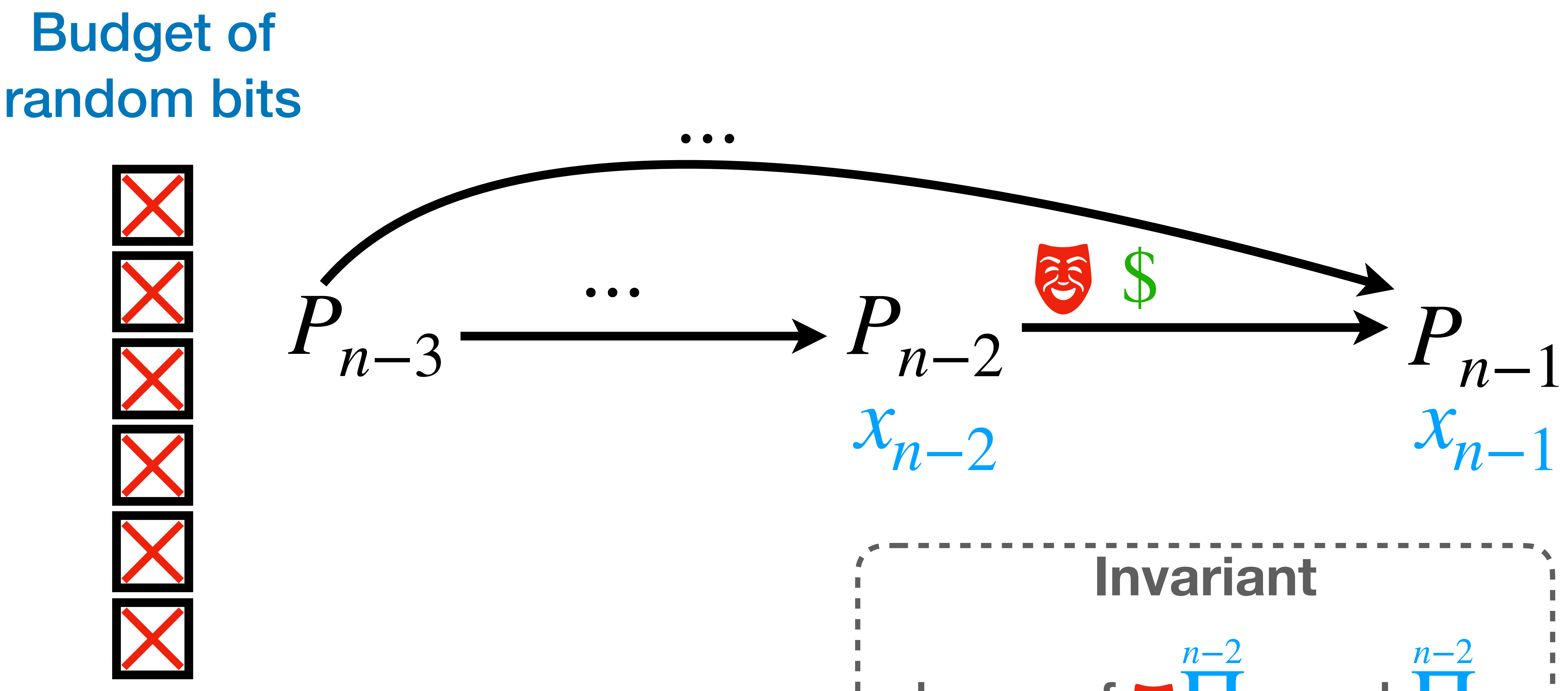

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Thank you for your attention!

Questions?