How to Generate Correlated Randomness from (variants of) LPN

Part I

Based on joint works with: Elette Boyle, Niv Gilboa, Yuval Ishai, Lisa Kohl, Srinivasan Raghuraman, Peter Rindal, Peter Scholl

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Stay tuned for part II :)

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What is Secure Computation?

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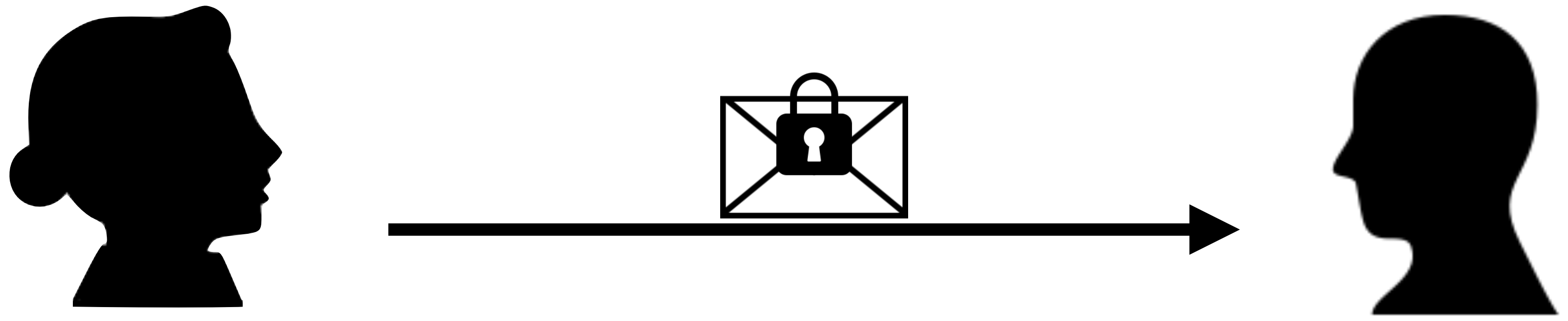
Secure communication

Goal: *communicating* a secret message



m

Output: Bob learns m

Security: Eve learns nothing

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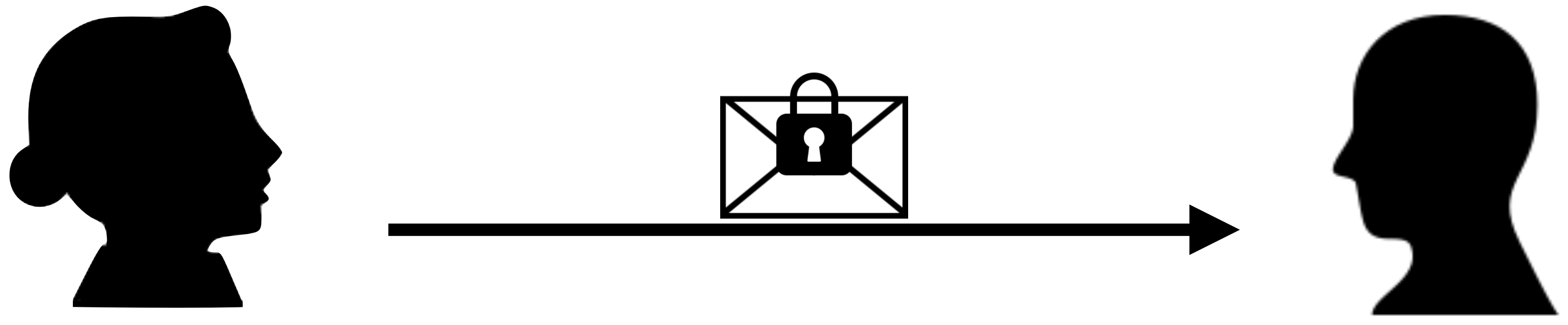

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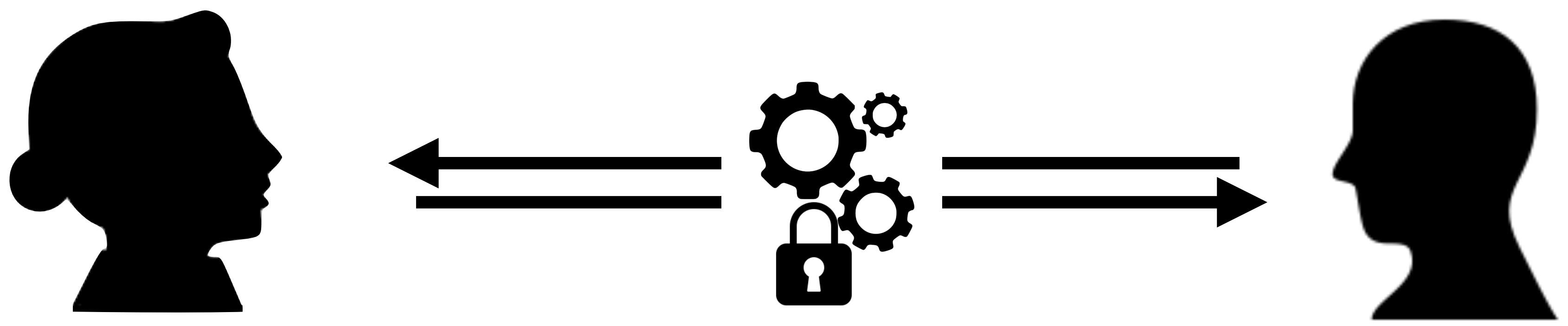
Secure computation

Goal: *computing* a (public) function on secret inputs

$\boxed{f_A(\cdot,\cdot), f_B(\cdot,\cdot)}$



x y

Output: Alice learns $f_A(x, y)$ and Bob learn $f_B(x, y)$

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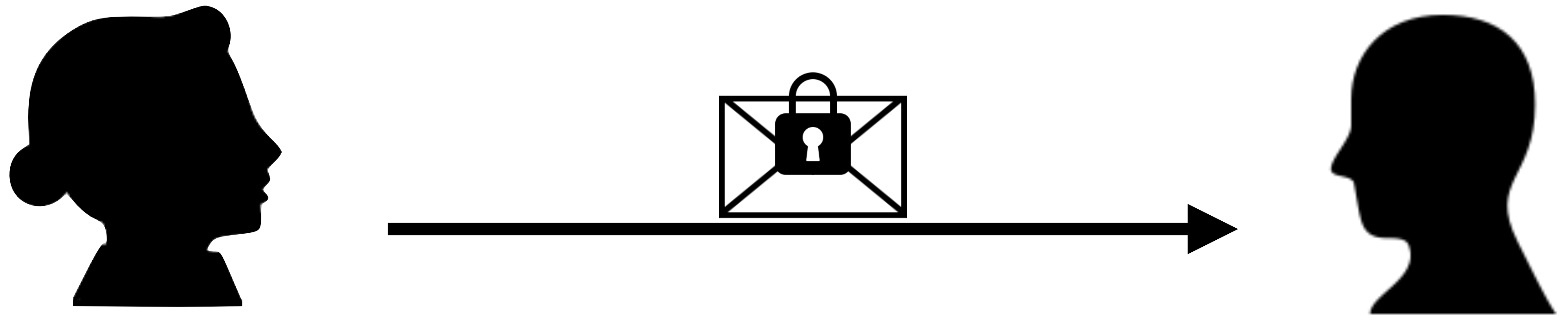

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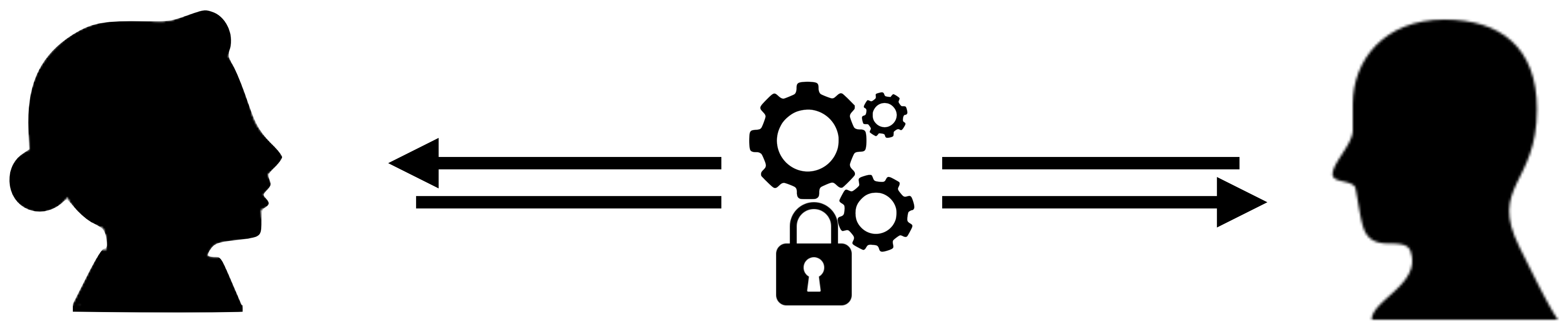

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Output: Alice learns $f_A(x,y)$ and Bob learn $f_B(x,y)$

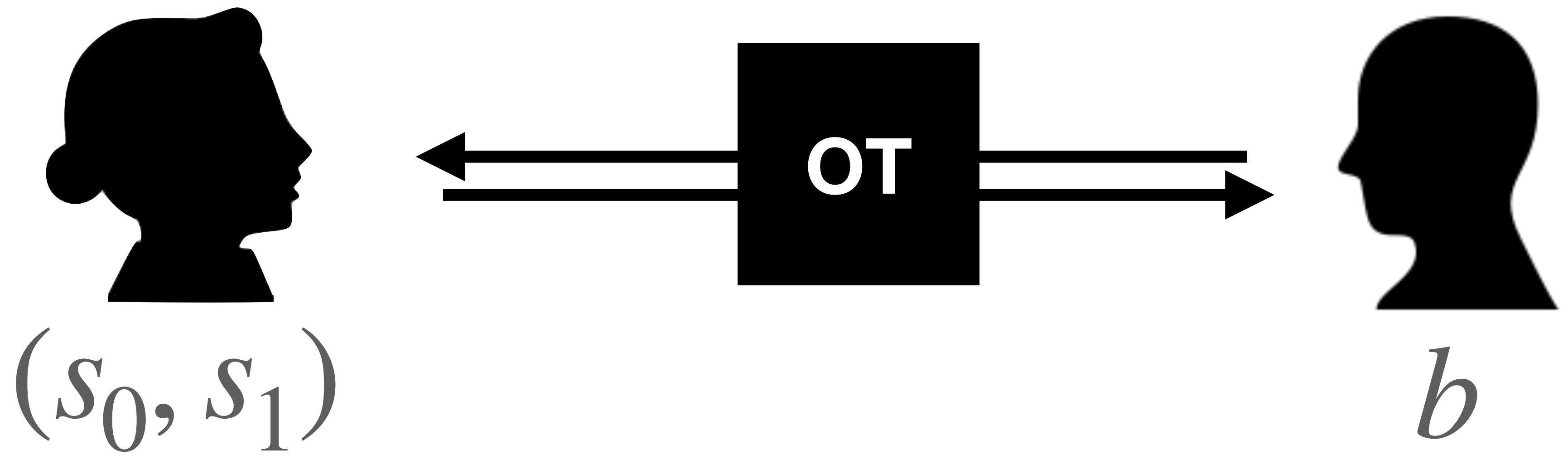
Security: Alice and Bob learn nothing else

- It's a more *fine-grained* approach to security: the function controls precisely what is learned (secure communication is *all or nothing*)
- It is much more demanding: now the adversary is *internal* (Alice must be protected against Bob, and Bob against Alice), and can influence the protocol!

Secure Computation from Oblivious Transfer

Oblivious Transfer

A minimal example of secure computation



Output: Bob learns s_b

Security: Alice does not learn b, Bob does not learn s_{1-b}.

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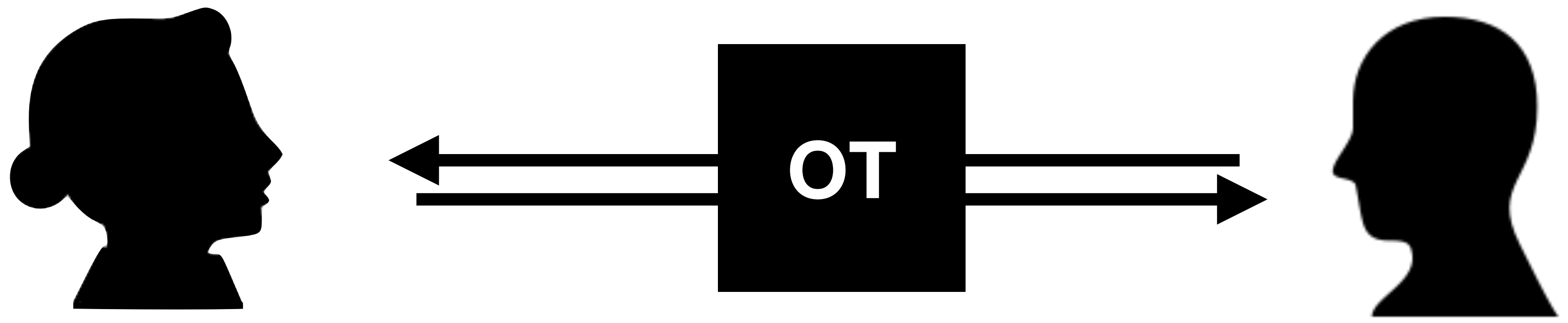

(s_0, s_1) b

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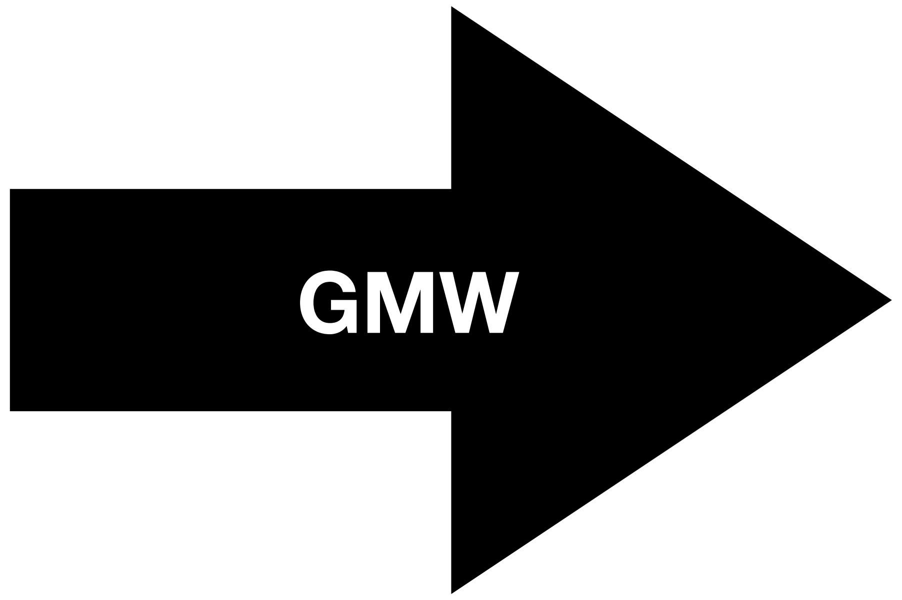
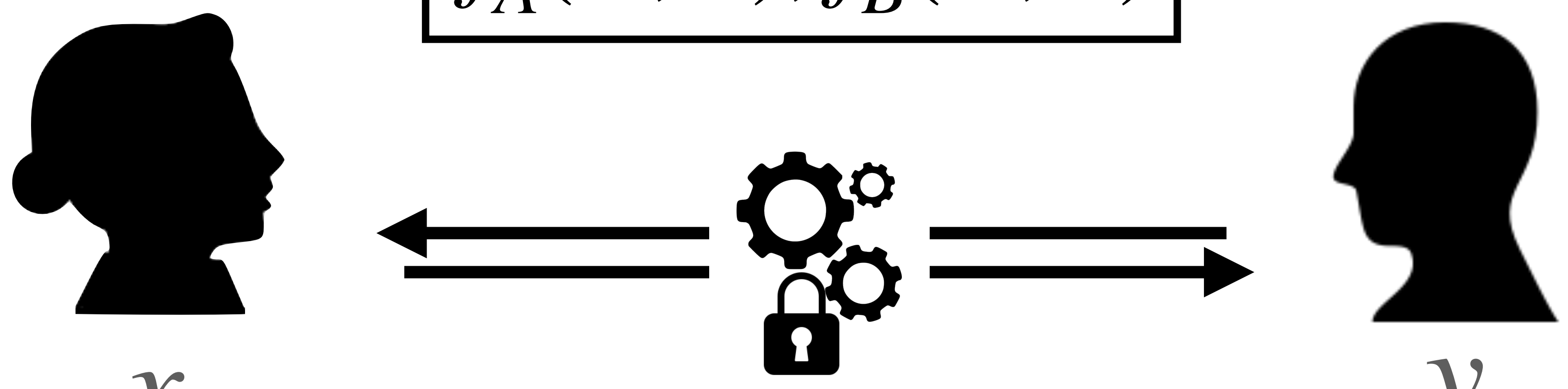


Secure Computation for all functions

$\boxed{f_A(\cdot,\cdot), f_B(\cdot,\cdot)}$



x y

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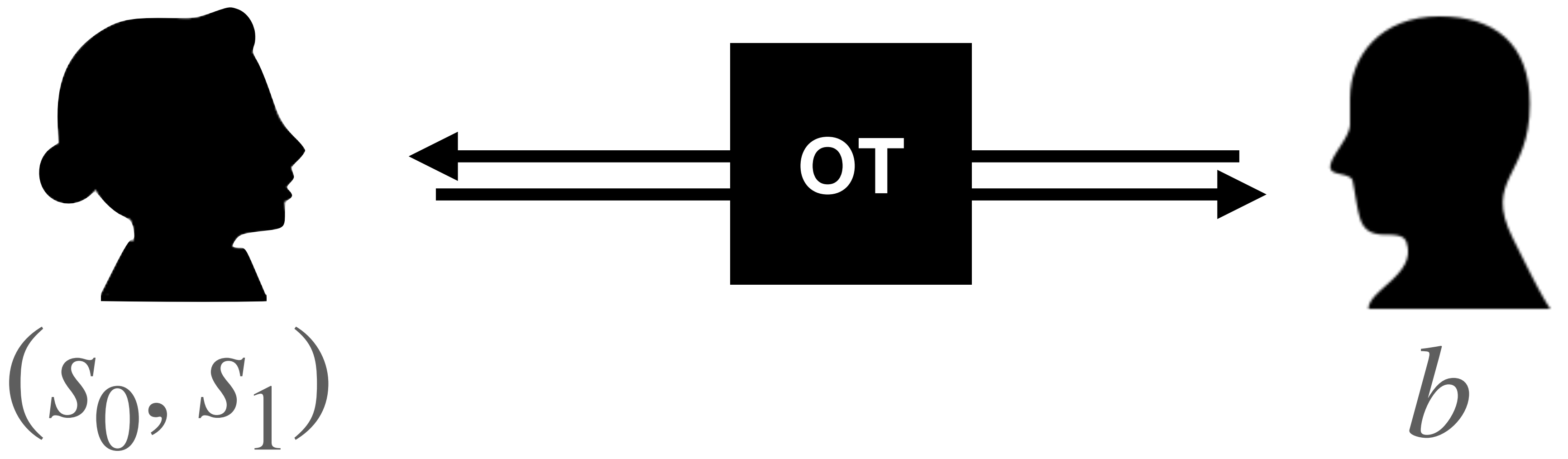

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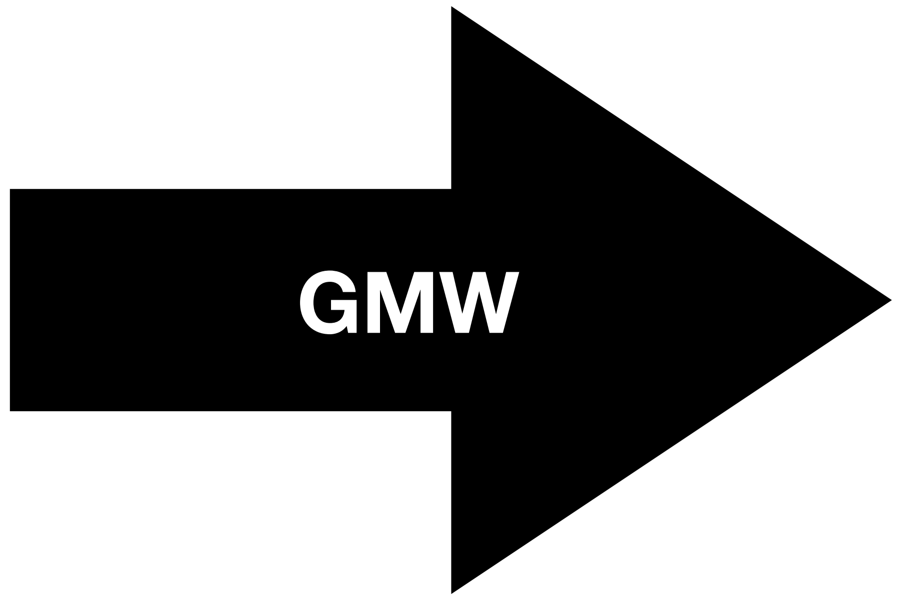

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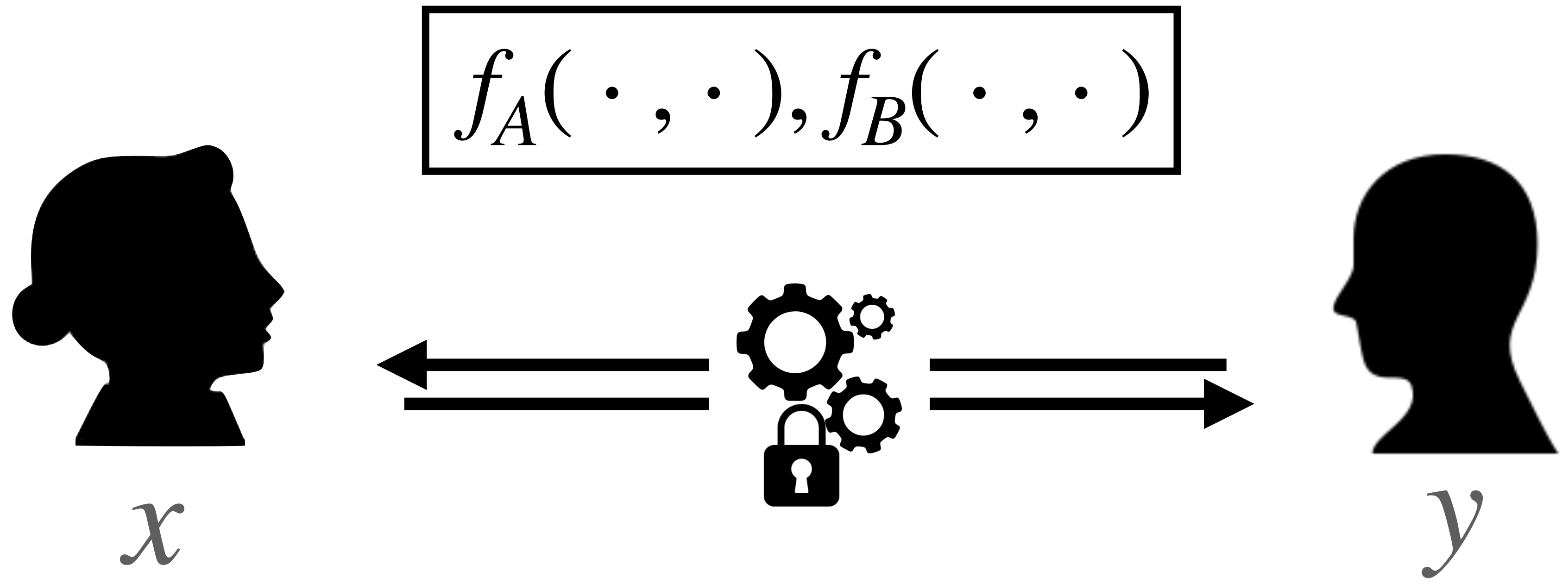

Output: Alice learns $f_A(x, y)$ and Bob learns $f_B(x, y)$

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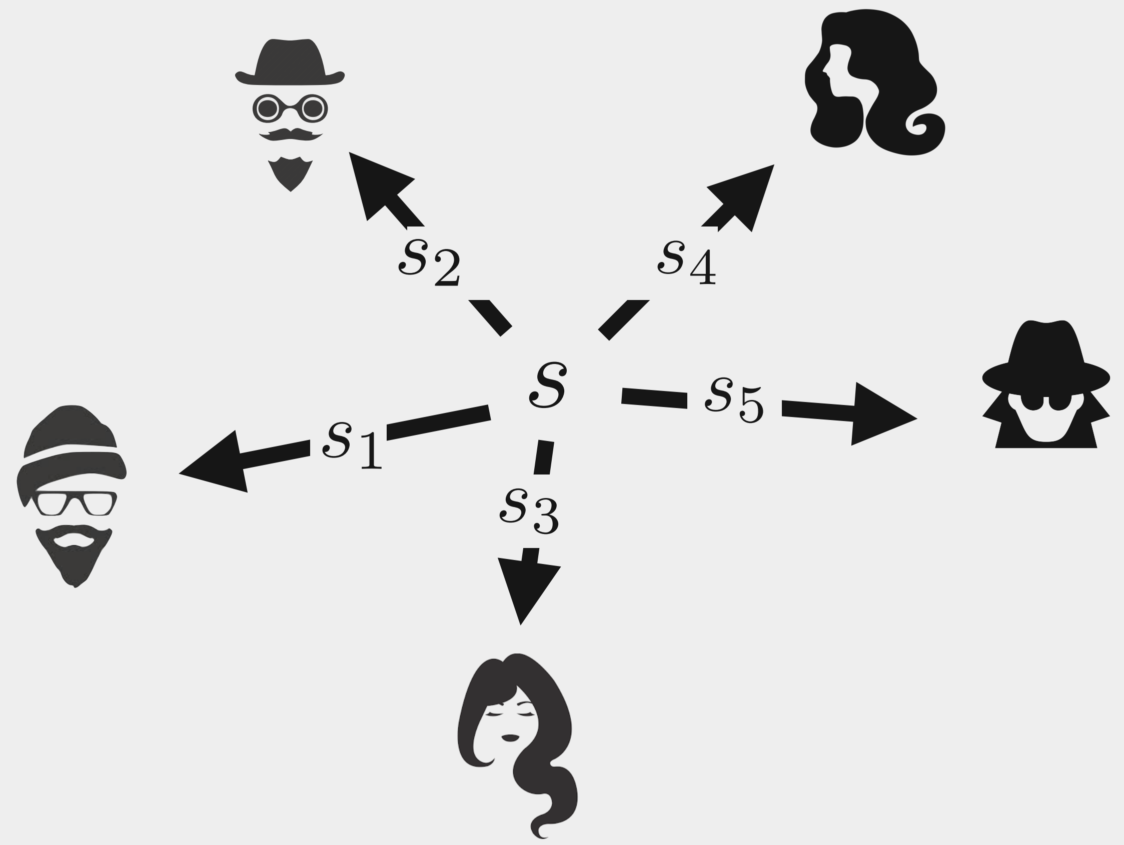
1. Use (additive) secret sharing



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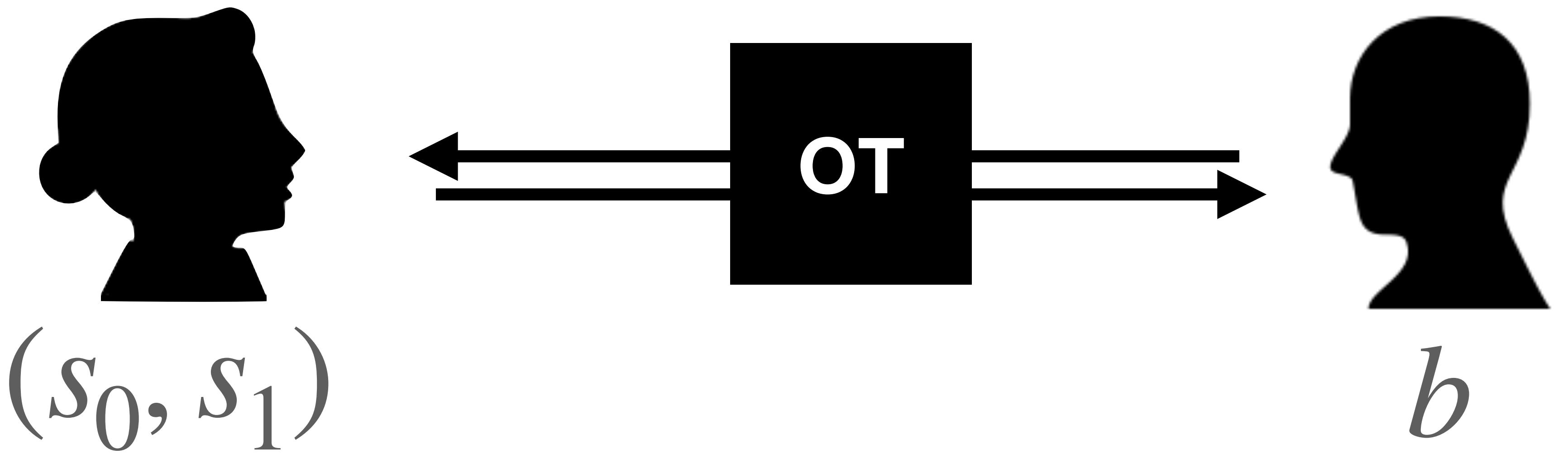

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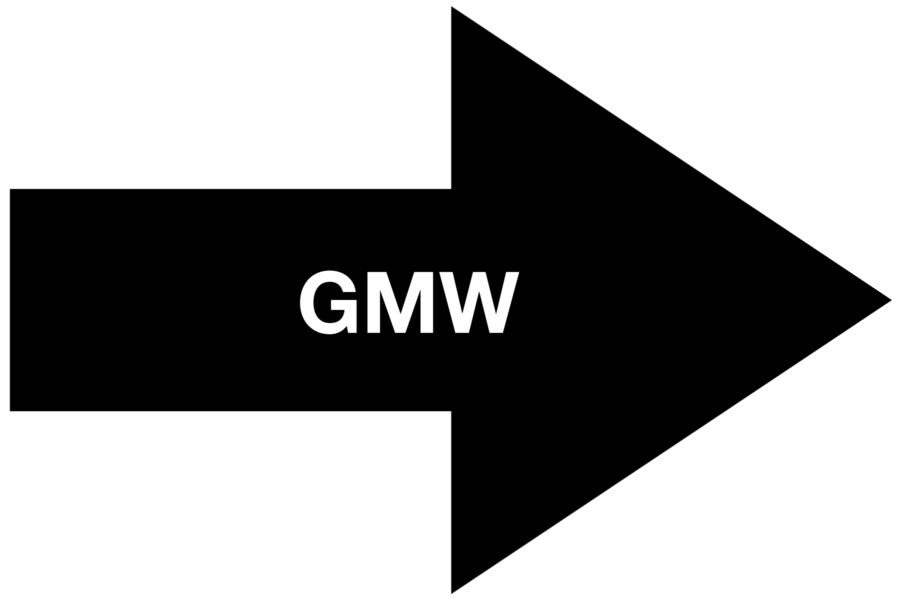

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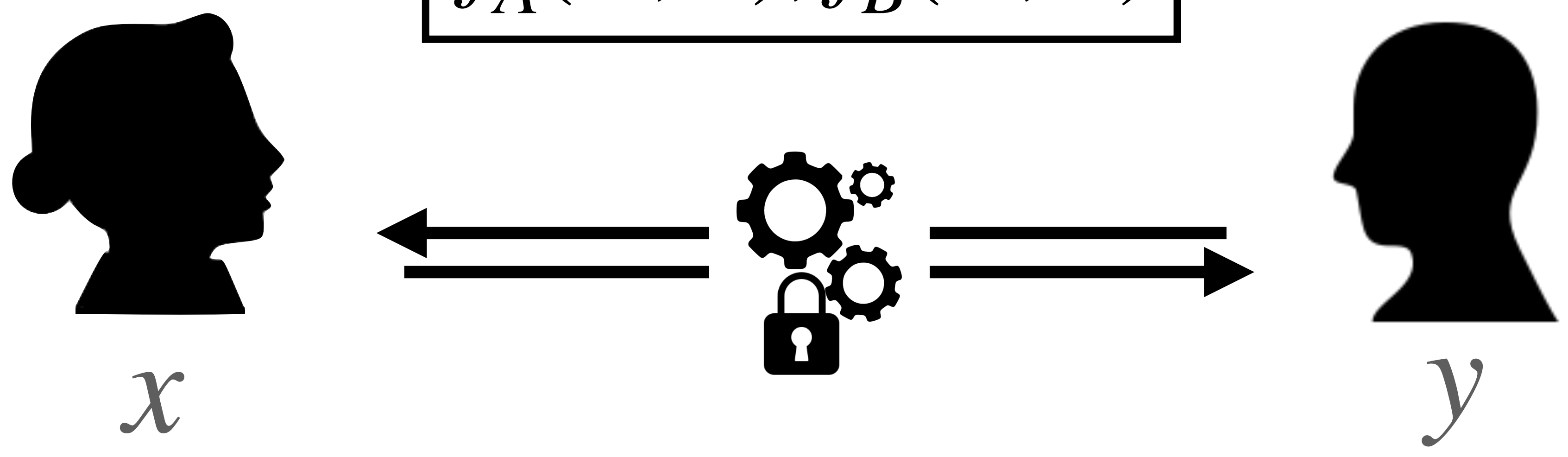

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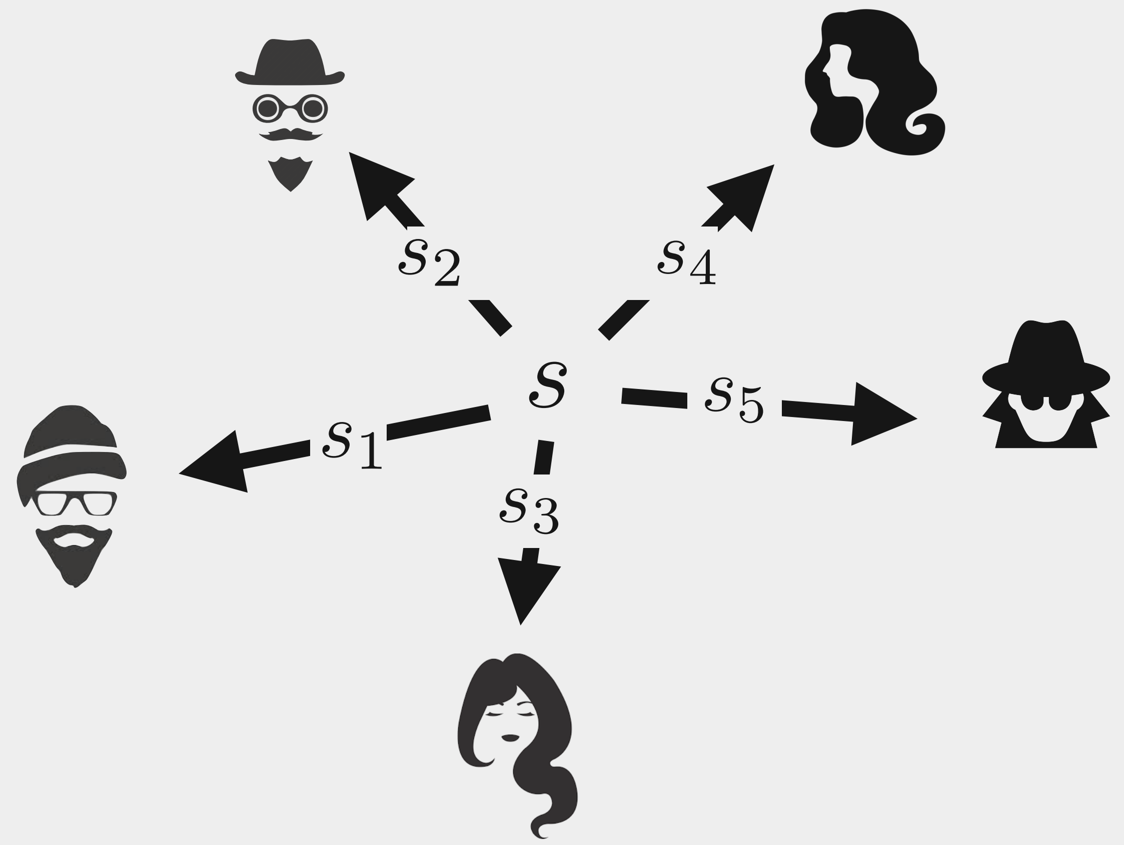
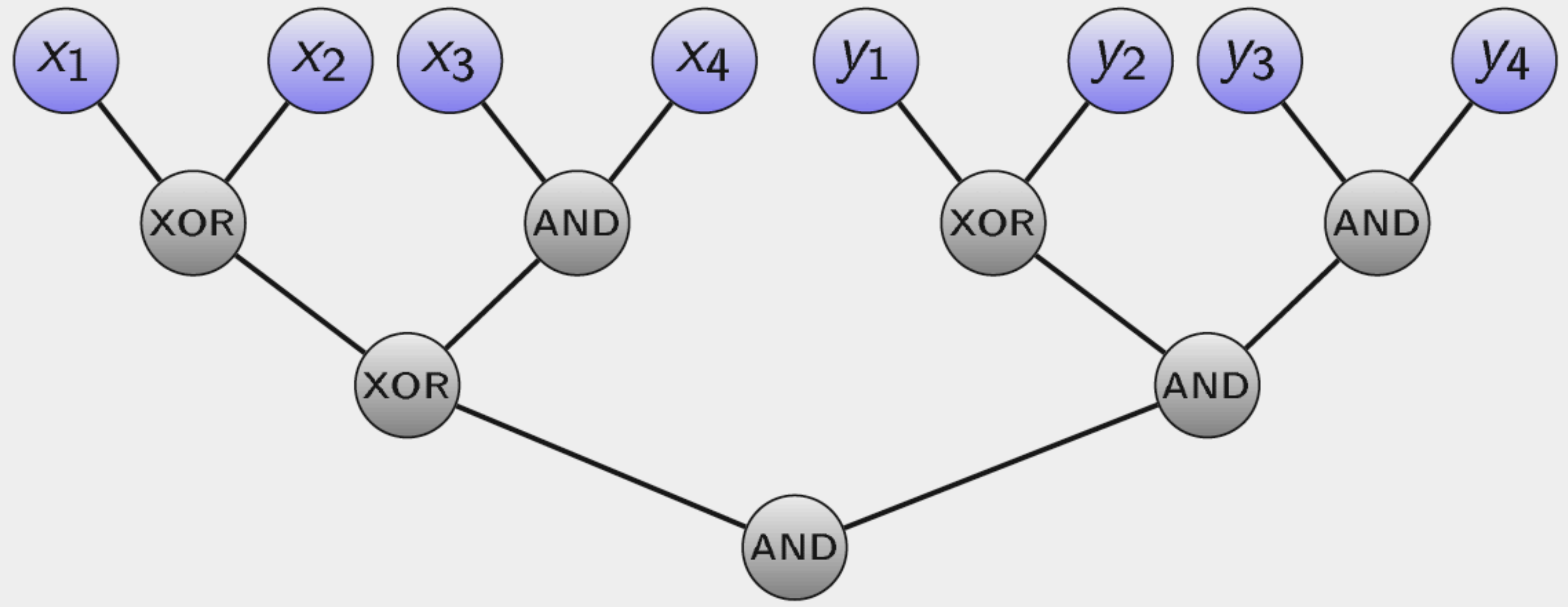


2. Write the function as a circuit



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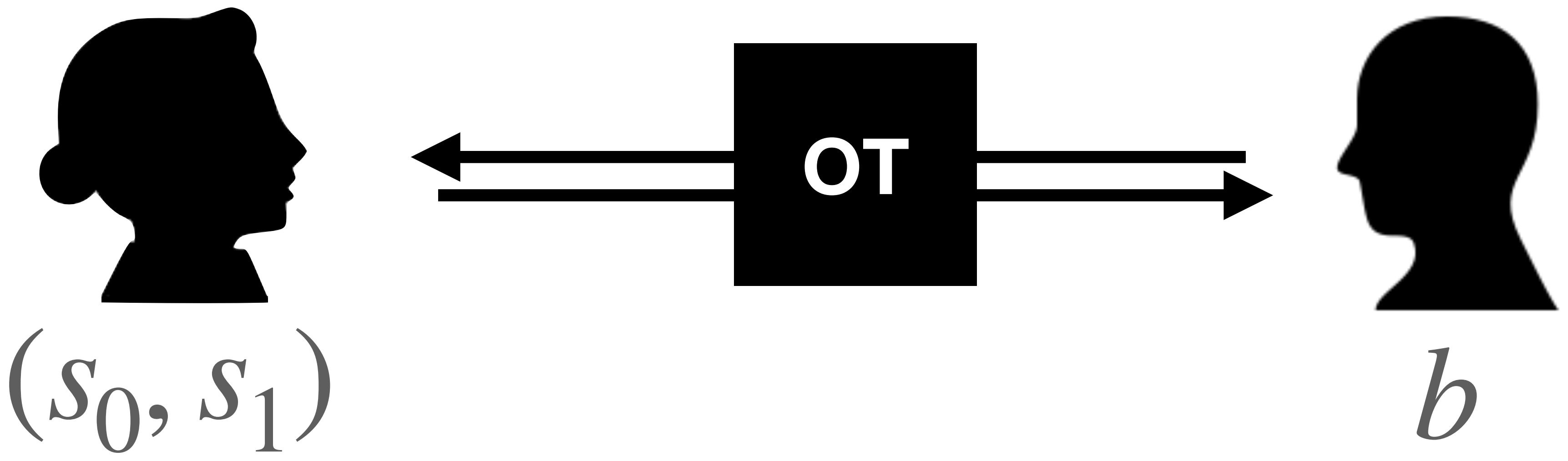

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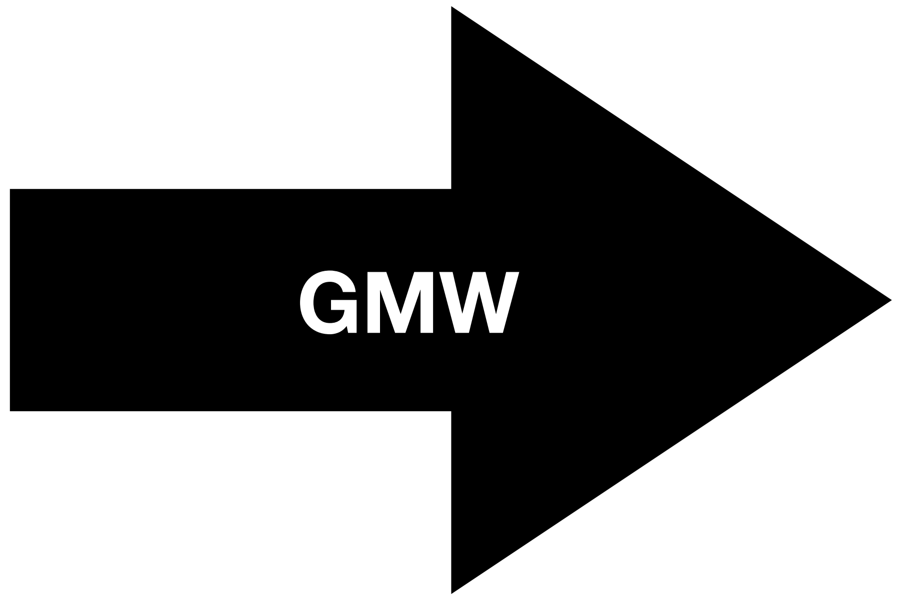

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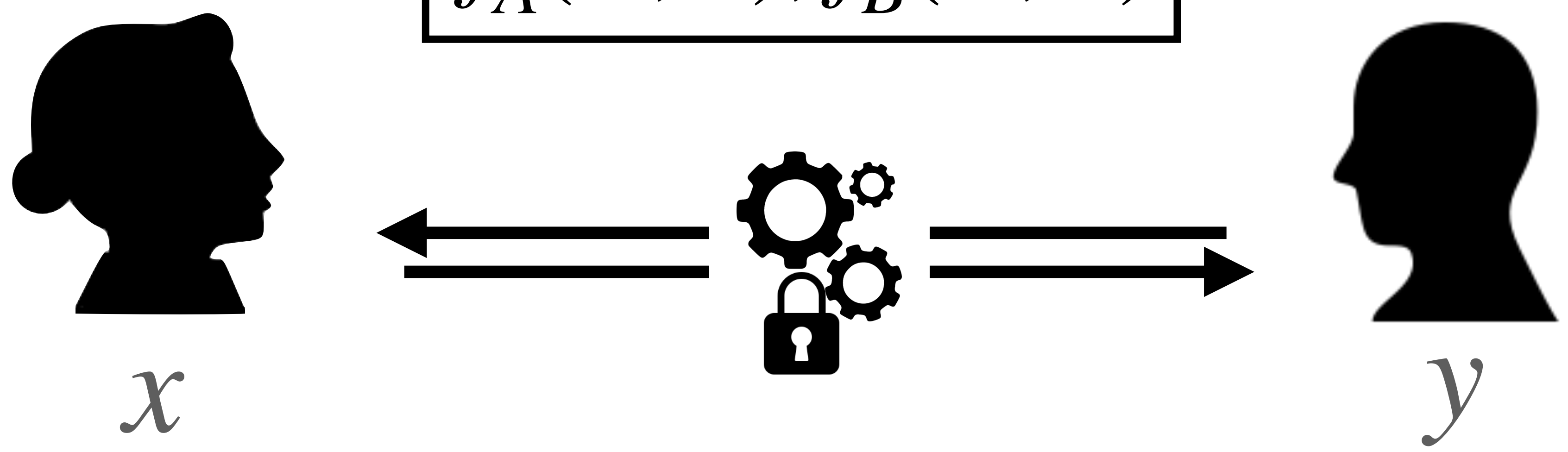

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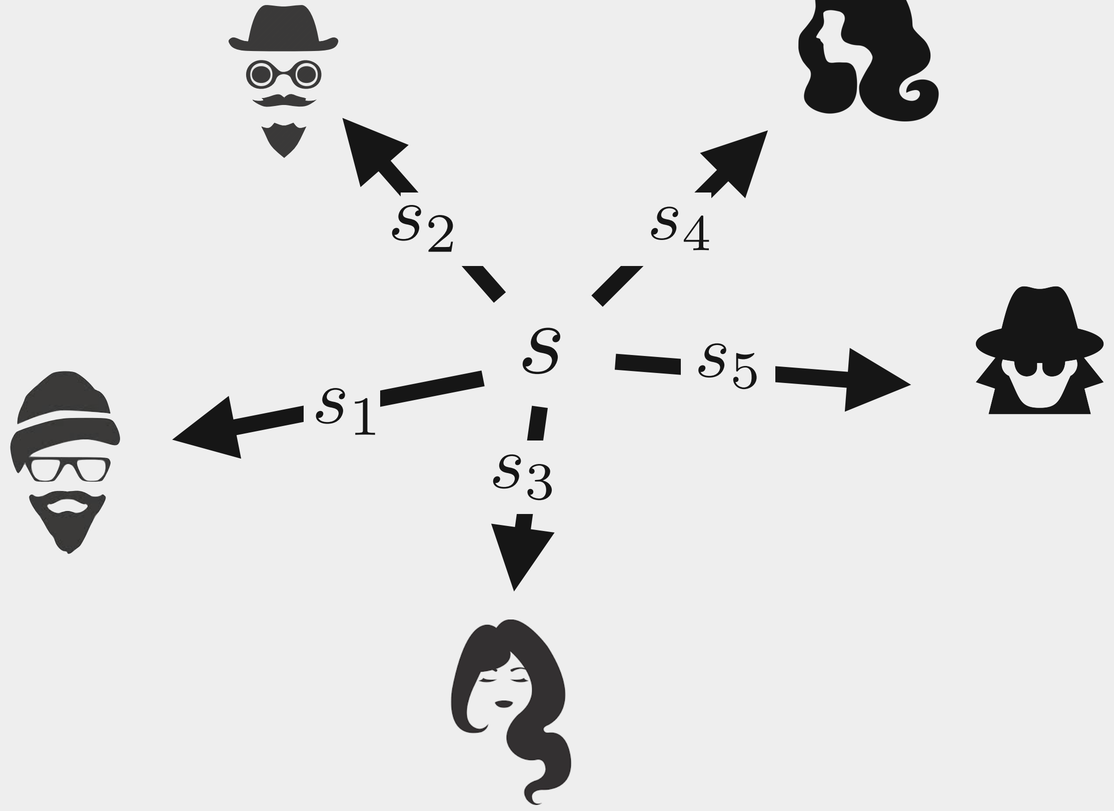

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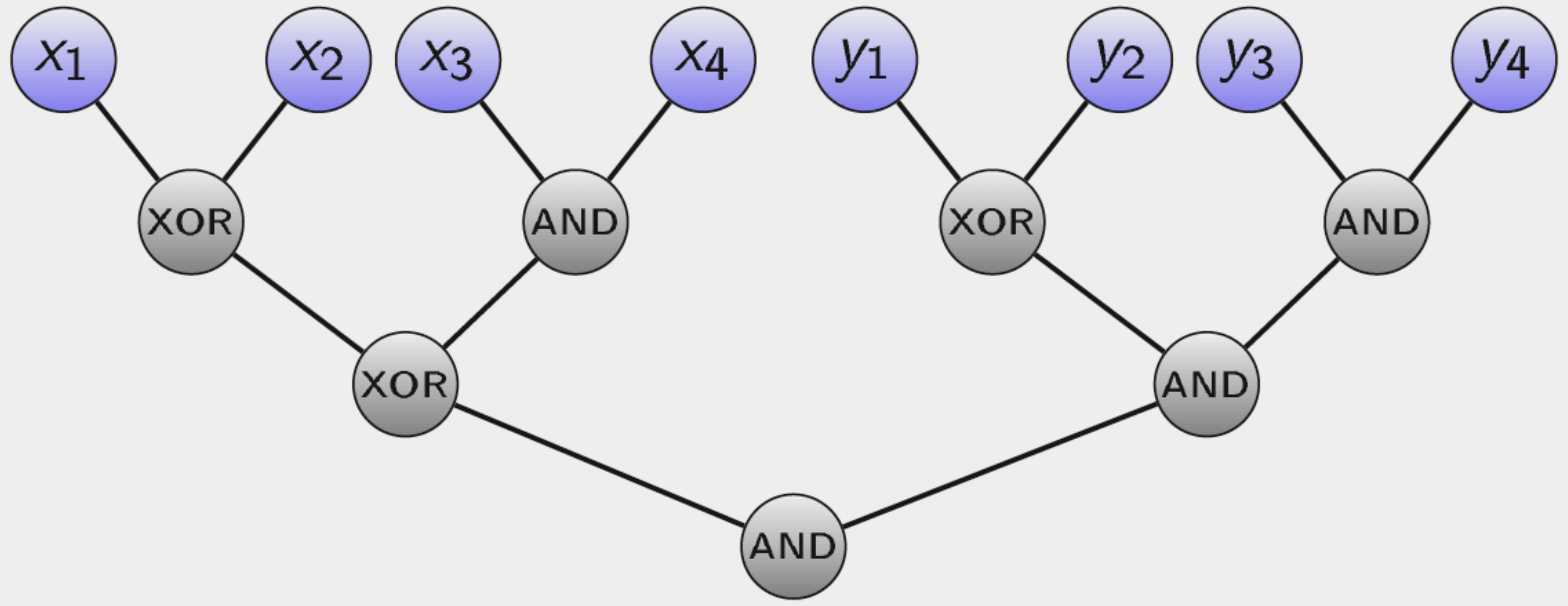


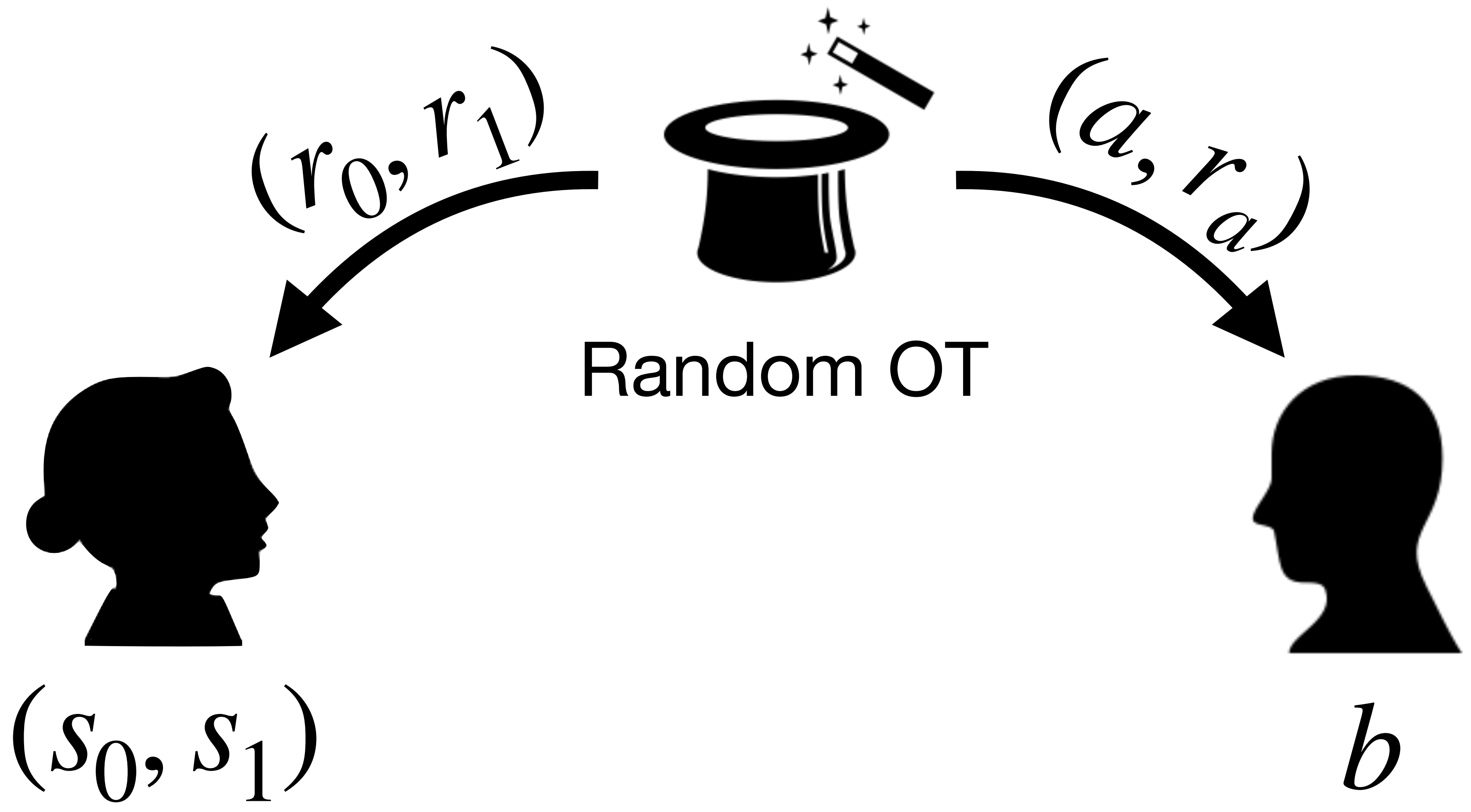
3. Use OT to compute the gates

$$\text{share}(x, y) \implies \text{share}(\text{GATE}(x, y))$$

I'll skip the details for now, but feel free to ask for them!

Secure Computation from Correlated Randomness

Suppose that, for some reason, the parties could already obtain the result of a *random* oblivious transfer prior to the protocol:



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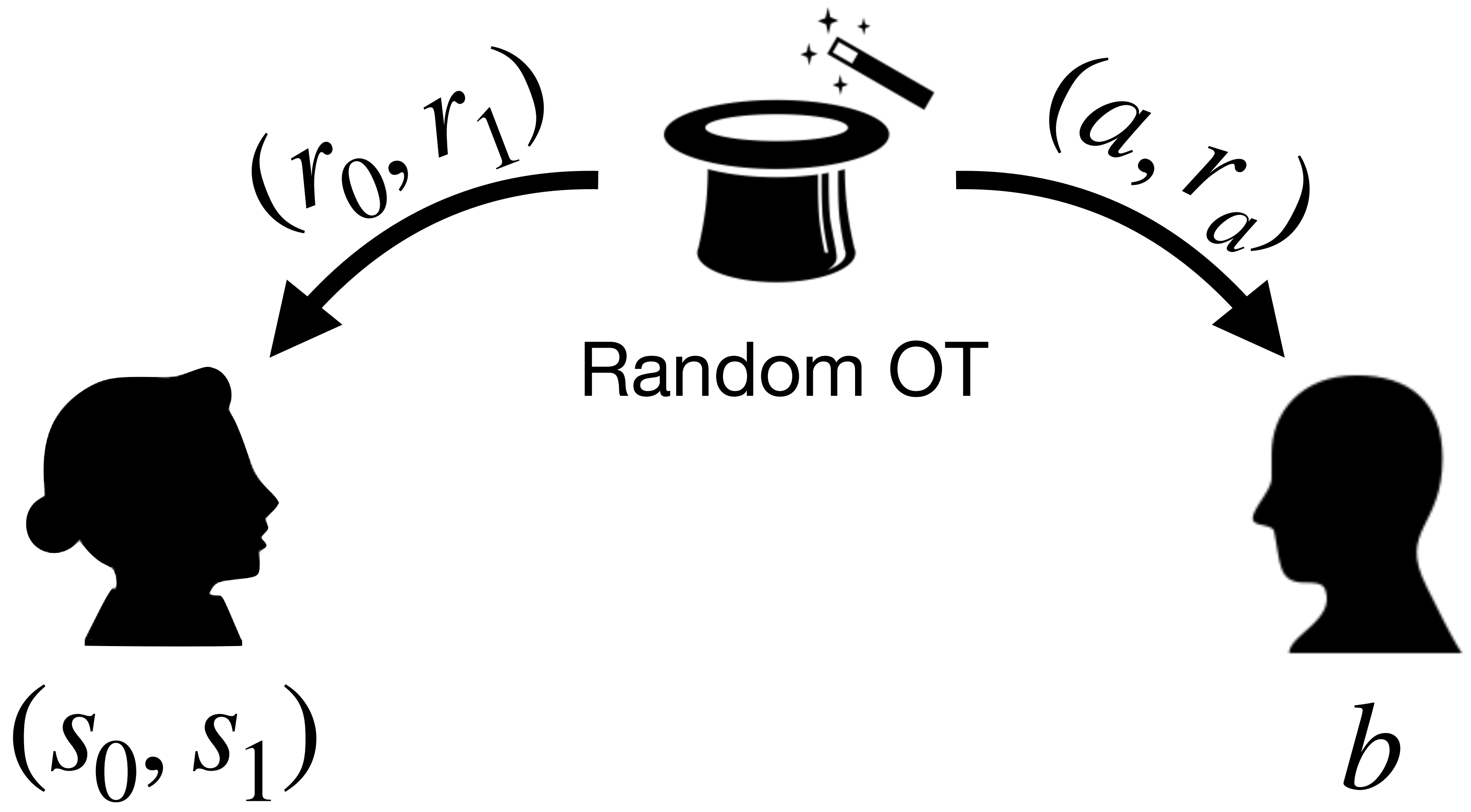

Then the parties can use it to perform an *arbitrary* oblivious transfer!

(Simple) protocol:

- If $a = b$ and Bob gets $(s_0 \oplus r_0, s_1 \oplus r_1)$, he can get $s_b = s_a$, since he knows only $r_b = r_a$.
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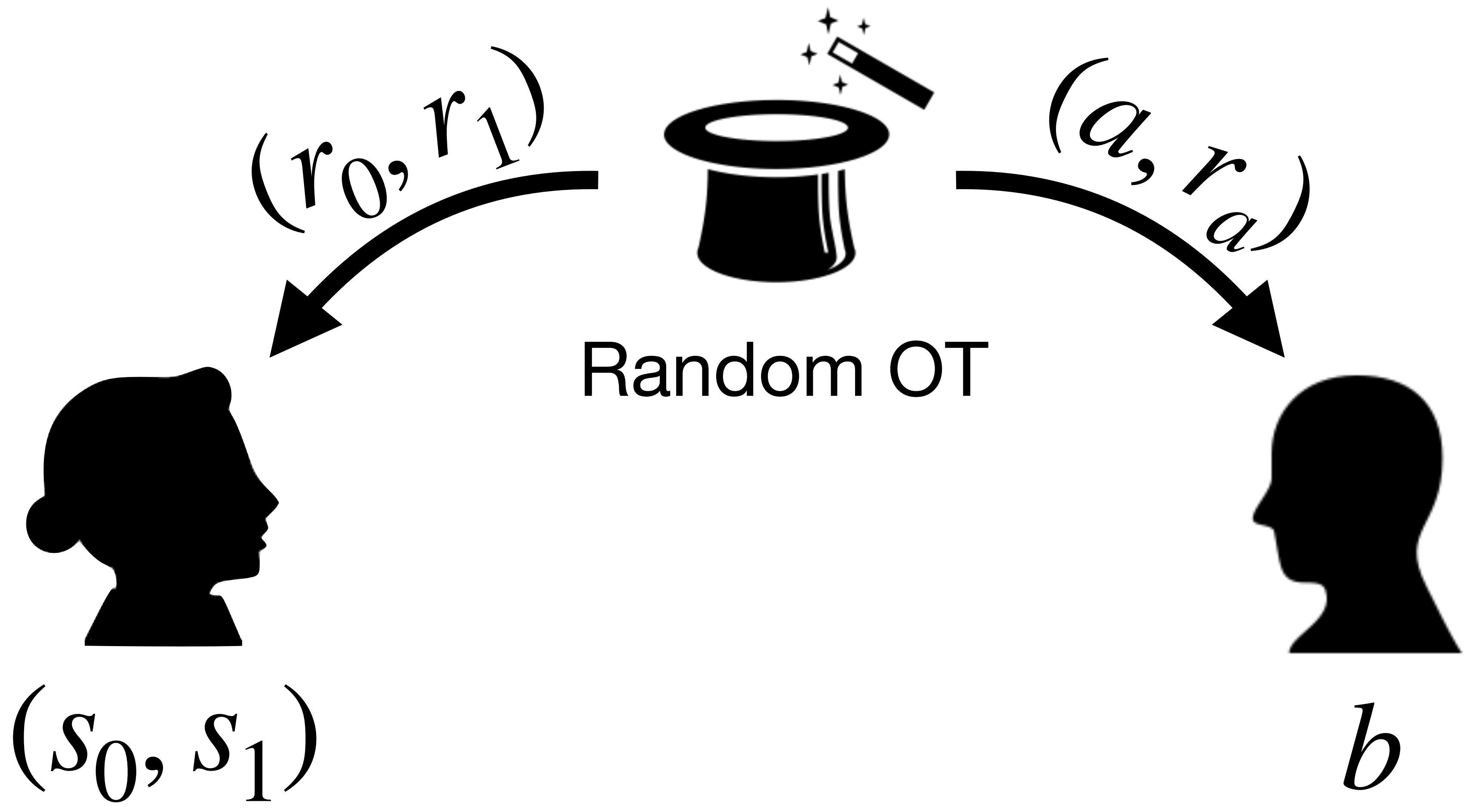

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Given *many* random OTs, one can compute an arbitrary function! The protocol is

- Information-theoretically secure
- Very fast: only three bits exchanged per OT! (In practice, this means 6 bits / AND gate, and 0 / XOR gate)

This is the *correlated randomness model*: fast, information-theoretically secure computation given access to a (trusted) source of correlated random coins.

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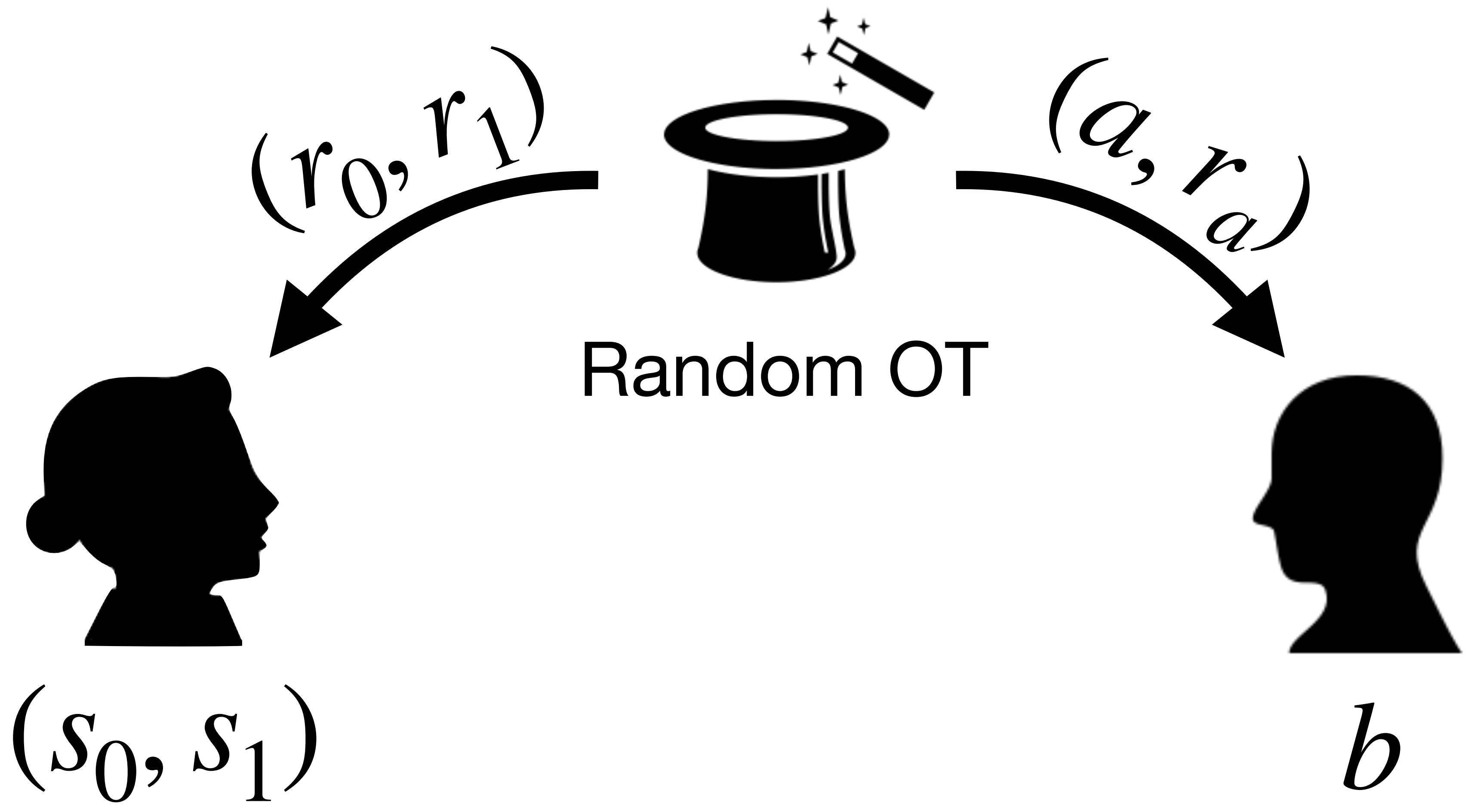

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Can we *efficiently* generate (securely) large amounts of correlated randomness?

Perhaps the most fundamental question in secure computation!

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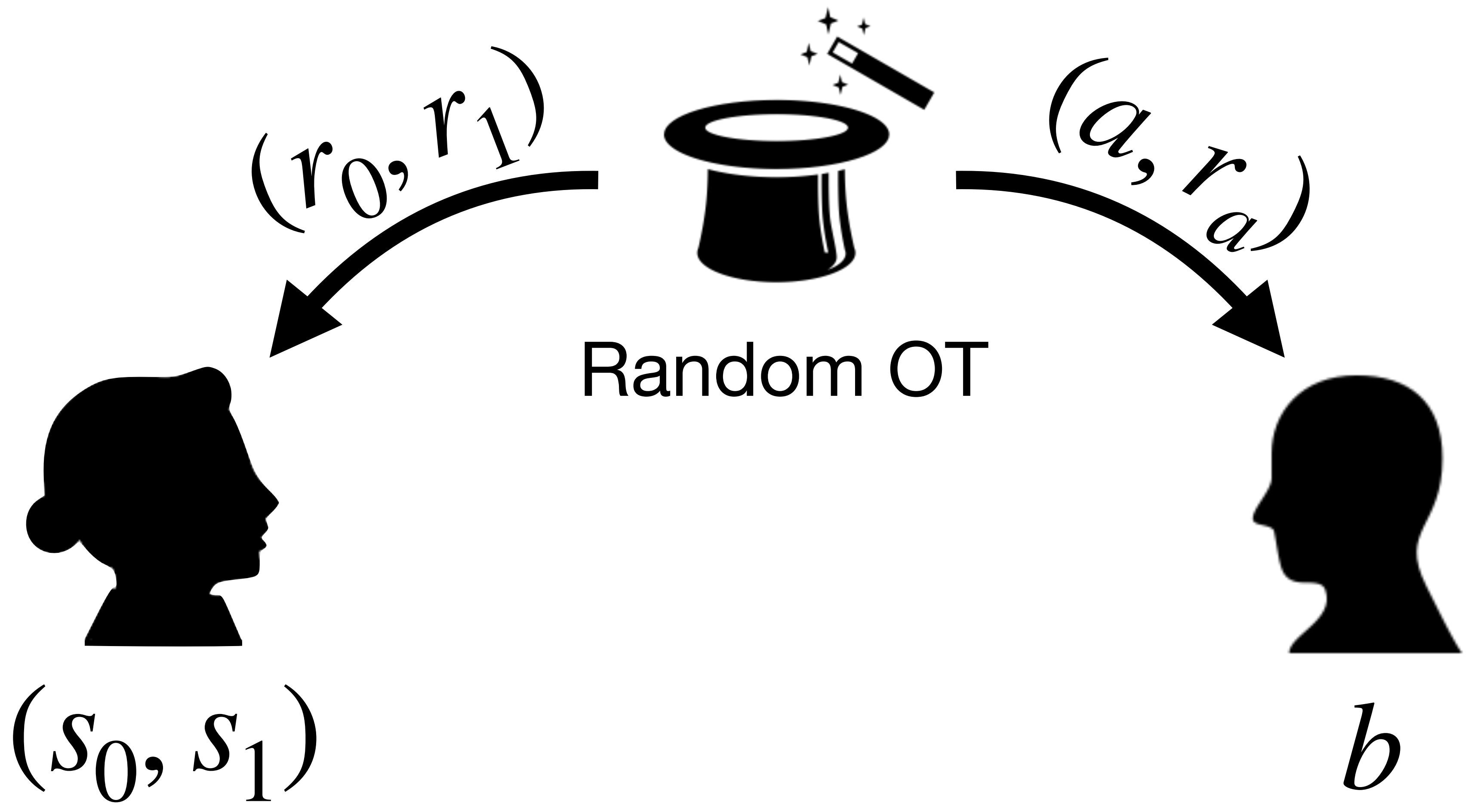

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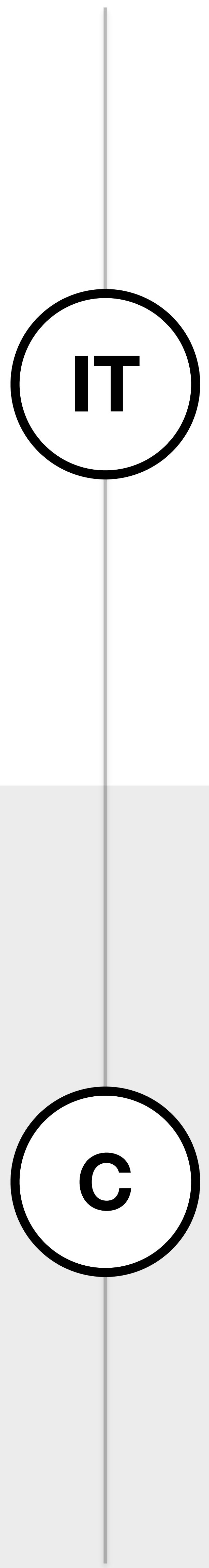
This talk:

Can we *compress* correlated randomness?

Turns out to be just the right way to ask the previous question.

Correlated Randomness in Cryptography

A source of secret *correlated* randomness is an extremely useful resource in secure protocols:



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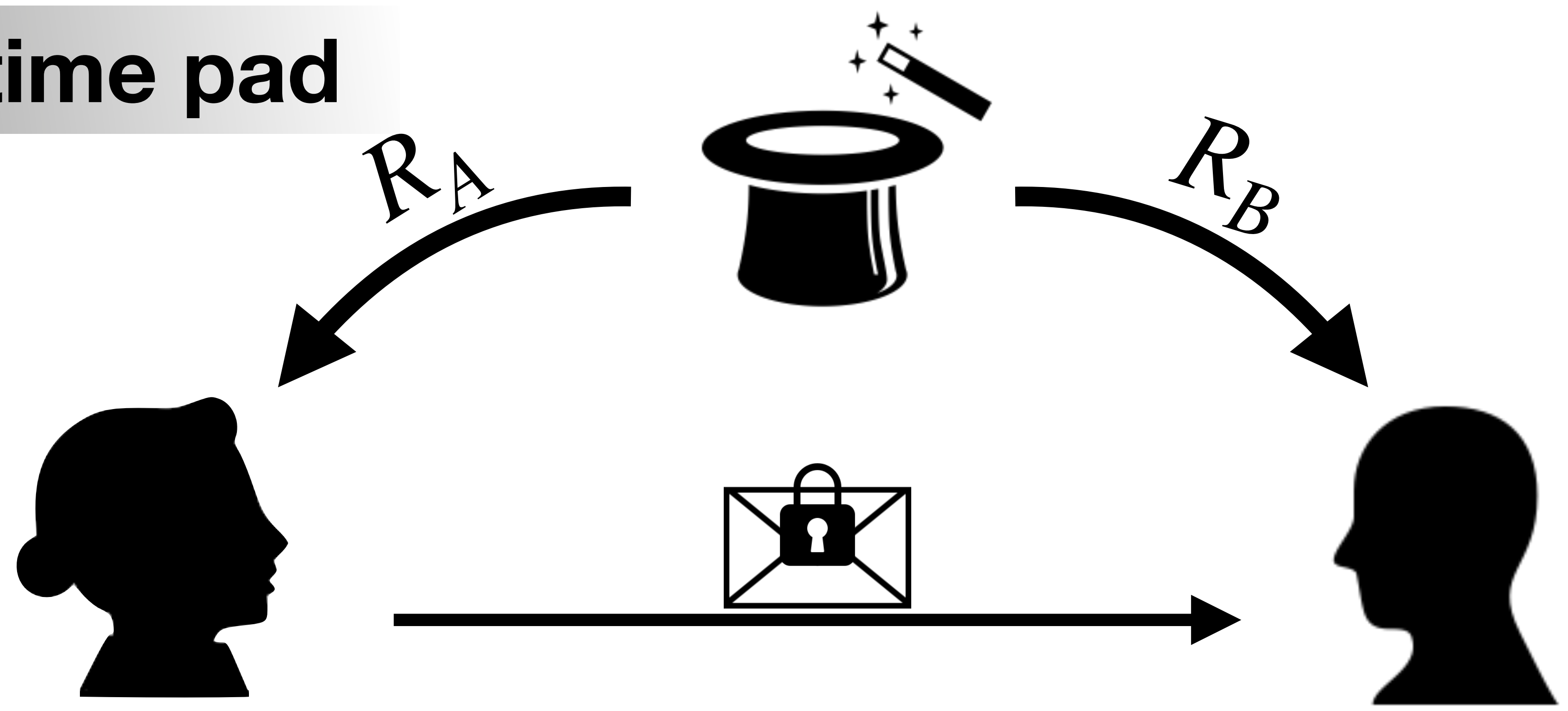
One-time pad



Correlation: $R_A = R_B$

(Equality correlation)

IT

C

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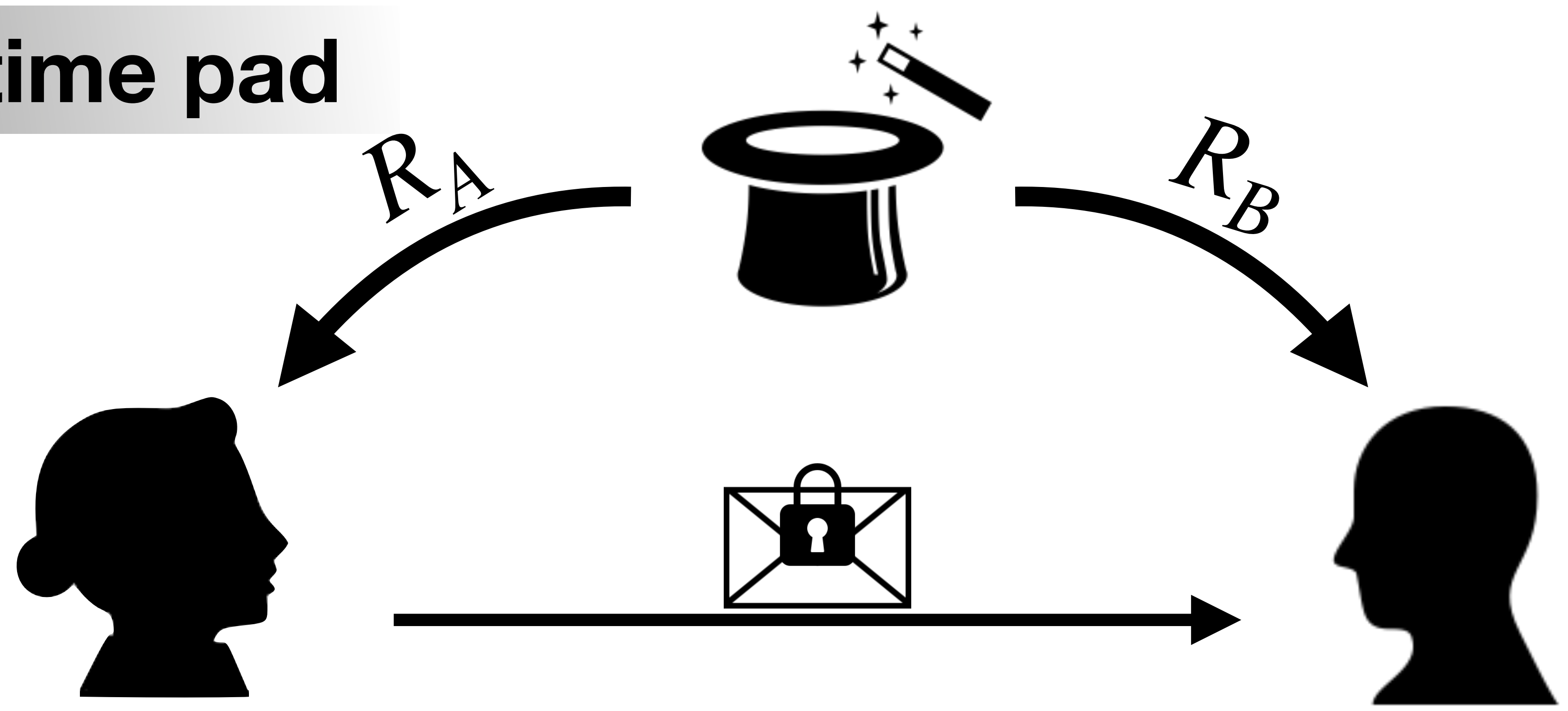

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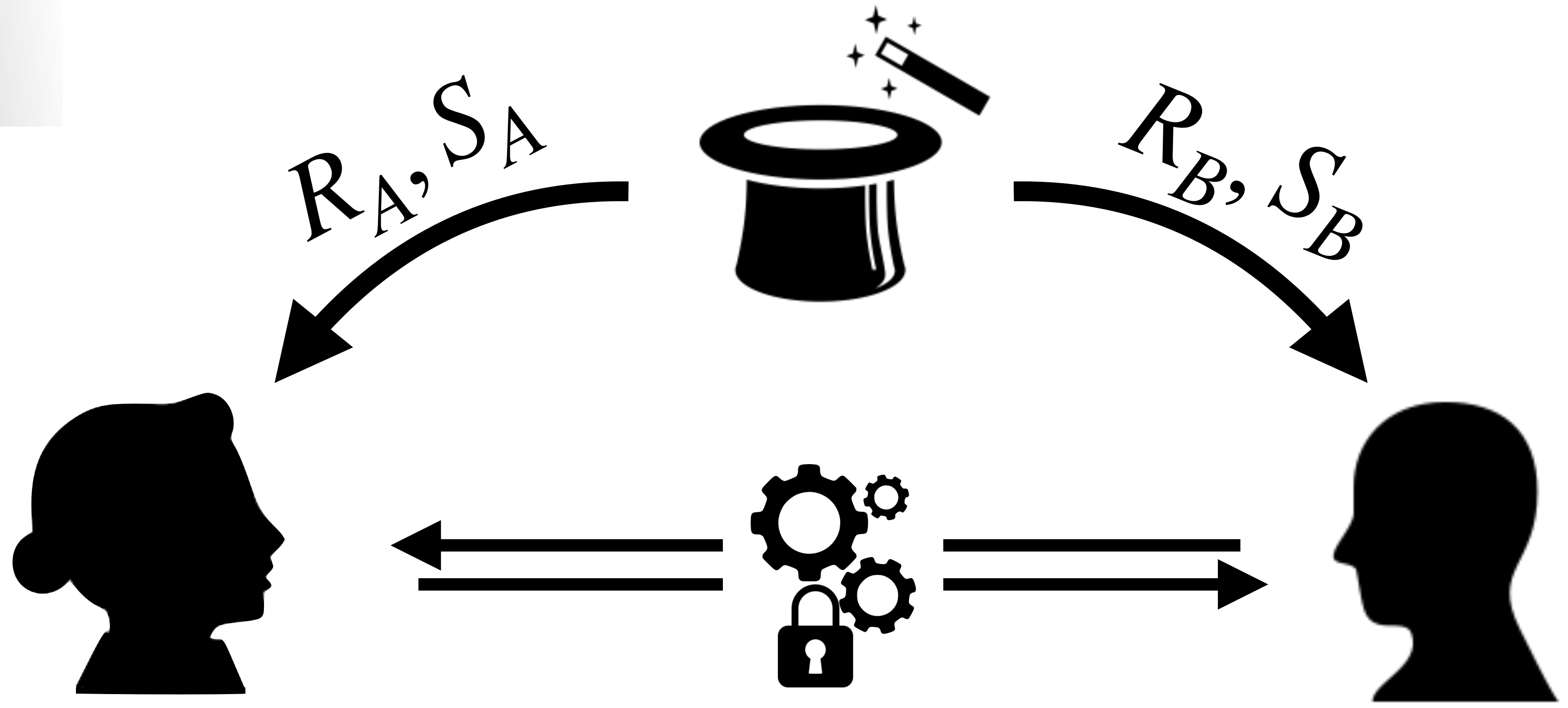

Correlation: $R_A + R_B = S_A \wedge S_B$

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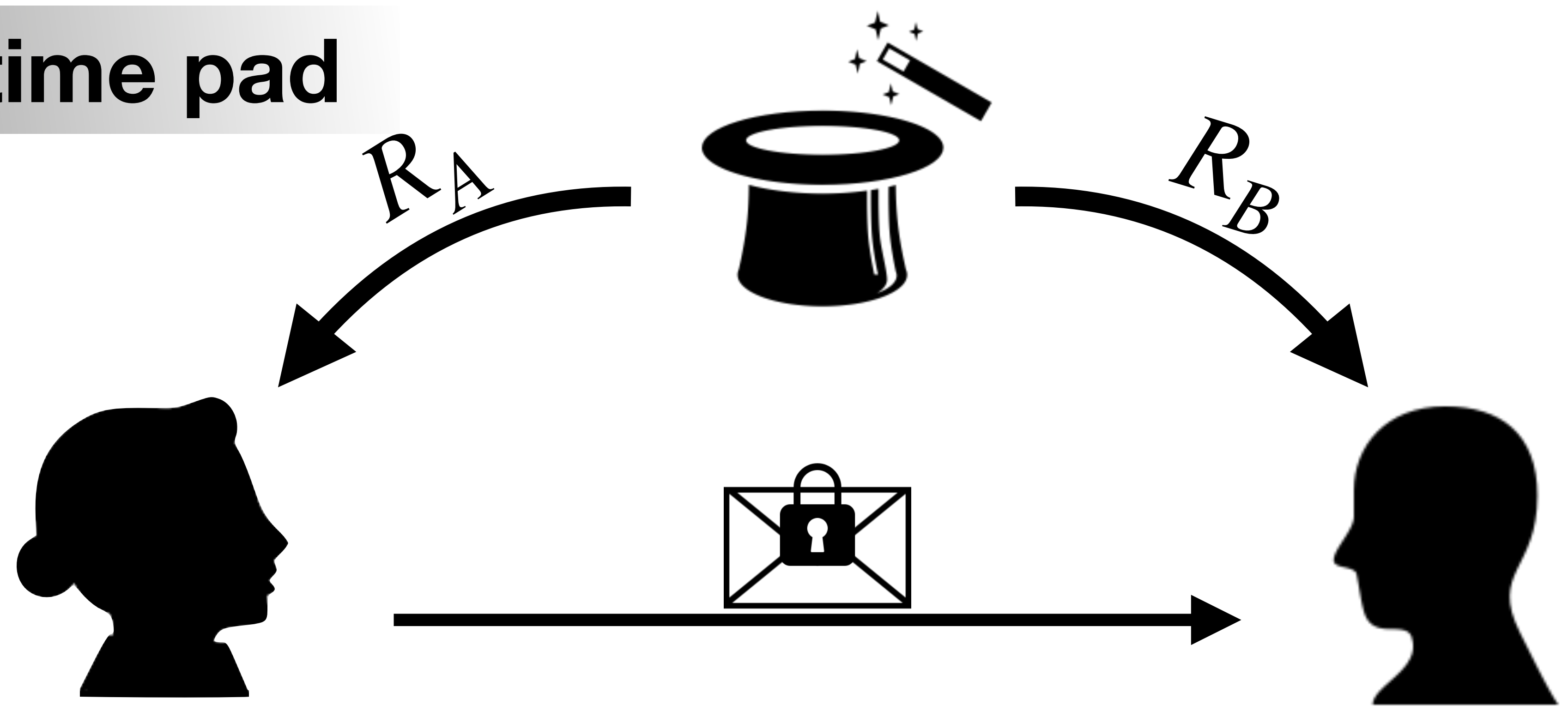

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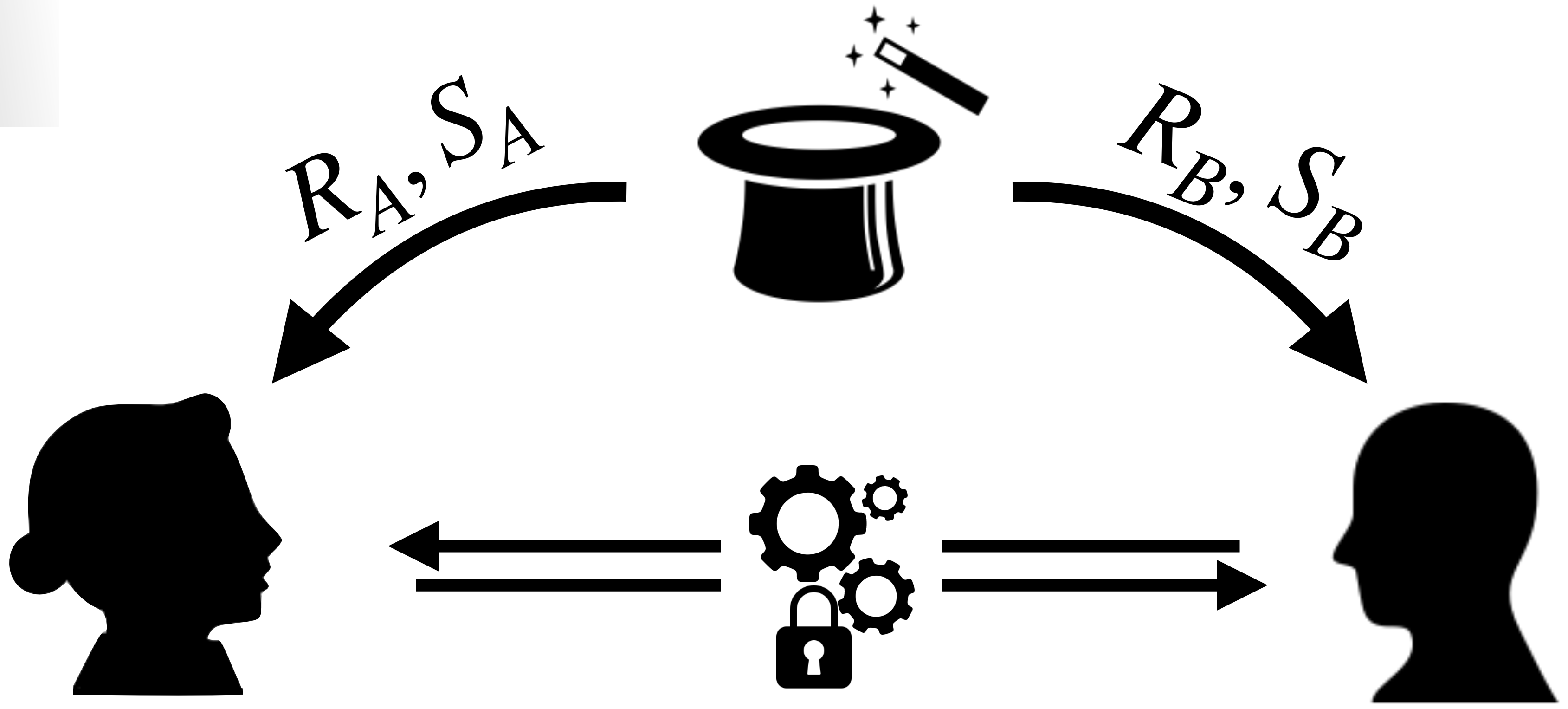

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In the computational world, can we *compress* correlated randomness?

C

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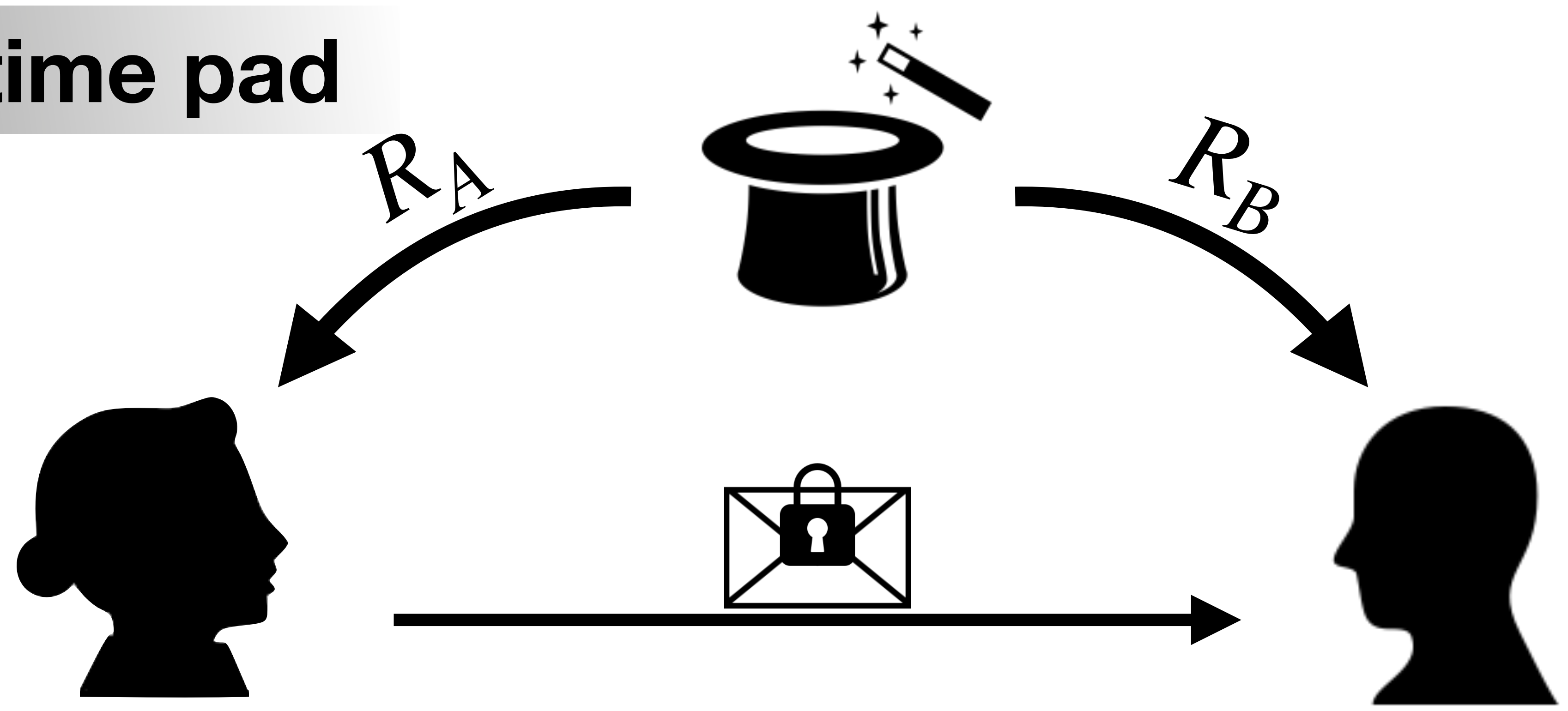

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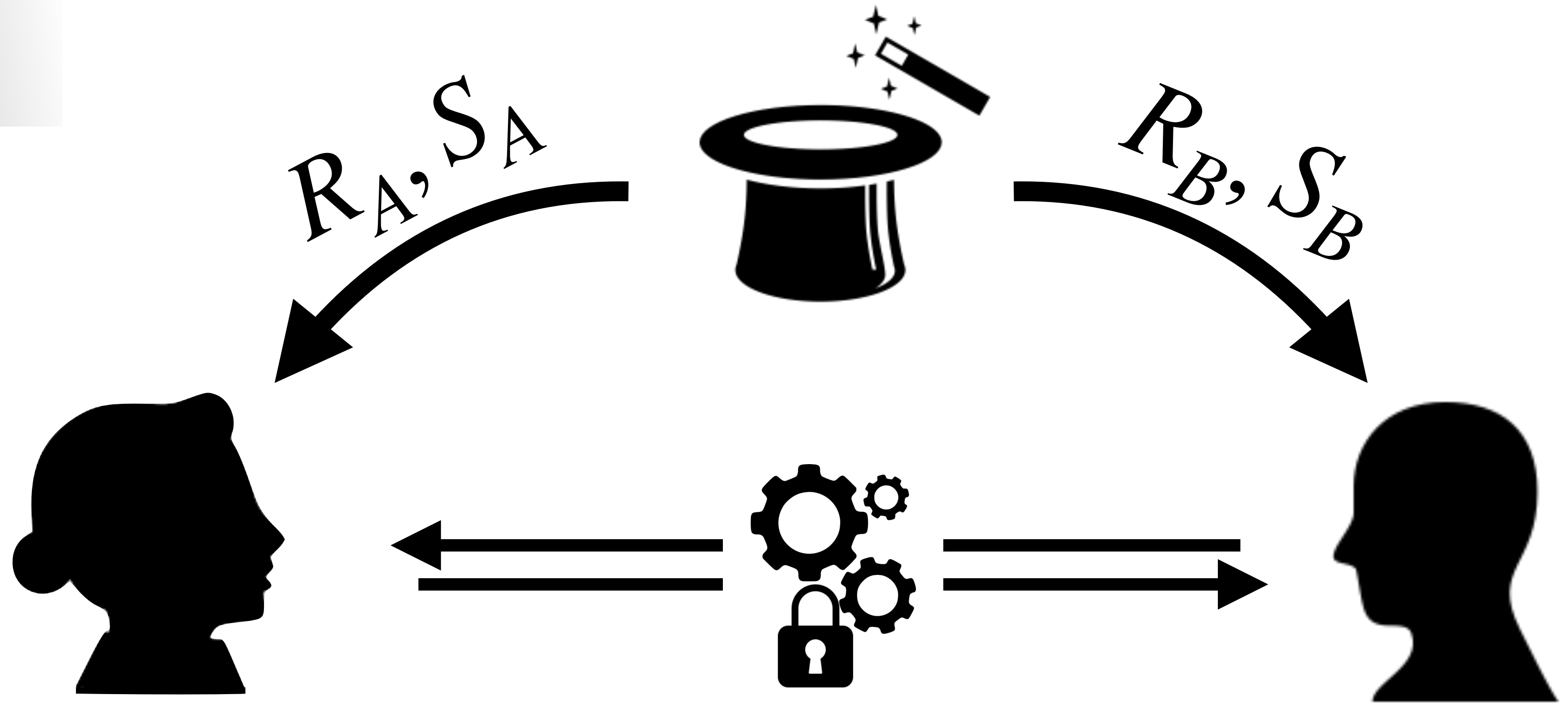

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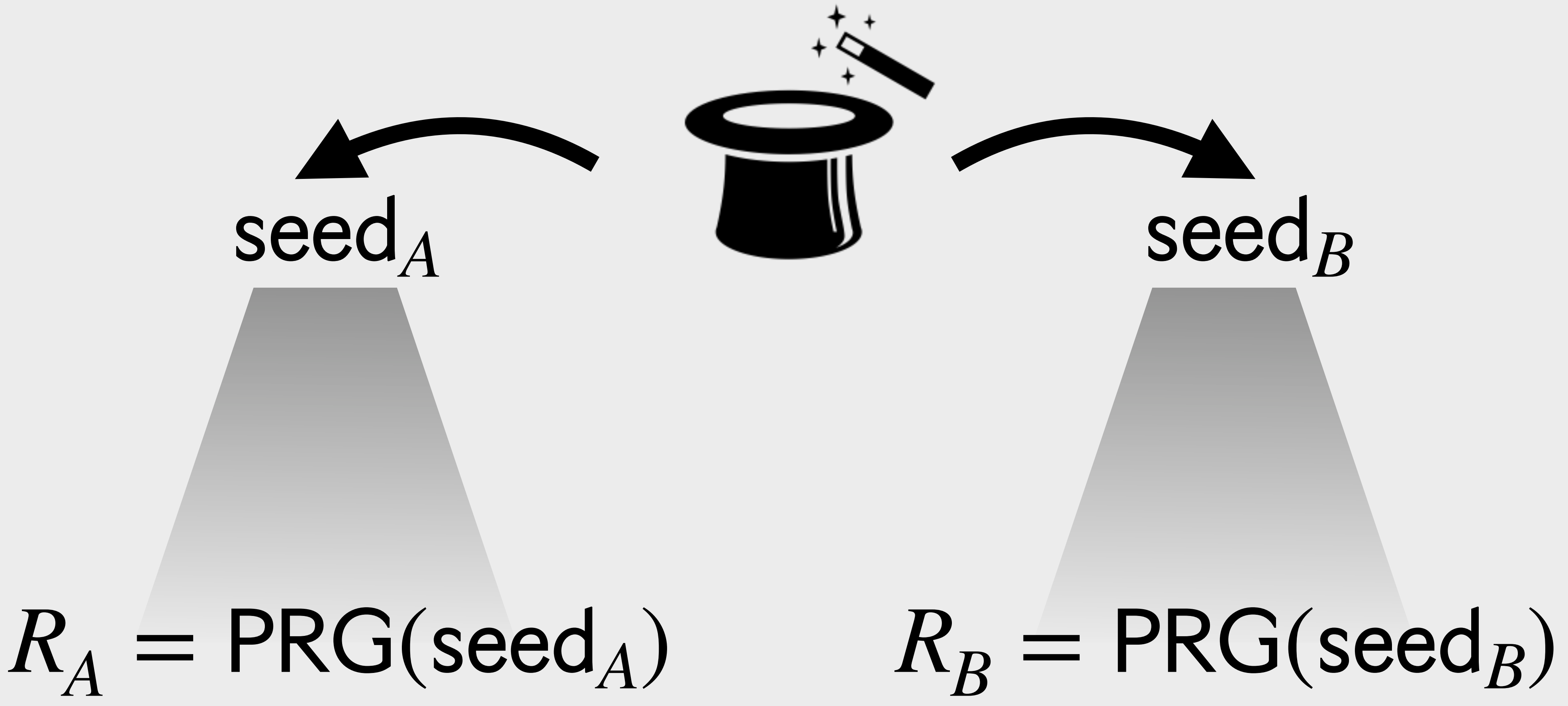
Equality correlations can be *compressed* using a PRG:



C

Correlated Randomness in Cryptography

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Secure Computation with Silent Preprocessing

Pseudorandom correlation generator: $\mathsf{Gen}(1^\lambda) \rightarrow (\mathsf{seed}_A, \mathsf{seed}_B)$ such that **(1)** $(\mathsf{Expand}(A, \mathsf{seed}_A), \mathsf{Expand}(B, \mathsf{seed}_B))$ looks like n samples from the target correlation, and **(2)** $\mathsf{Expand}(A, \mathsf{seed}_A)$ looks 'random conditioned on satisfying the correlation with $\mathsf{Expand}(B, \mathsf{seed}_B)$' to Bob (similar property w.r.t. Alice).

Preprocessing phase

Online phase

Secure Computation with Silent Preprocessing

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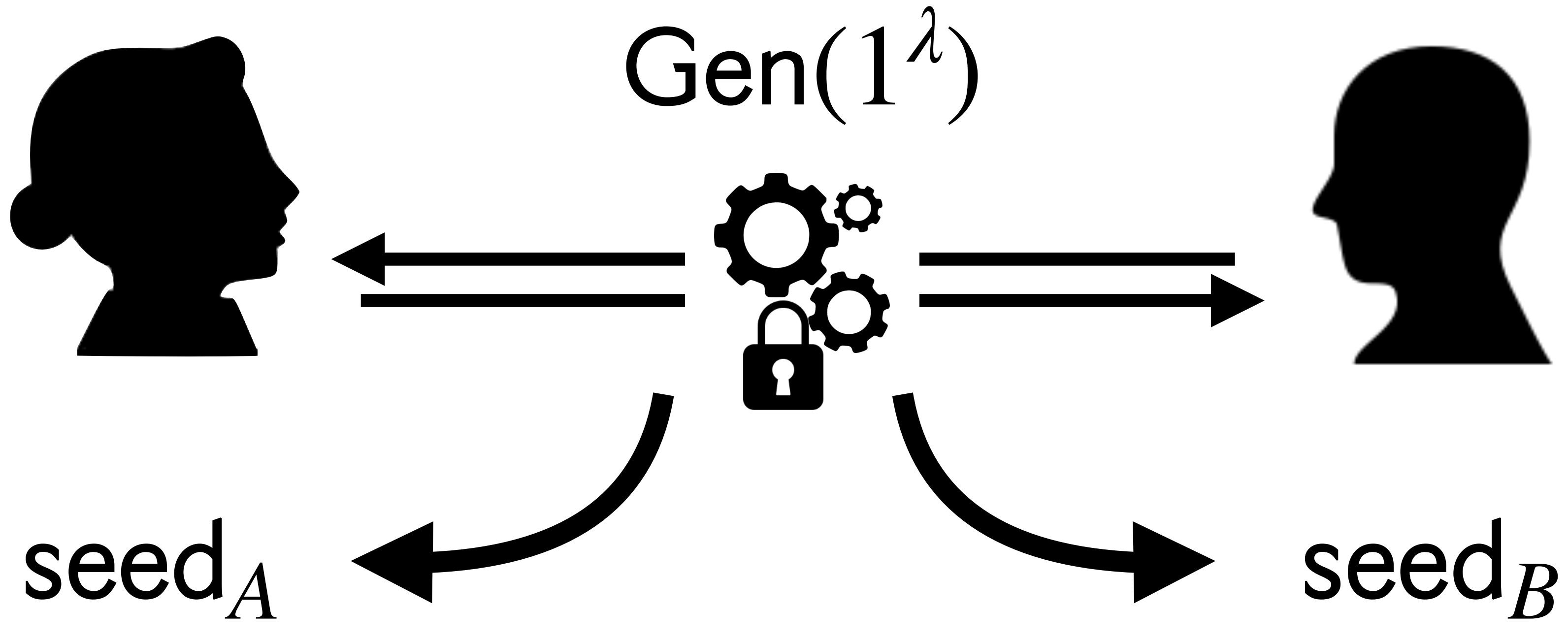
One-time short interaction



Interactive protocol with short communication and computation; Alice and Bob store a small seed afterwards.

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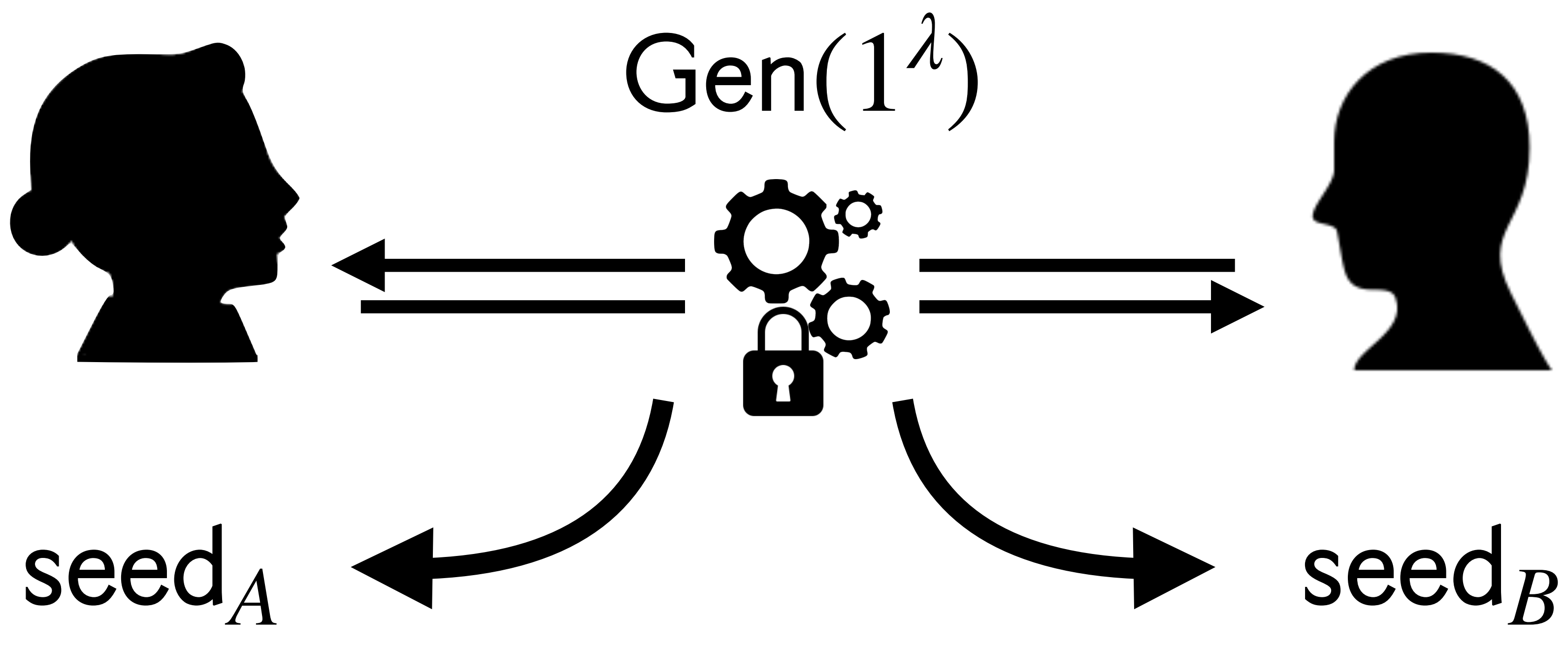

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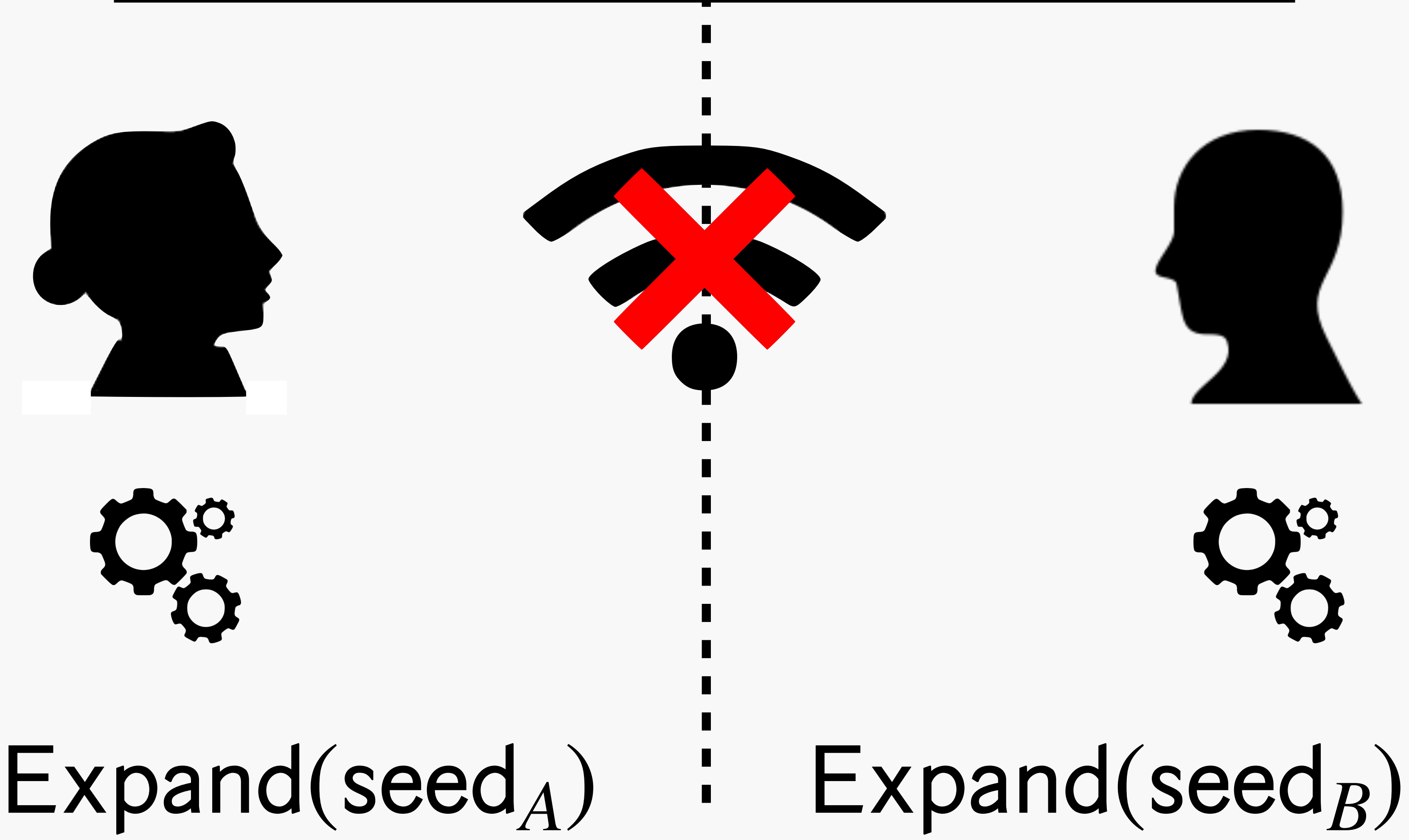

The bulk of the preprocessing phase is offline: Alice and Bob stretch their seeds into large pseudorandom correlated strings.

Preprocessing phase

Online phase

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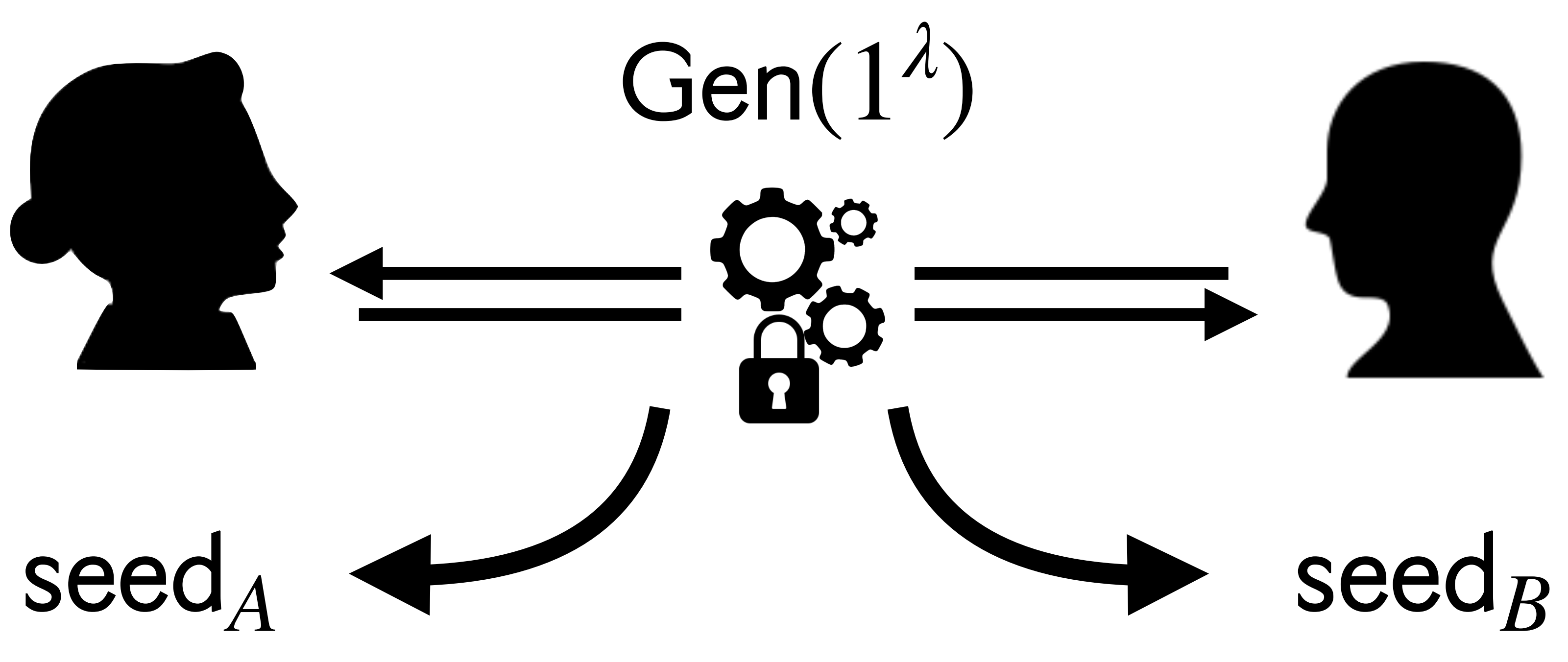

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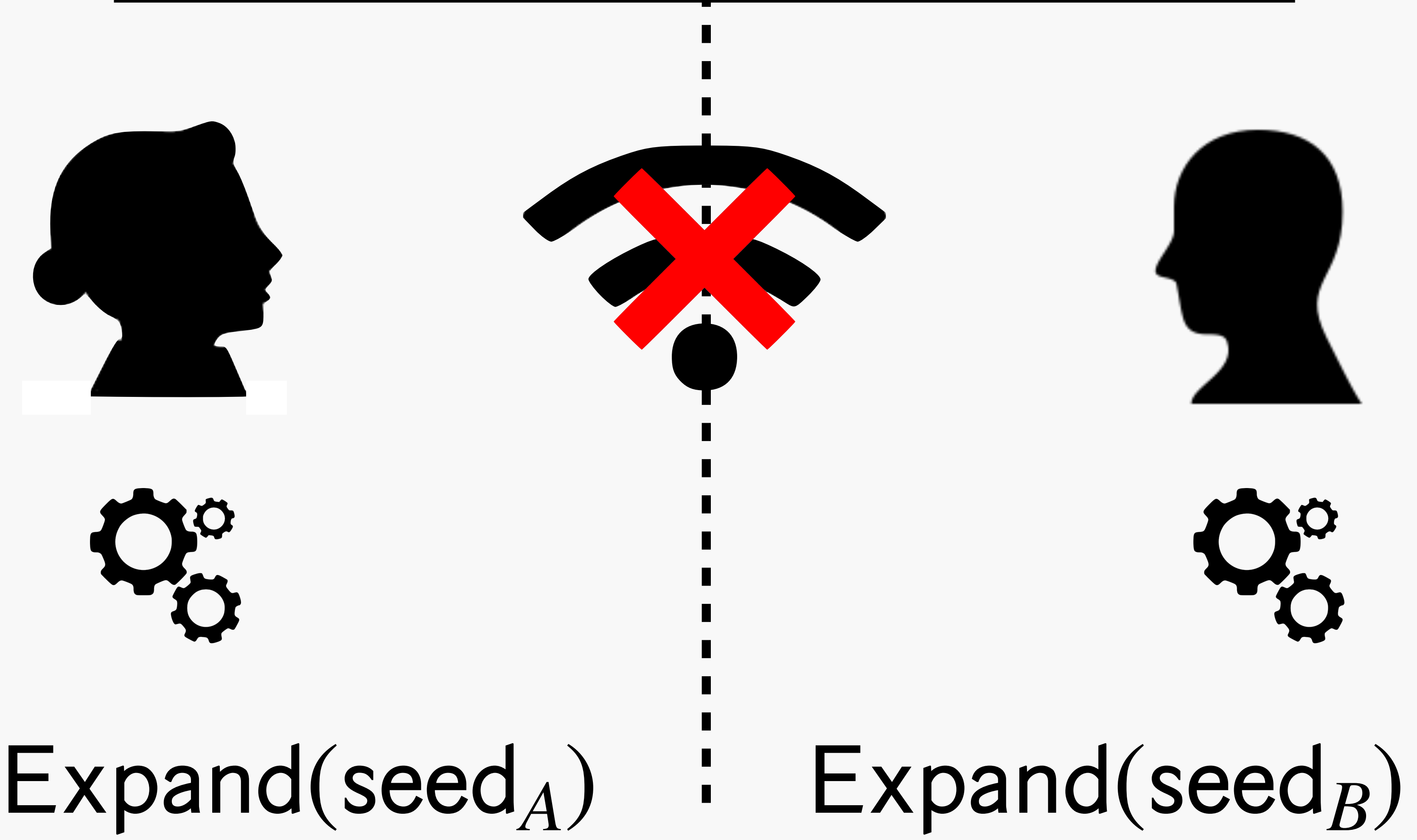

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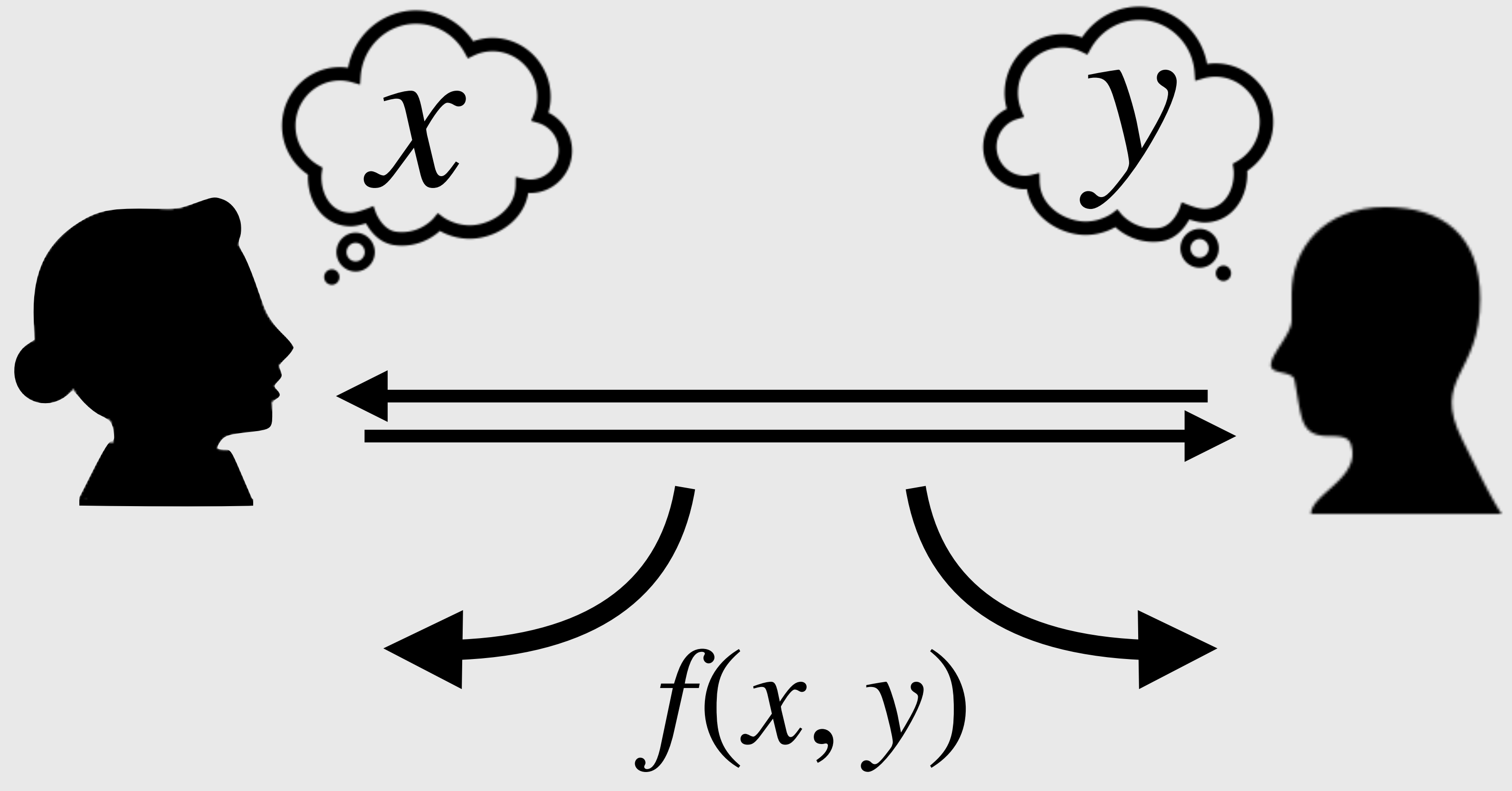
Preprocessing phase

Non-cryptographic

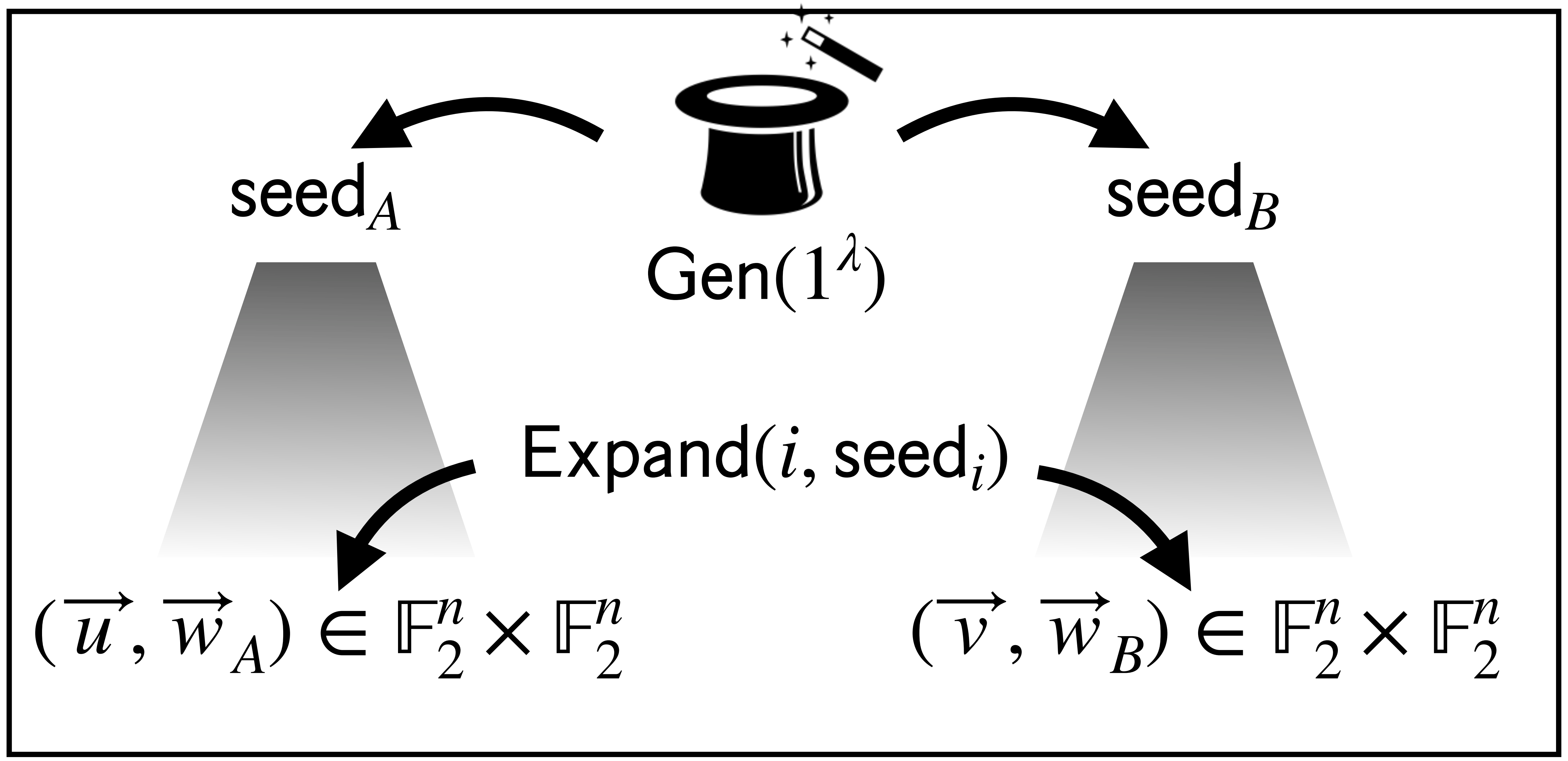


Alice and Bob consume the preprocessing material in a fast, non-cryptographic online phase.

Online phase

Pseudorandom Correlation Generators - Walkthrough

A quick reminder of what we want: Gen generates *short correlated seeds* which can be *locally expanded* into pseudorandom instances of a target correlation.



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A construction from LPN

0. Rewriting the ‘many OTs correlation’

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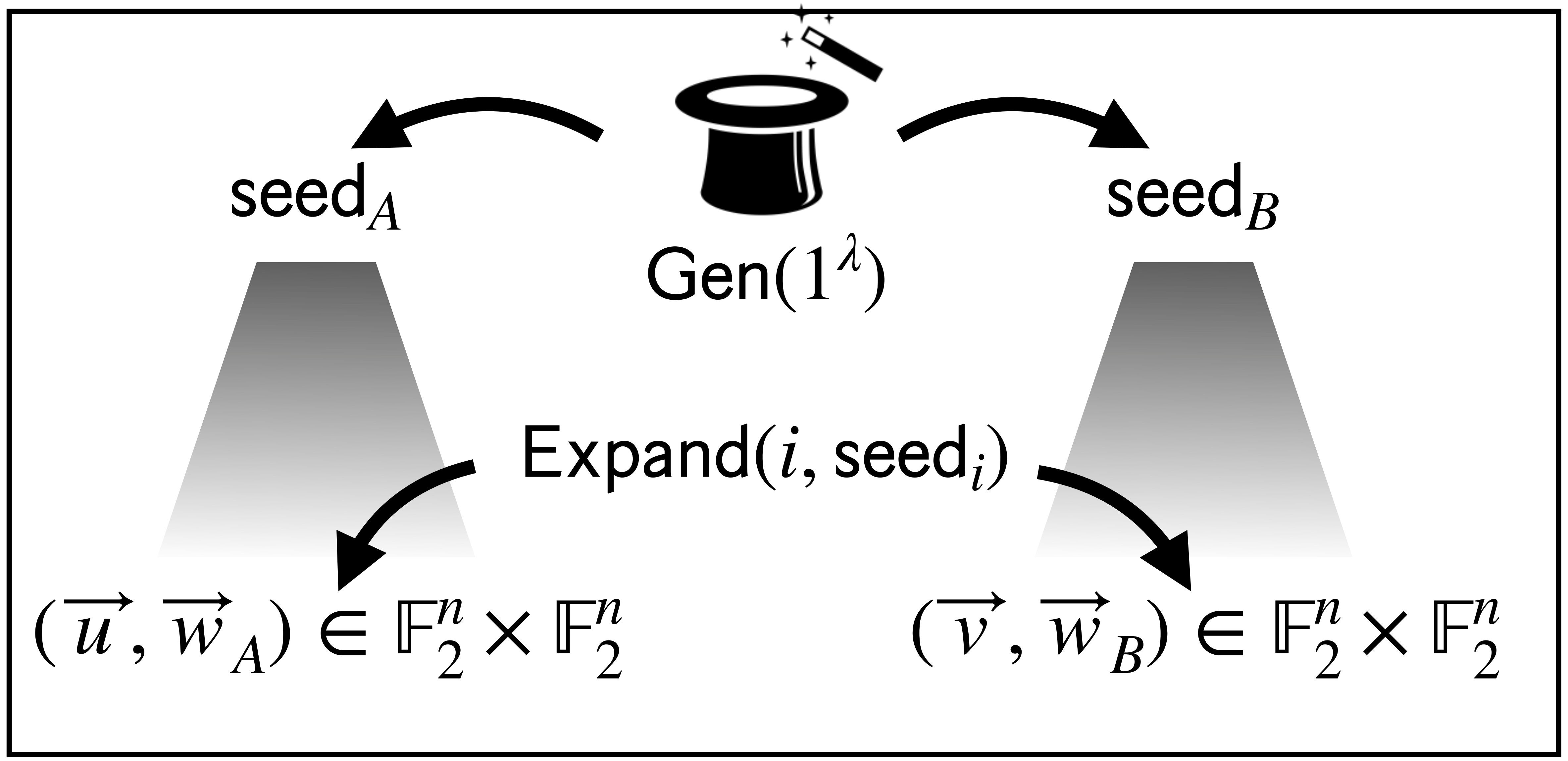

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seed$_A$ Gen(1^λ) seed$_B$

Expand(i, seed_i)

$(\vec{u}, \vec{w}_A) \in \mathbb{F}_2^n \times \mathbb{F}_2^n$ $(\vec{v}, \vec{w}_B) \in \mathbb{F}_2^n \times \mathbb{F}_2^n$

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1. Reduction to subfield-VOLE

[IKNP03]: *subfied vector-OLE* correlation + *correlation-robust hash functions* gives (pseudorandom) OT correlations.

Subfield vector-OLE

$(\vec{u}, \vec{w}_A) \in \mathbb{F}_2^n \times \mathbb{F}_{2^\lambda}^n$ $(x, \vec{w}_B) \in \mathbb{F}_{2^\lambda} \times \mathbb{F}_{2^\lambda}^n$

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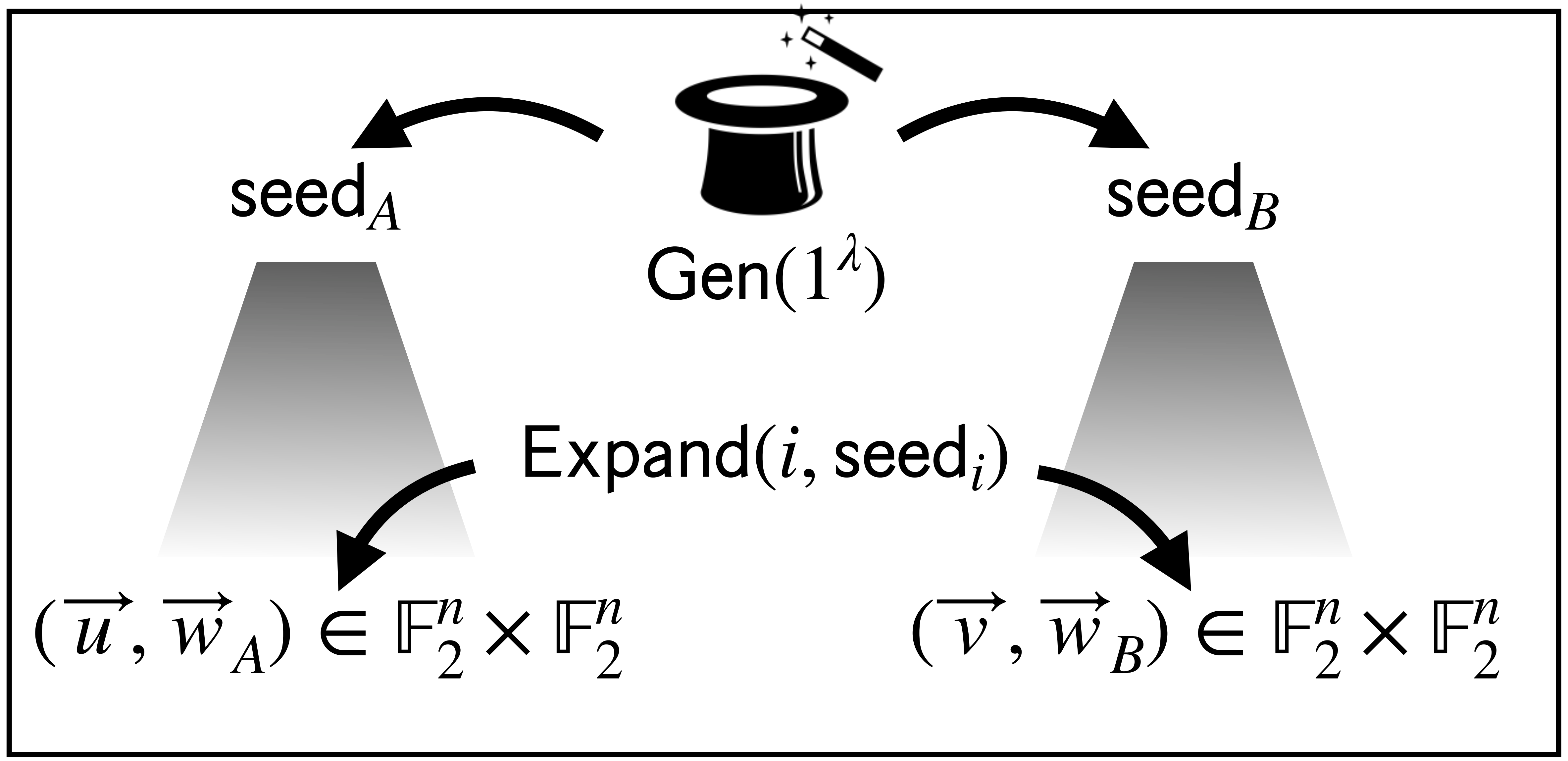

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Intuition. the i-th (string-) OT is:

- $(s_0, s_1) = (H(-w_{B,i}), H(x - w_{B,i}))$
- $(b, s_b) = (u_i, H(w_{A,i}))$

where H is a correlation-robust hash function.

Pseudorandom Correlation Generators - Walkthrough

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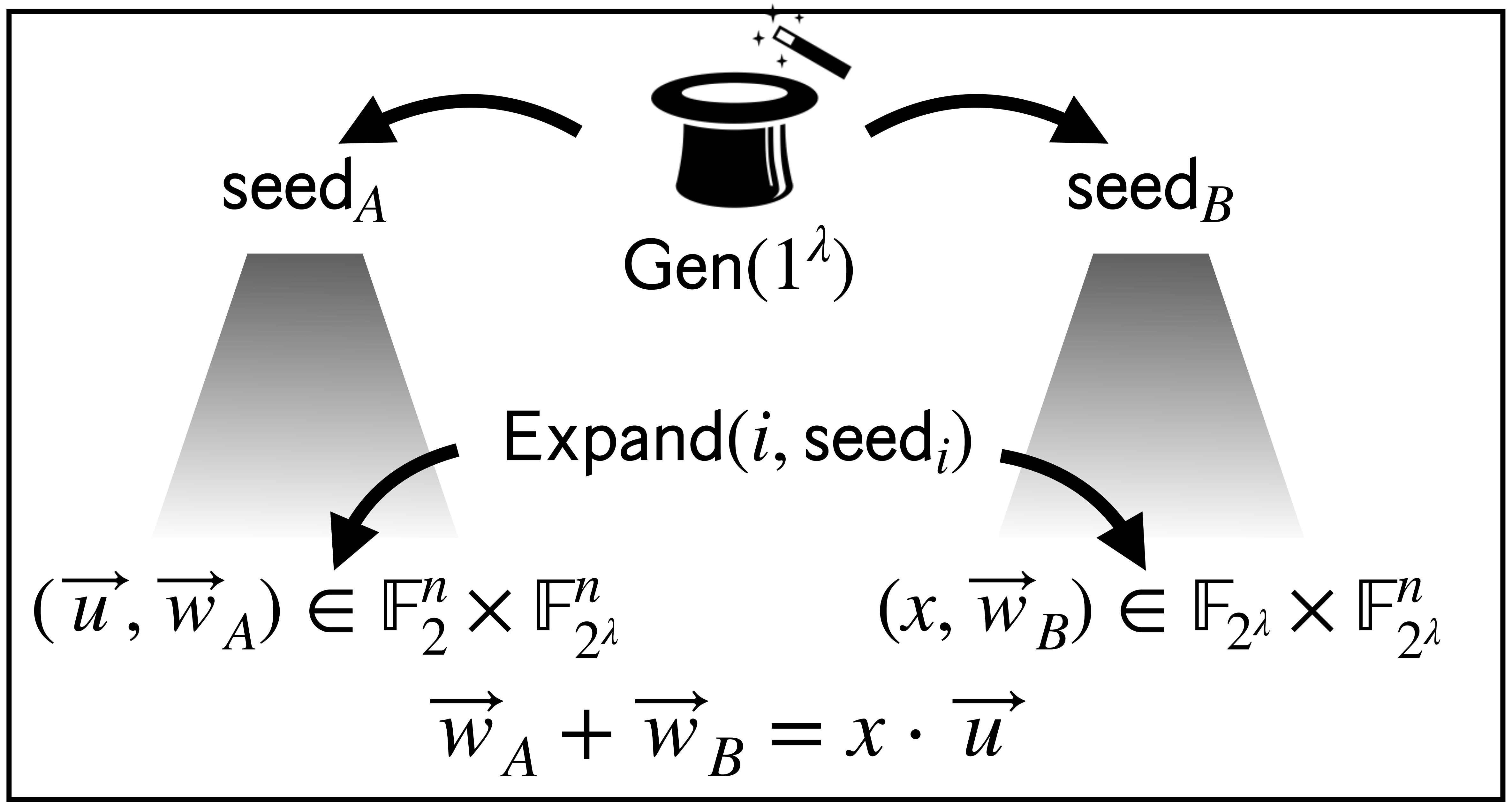

New target

A construction from LPN

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1. Reduction to subfield-VOLE

2. Constructing a PCG for subfield-VOLE

Three steps:

1. Construction for a random *unit vector* $\vec{u}$
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Pseudorandom Correlation Generators - Walkthrough

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$\mathsf{seed}_A = (x,$

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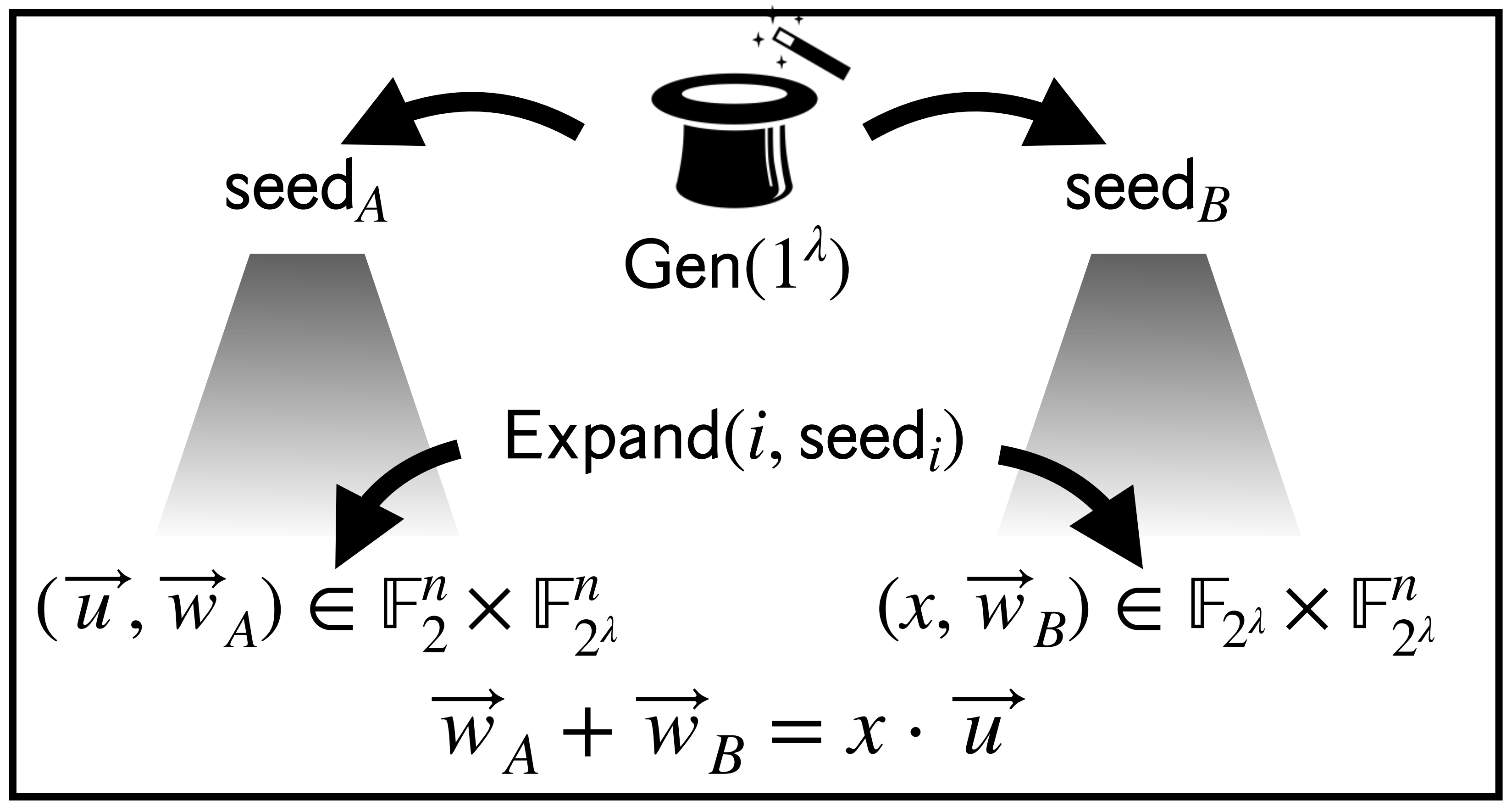

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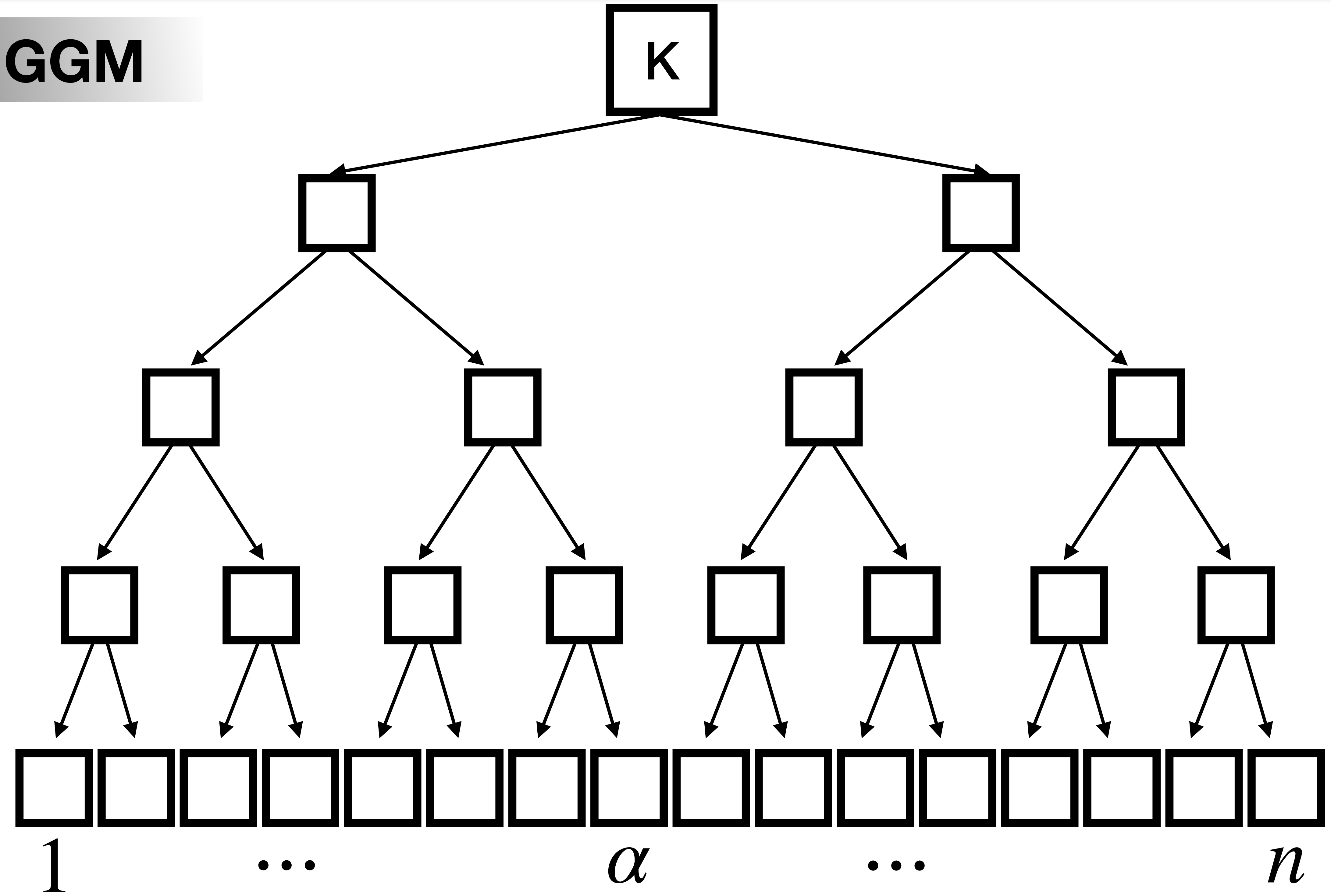
GGM



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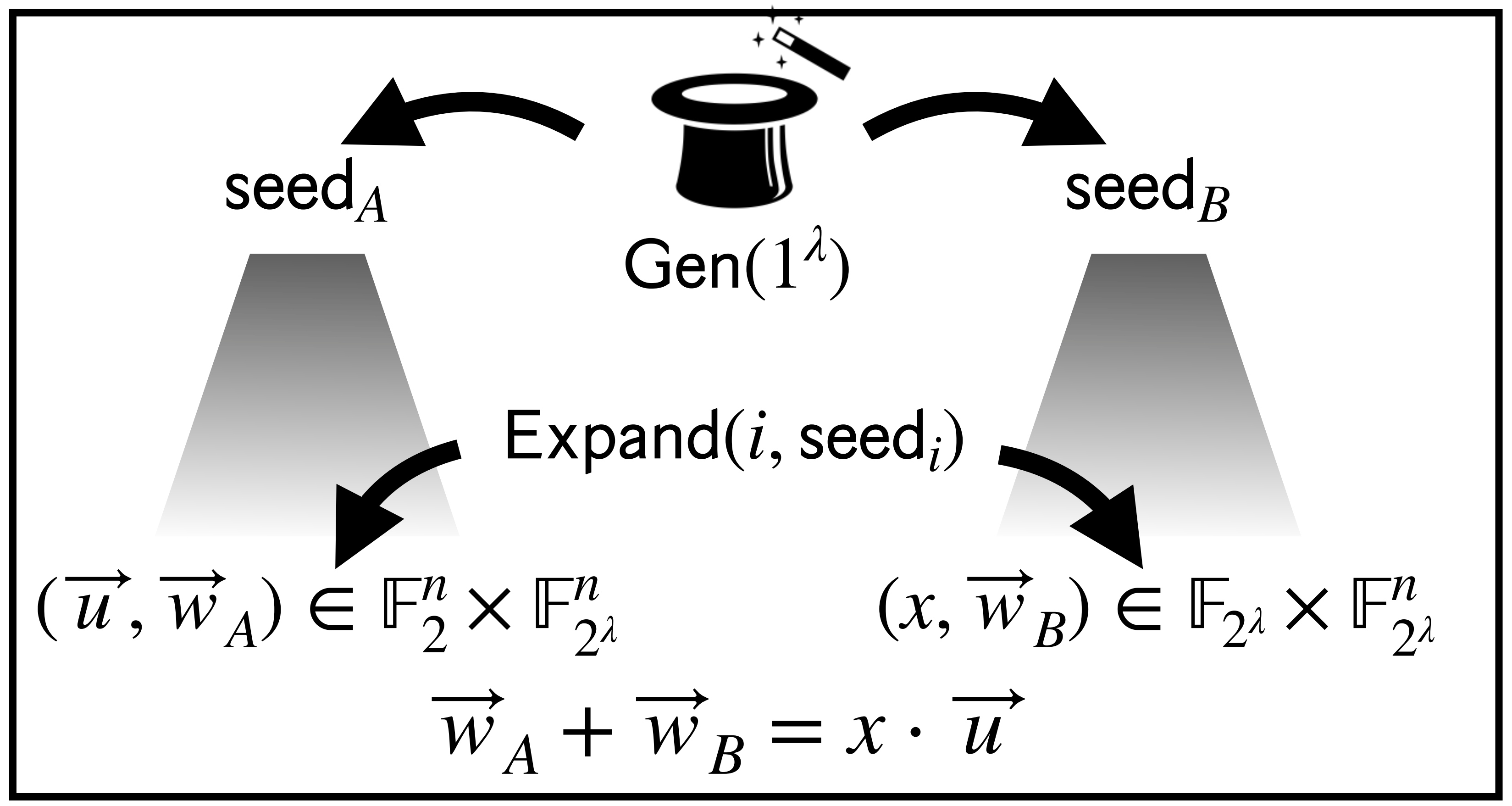

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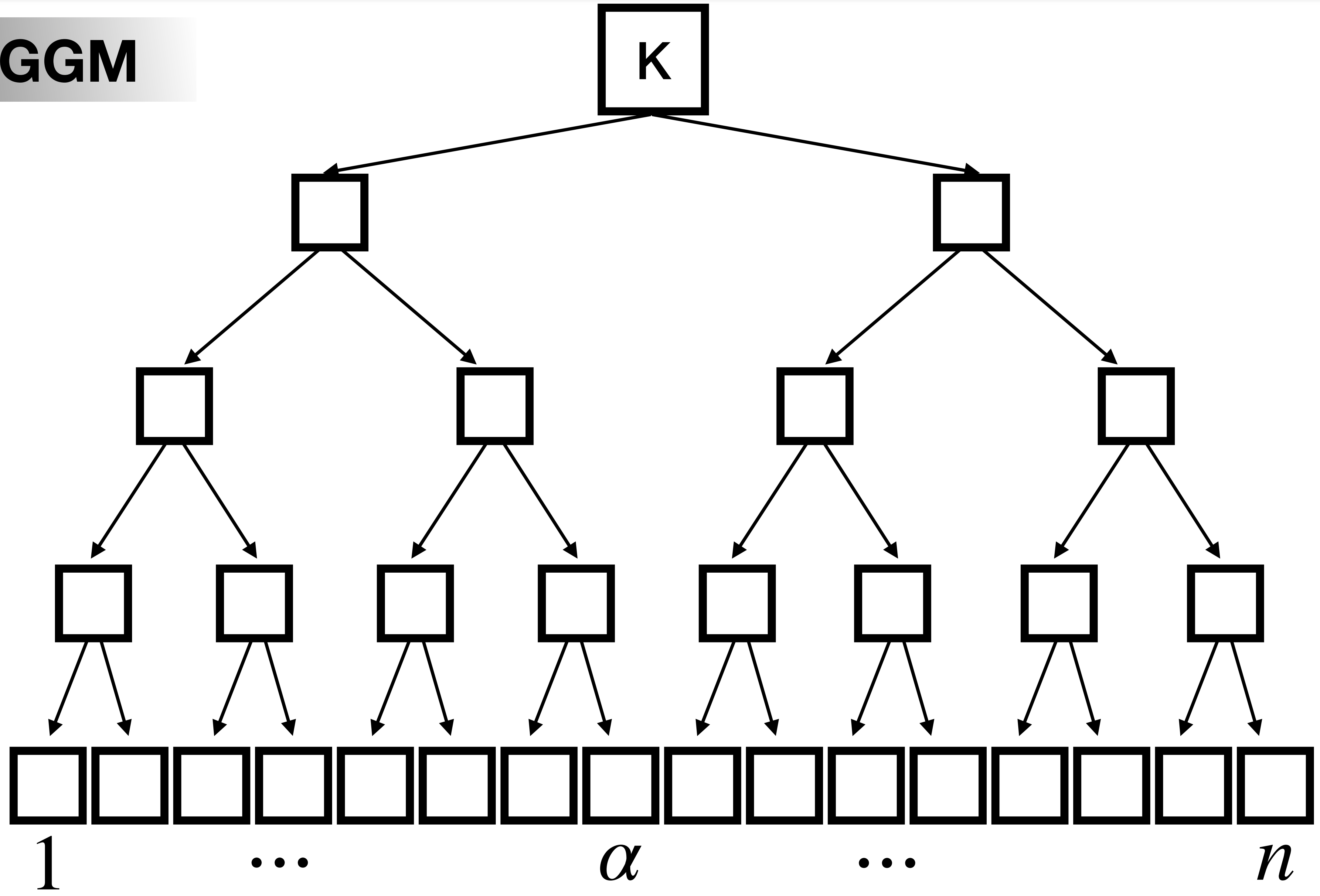

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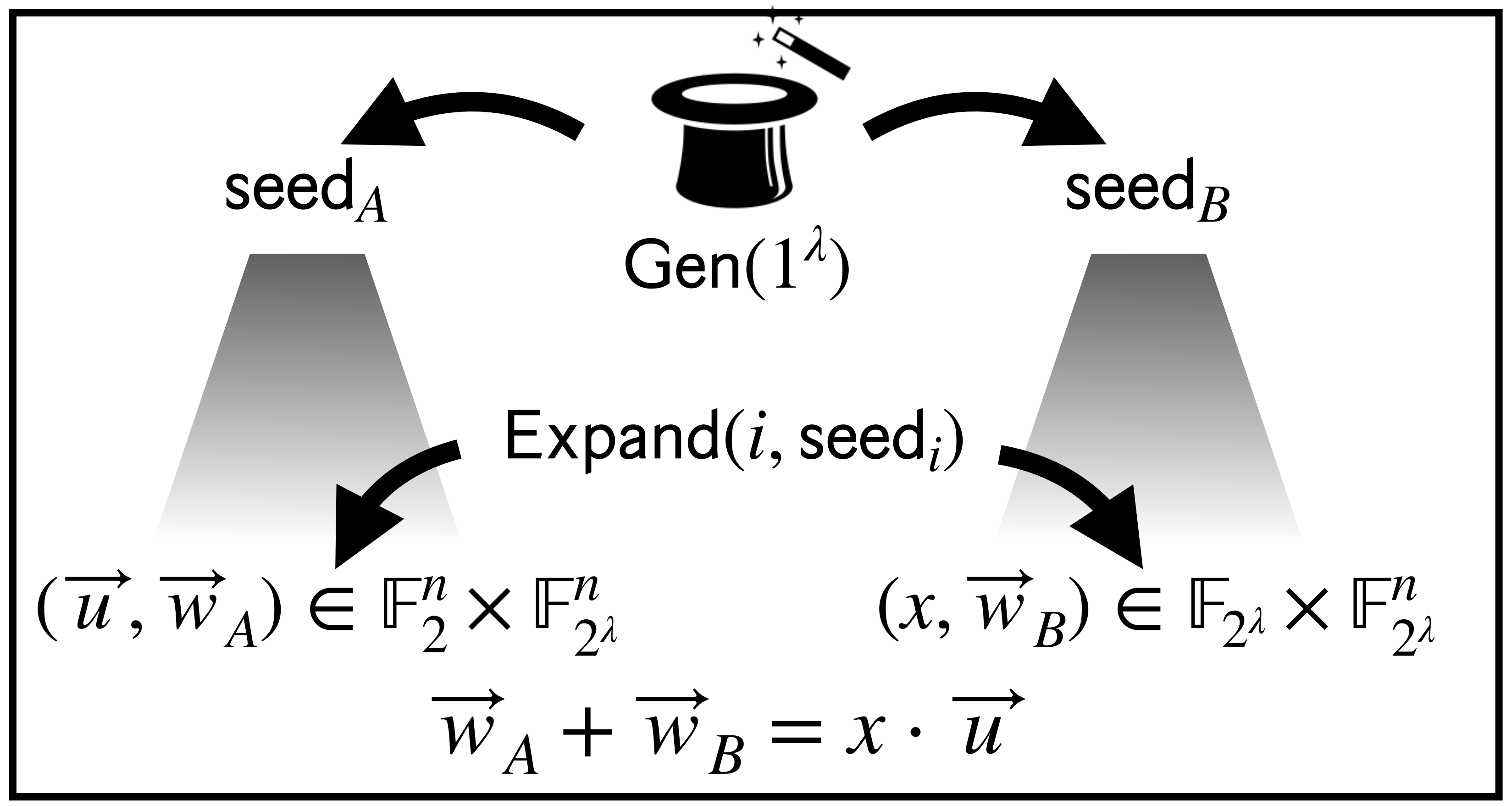

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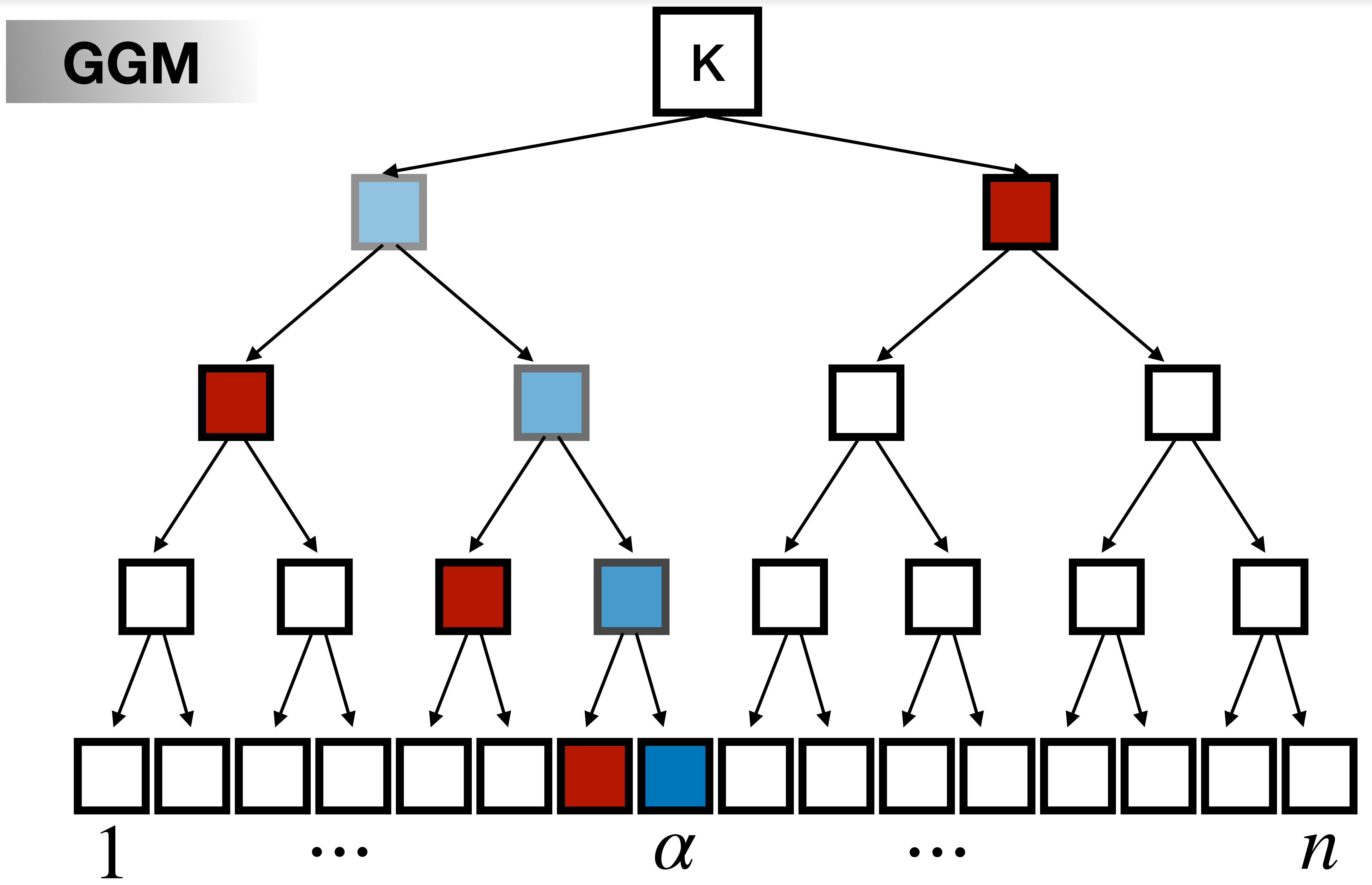

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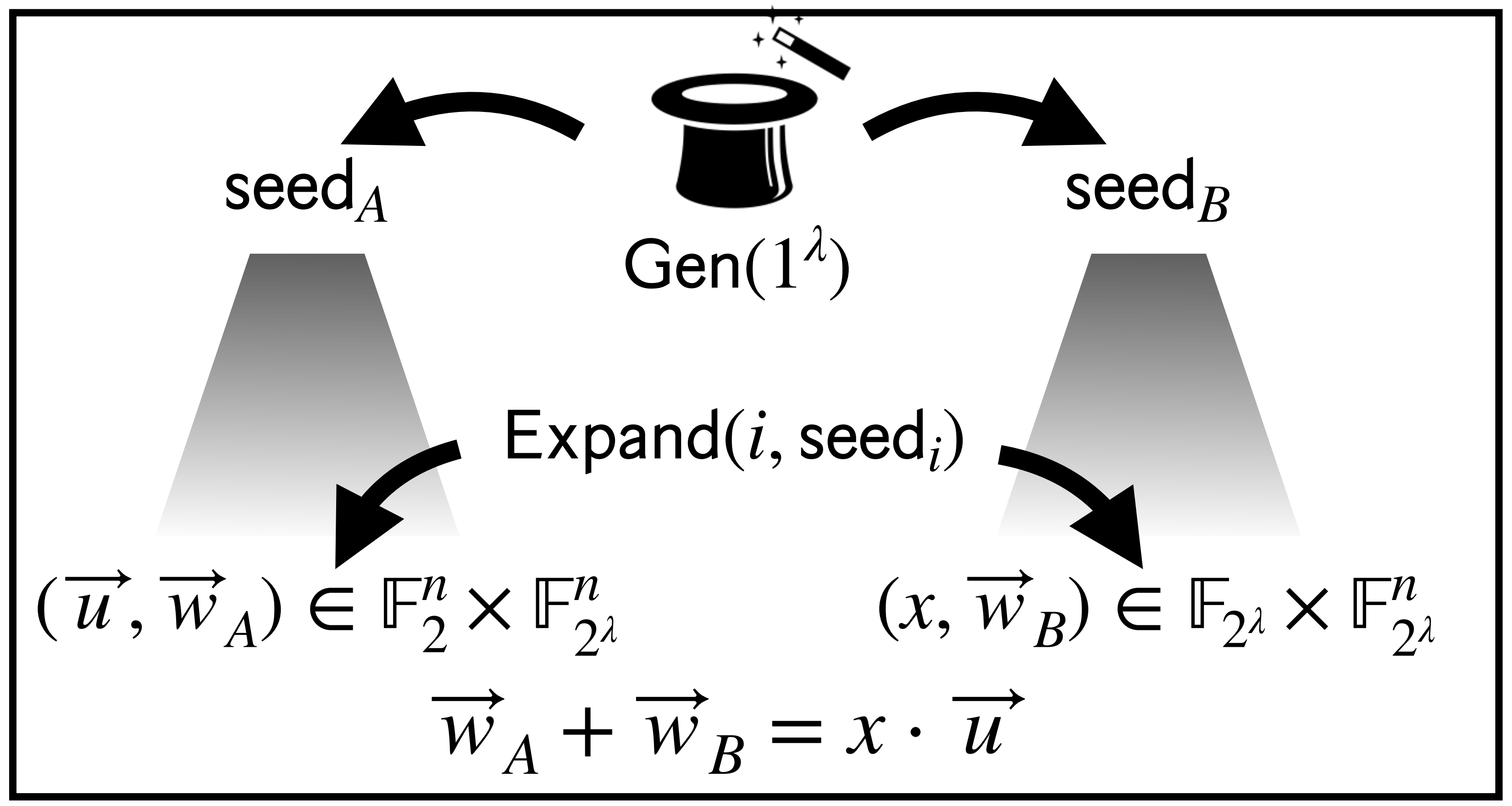

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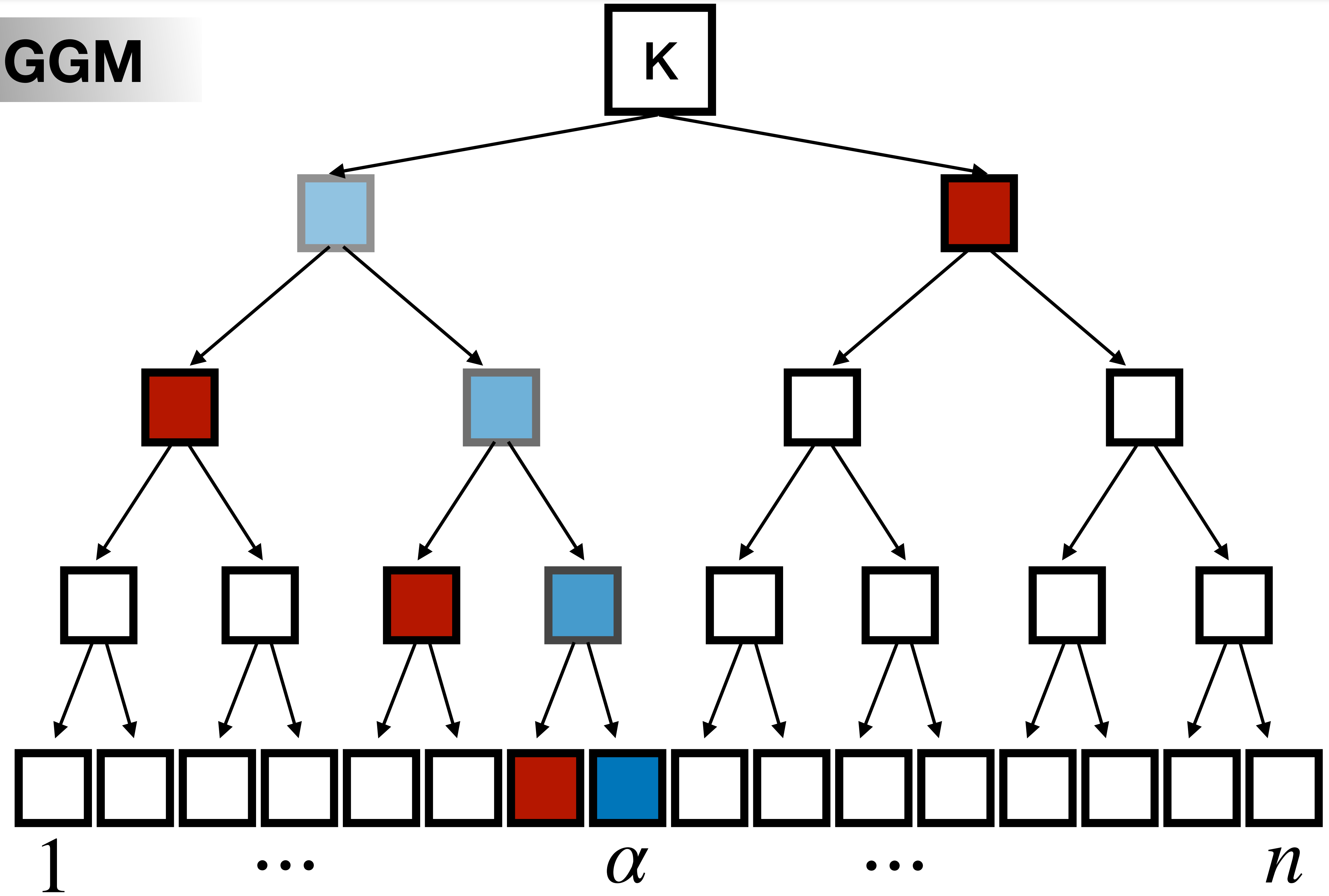

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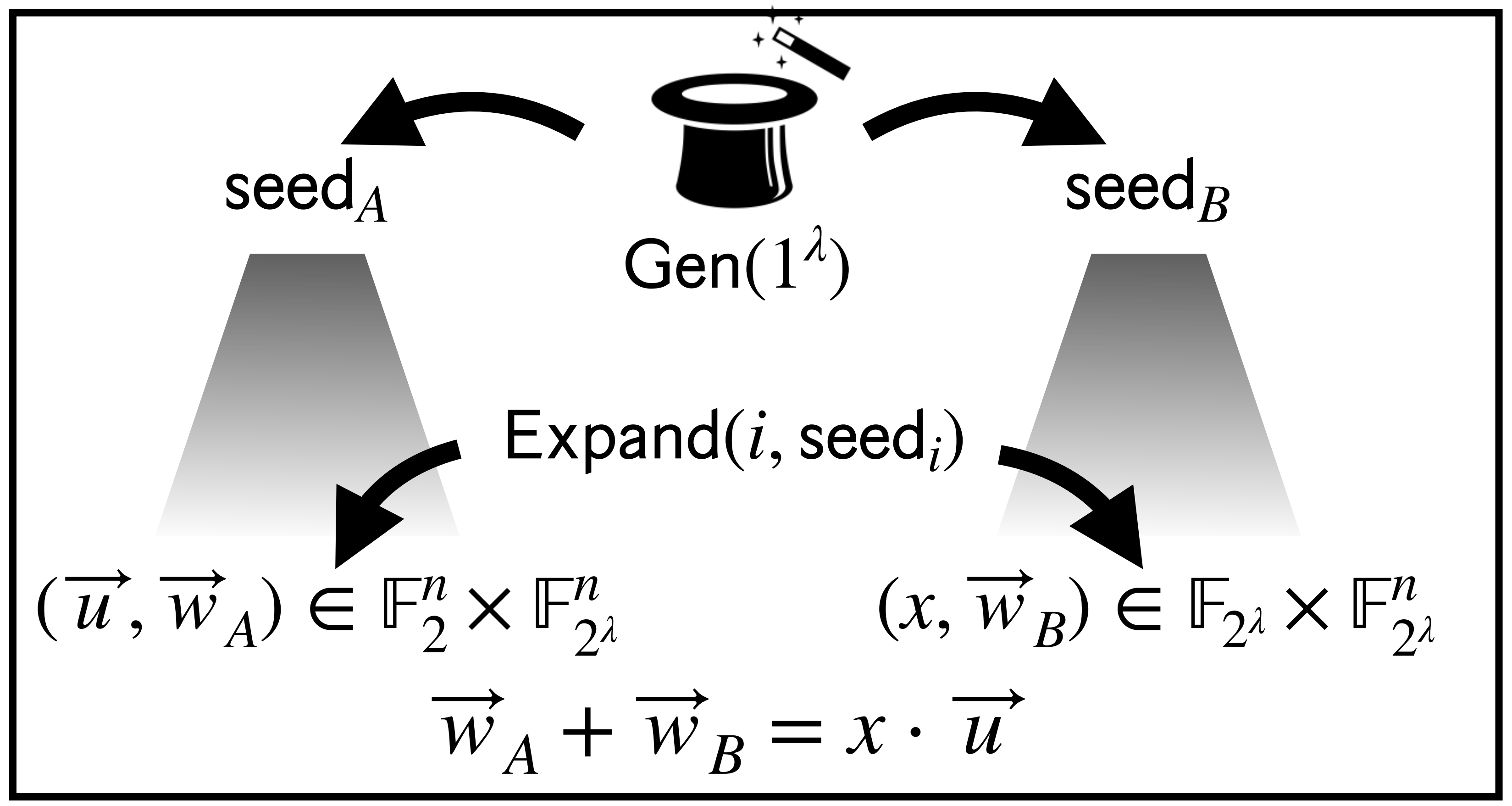

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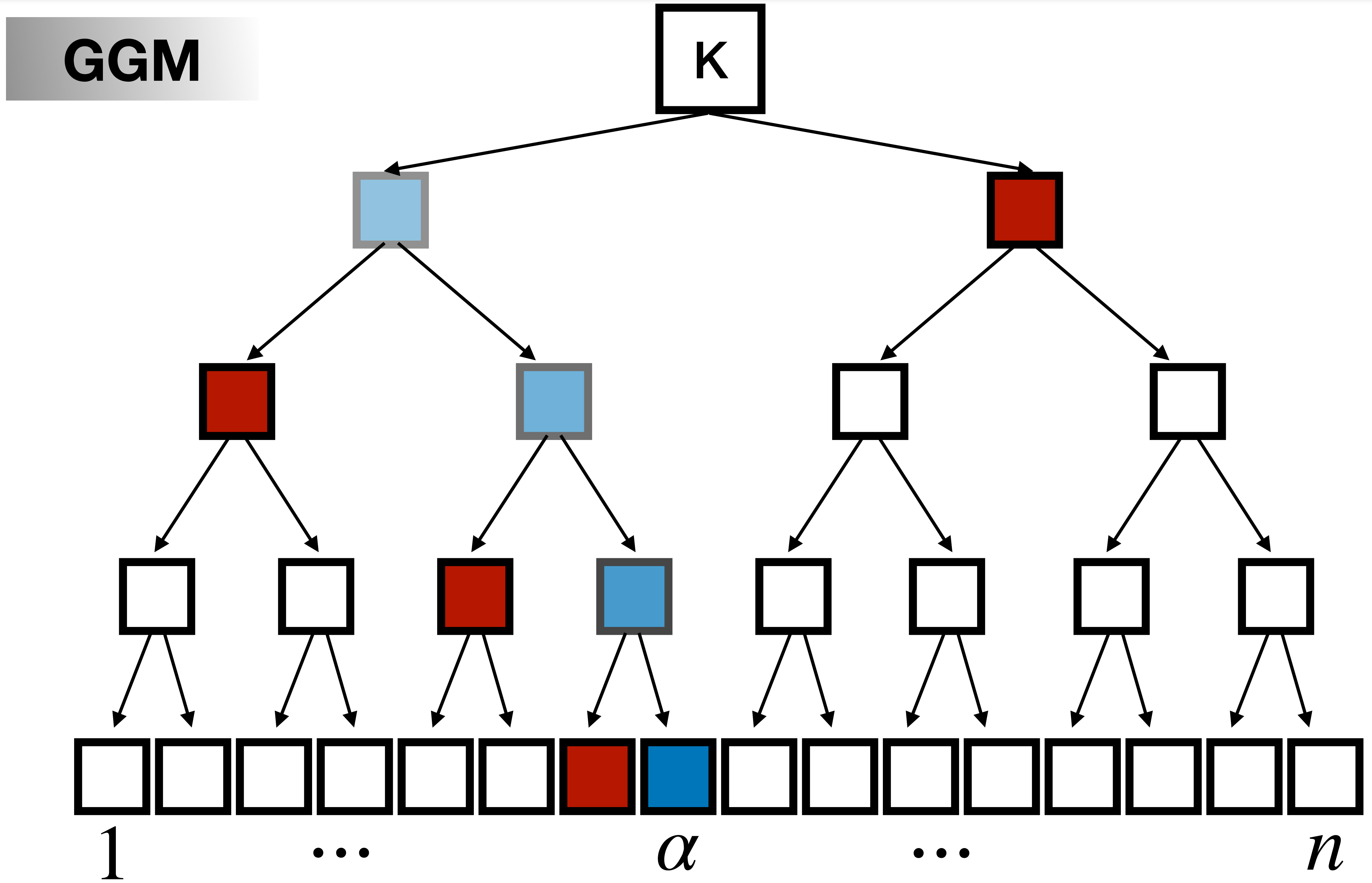
$\mathsf{seed}_A = (x, K)$ $\mathsf{seed}_B = (\vec{u}, \blacksquare\blacksquare\blacksquare\blacksquare, \blacksquare \oplus x)$

$\vec{w}_A \leftarrow \mathsf{FullEval}(K)$ $\vec{w}_B \leftarrow \mathsf{Insert}(x \oplus F_K(\alpha), \mathsf{FullEval}(K_{\{\alpha\}}))$



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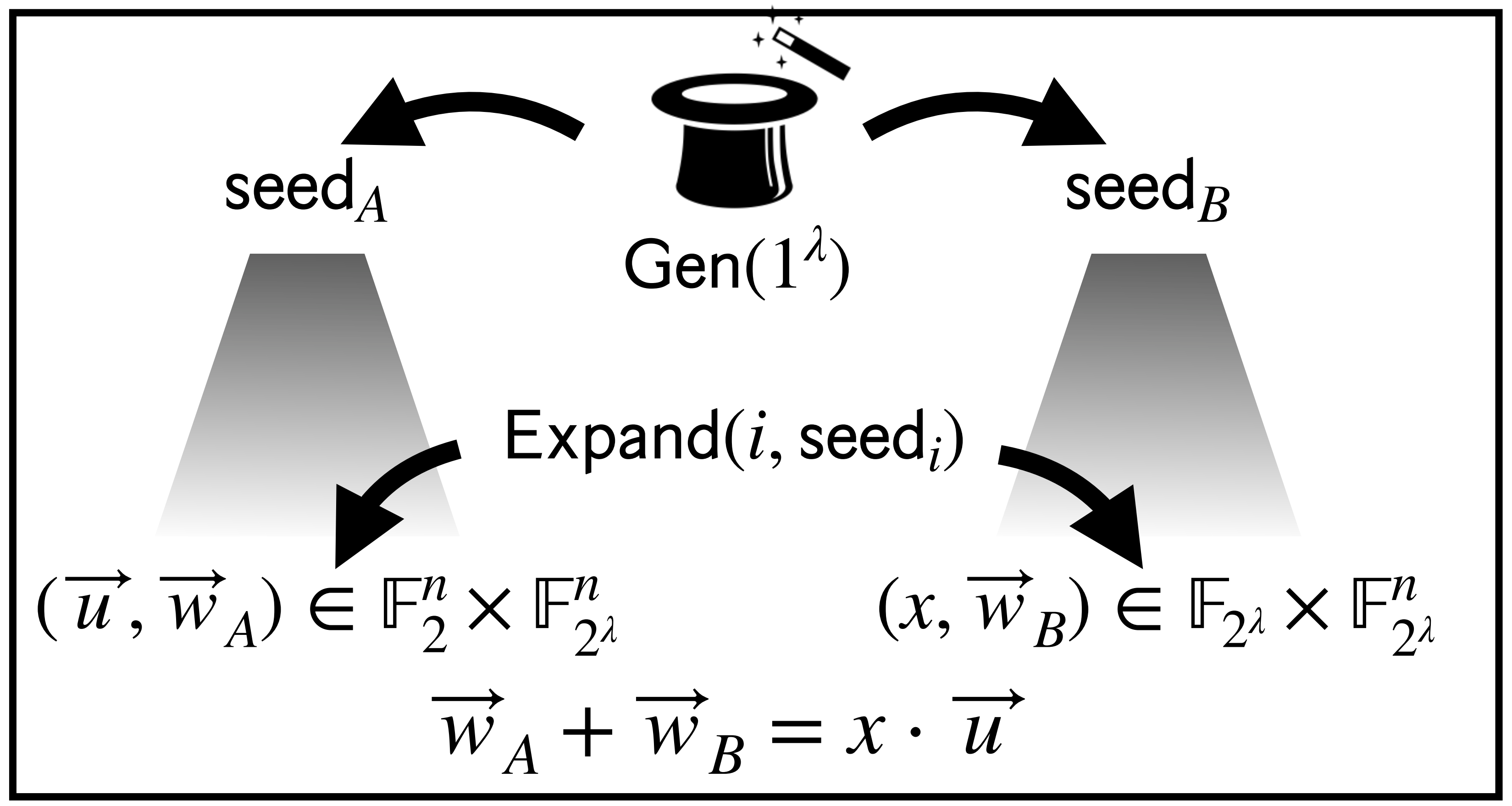

Pseudorandom Correlation Generators - Walkthrough

2 Construction for a random *t-sparse vector* $\vec{u}$
via t parallel repetitions of (1)

$(\alpha : \ u_\alpha = 1)$

$$\mathsf{seed}_A = (x, K^1) \qquad \mathsf{seed}_B = (\vec{u}_1, K^1_{\{\alpha_1\}}, F_{K^1}(\alpha_1) \oplus x)$$

$$K^2 \qquad (\vec{u}_2, K^2_{\{\alpha_1\}}, F_{K^2}(\alpha_2) \oplus x)$$

$$\vdots \qquad \vdots$$

$$K^t \qquad (\vec{u}_t, K^t_{\{\alpha_t\}}, F_{K^t}(\alpha_t) \oplus x)$$

- Write $\vec{u}$ as a sum of t unit vectors $\vec{u}_1 \cdots \vec{u}_t$
- Apply the previous construction t times (with the same x)
- After expansion, the parties locally sum their shares:

$$\left(\bigoplus_{i=1}^{t} \vec{w}_A^i\right) \oplus \left(\bigoplus_{i=1}^{t} \vec{w}_B^i\right) = x \cdot \bigoplus_{i=1}^{t} \vec{u}_i = x \cdot \vec{u}$$

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Pseudorandom Correlation Generators - Walkthrough

3 Construction for a pseudorandom vector $\vec{u}$
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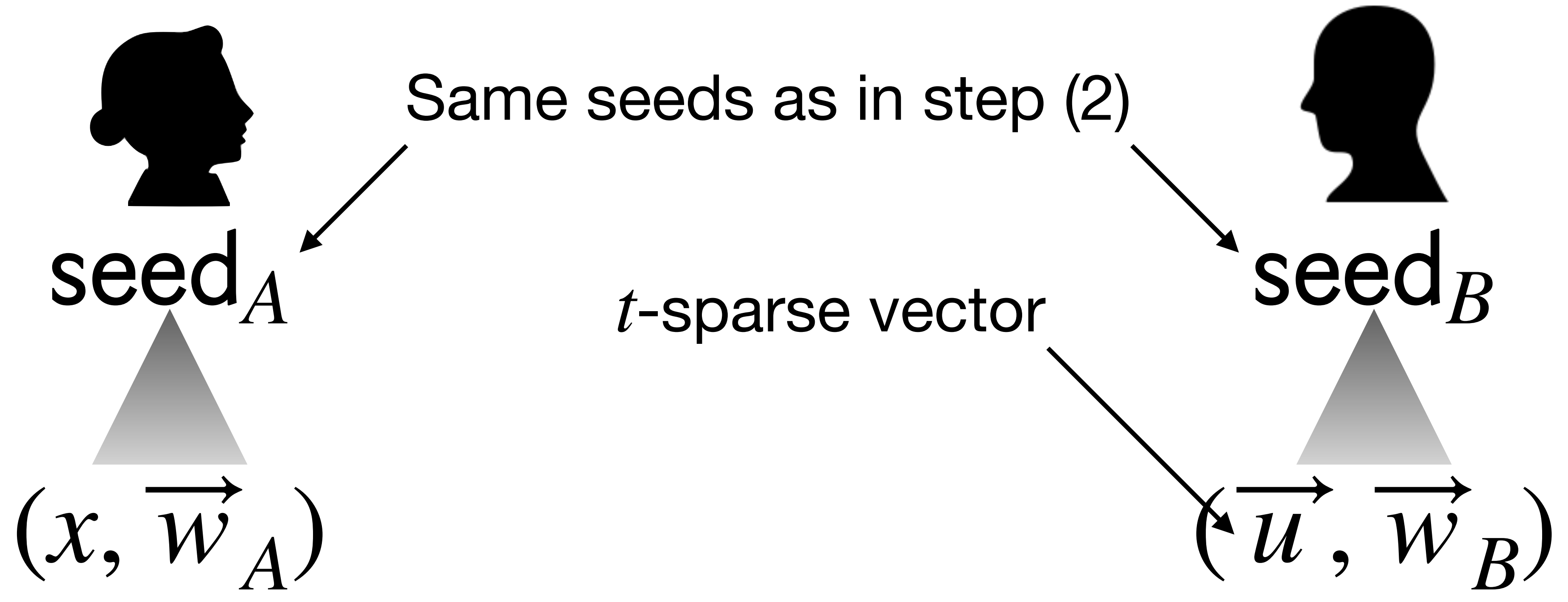

The LPN assumption - primal

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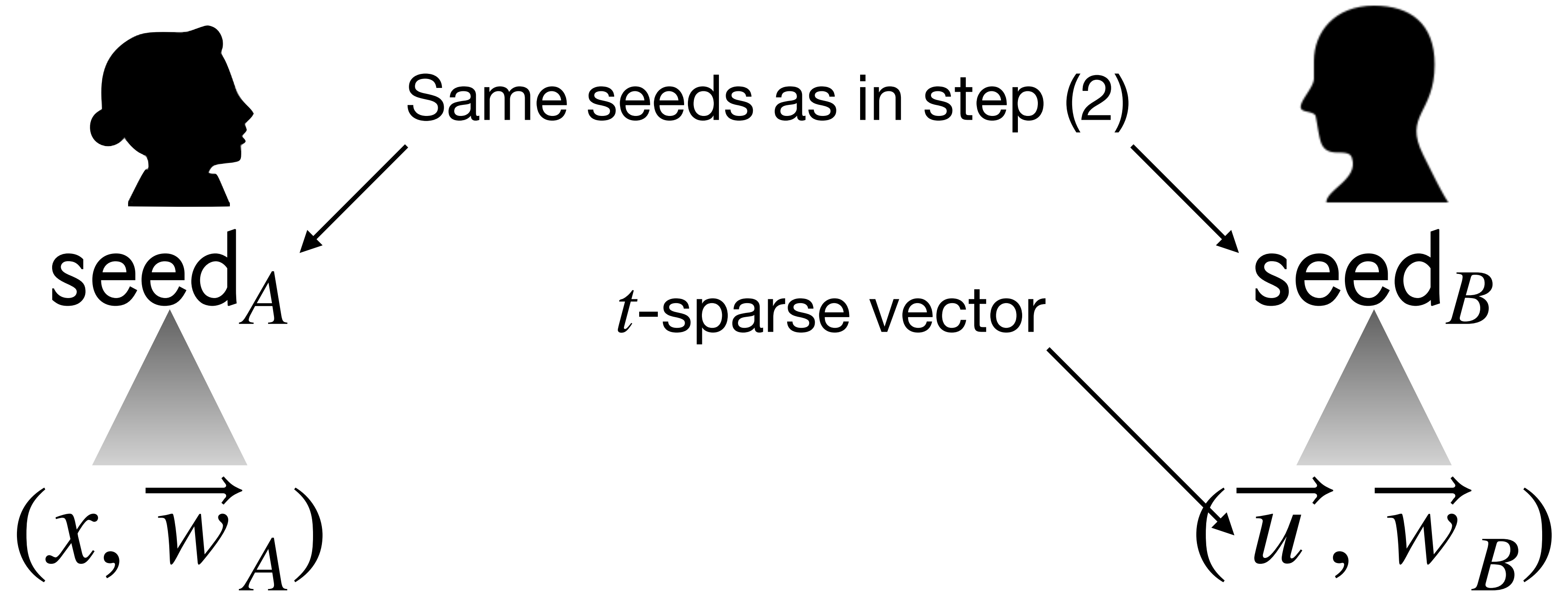

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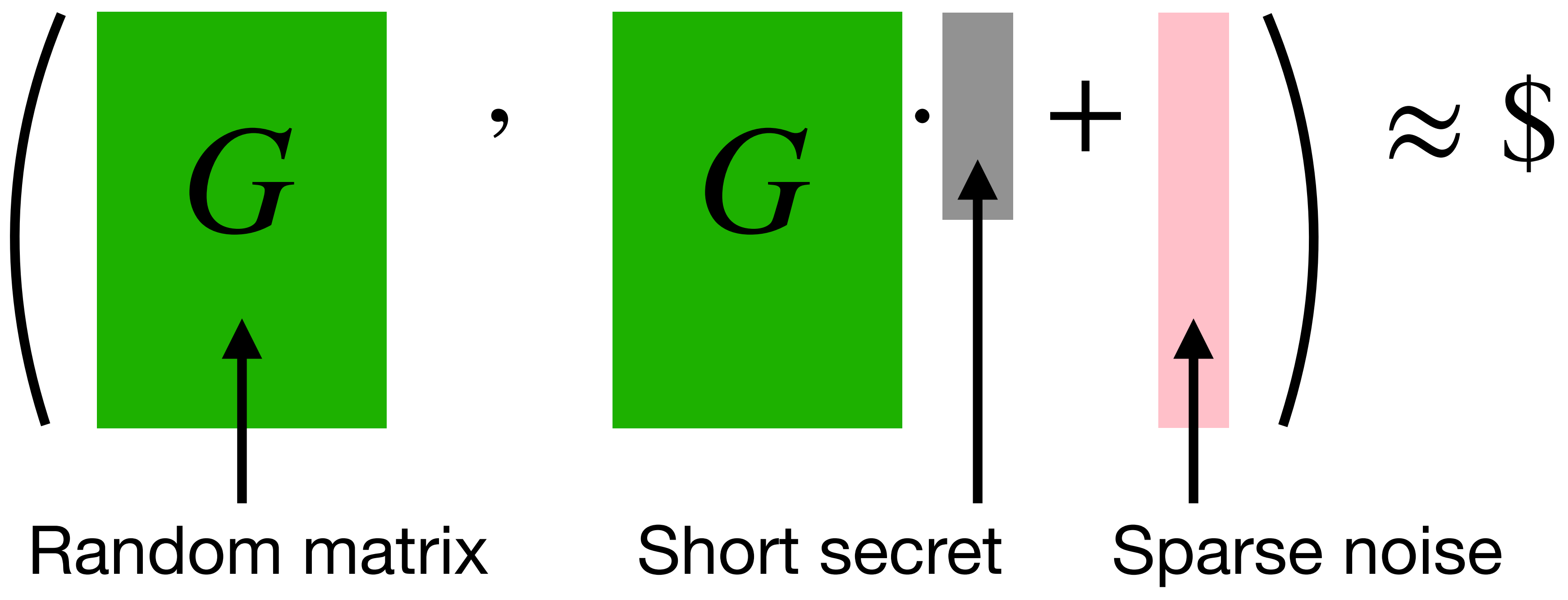

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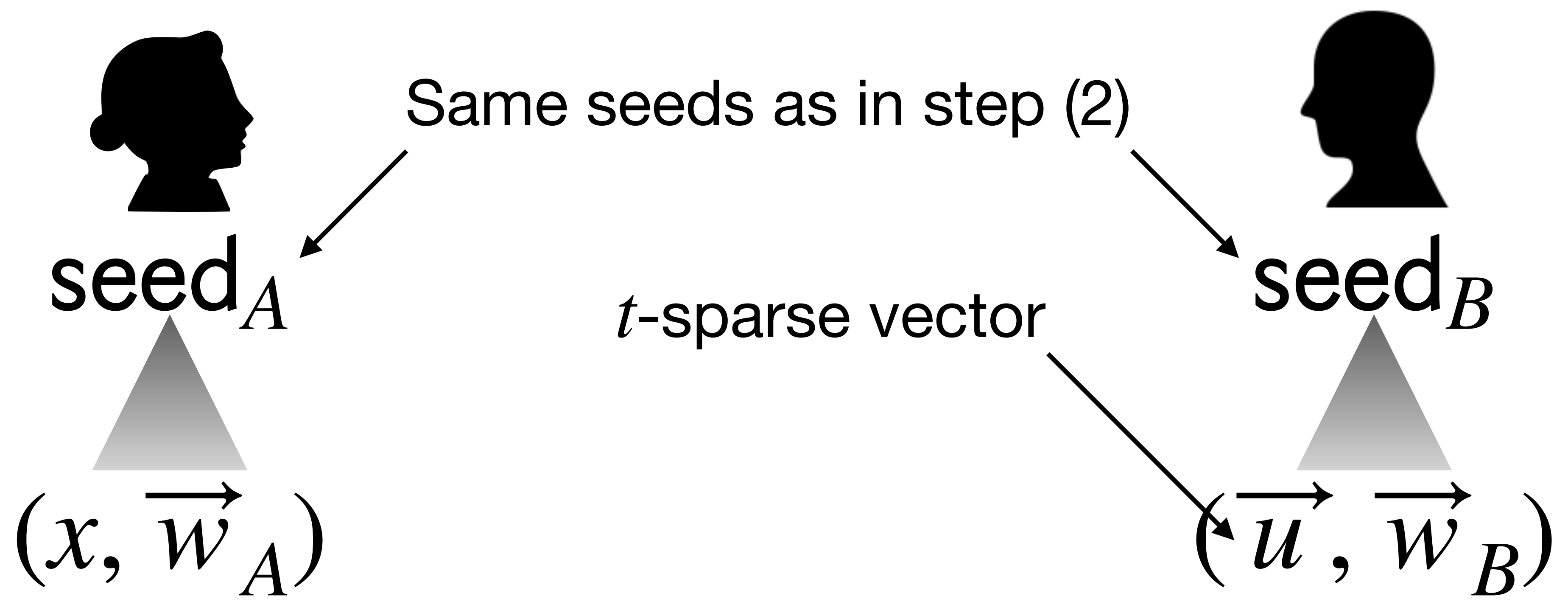

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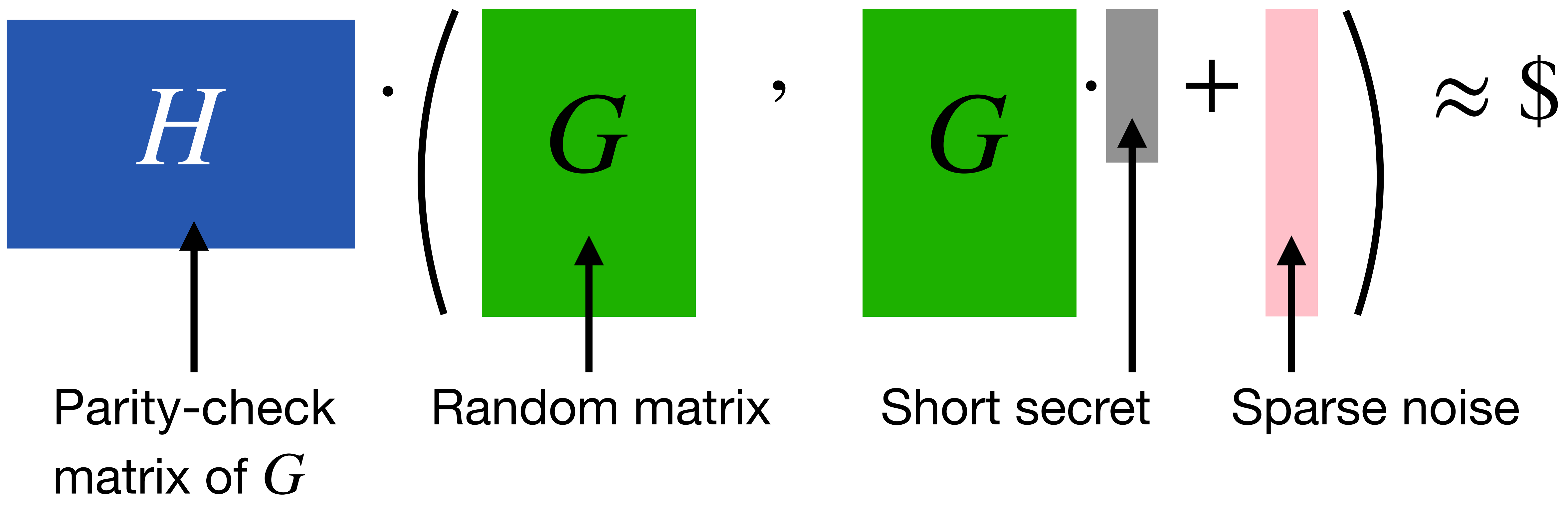

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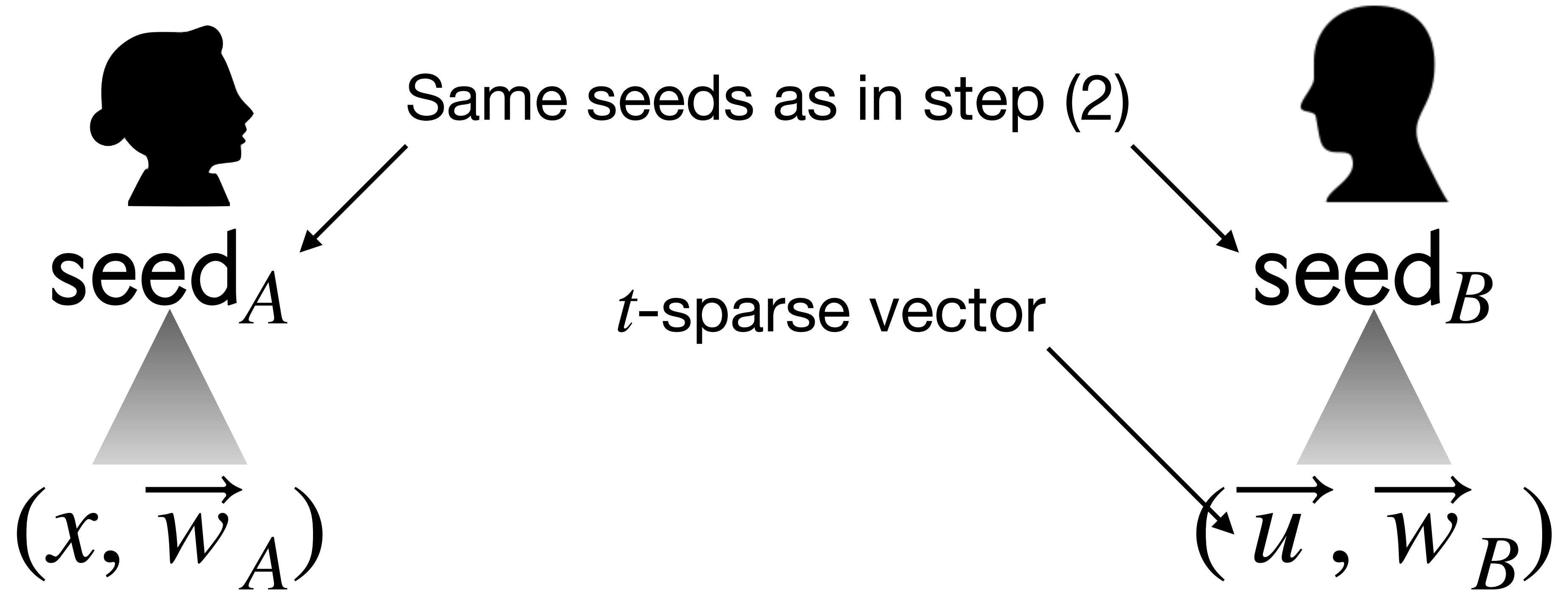

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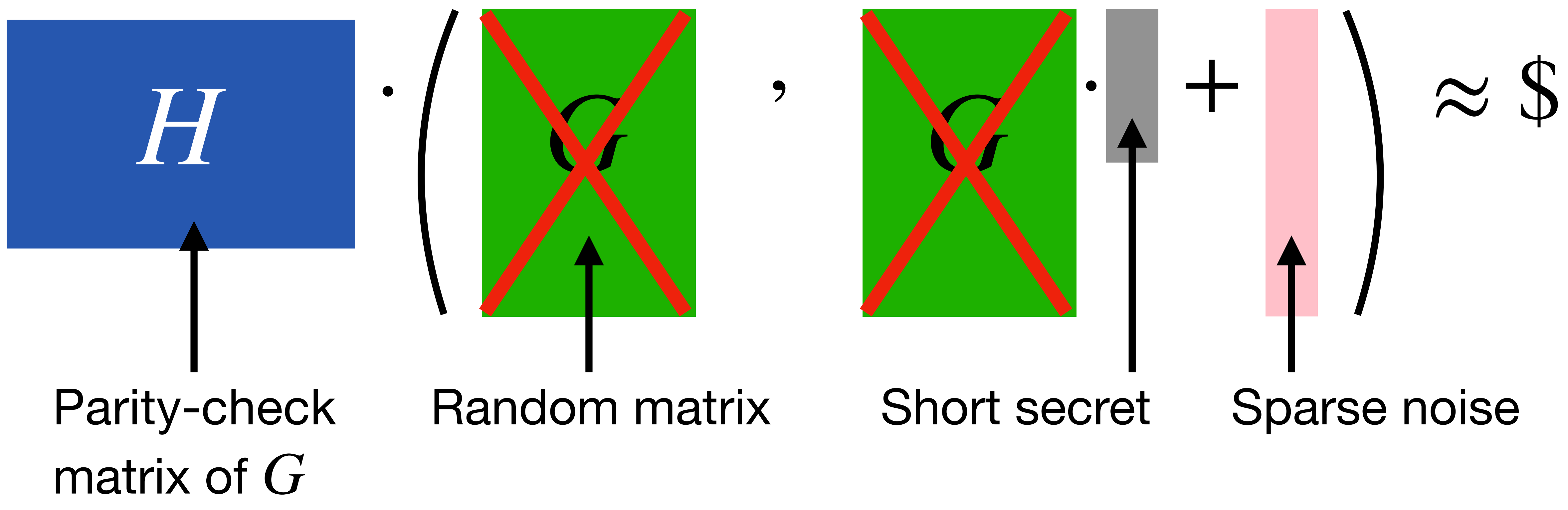

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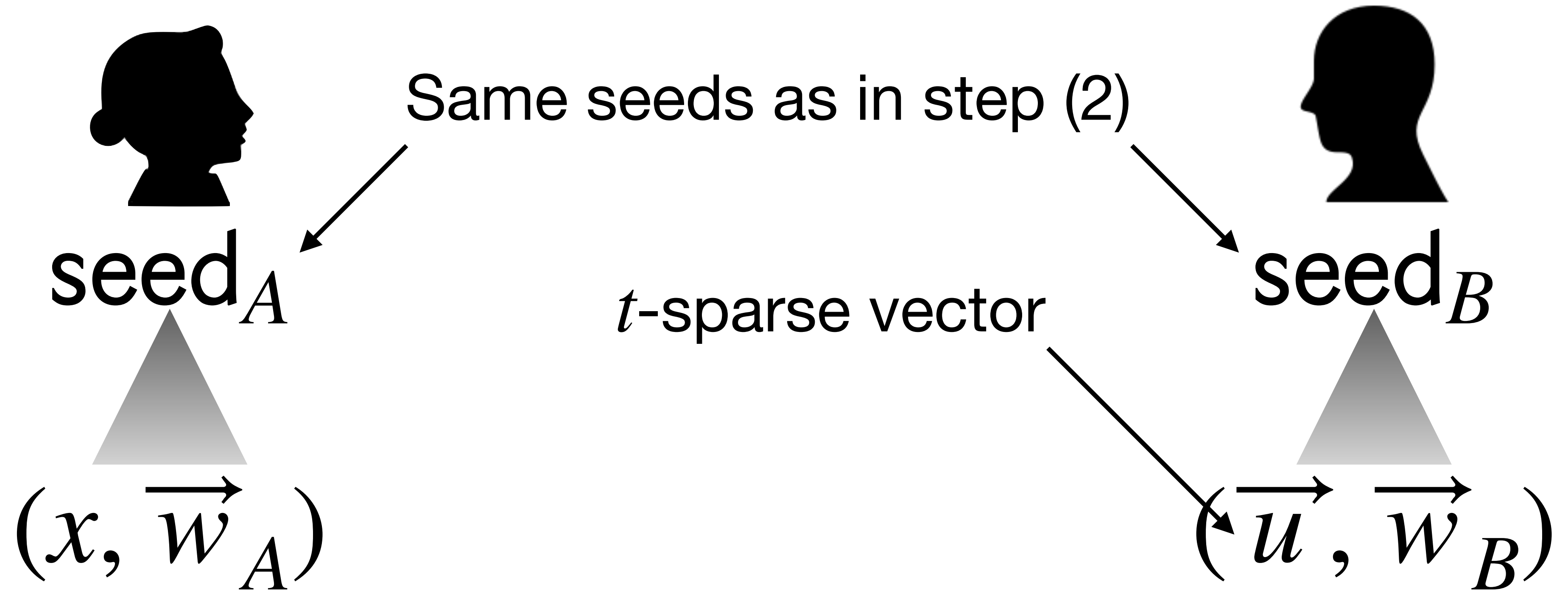

The LPN assumption - dual

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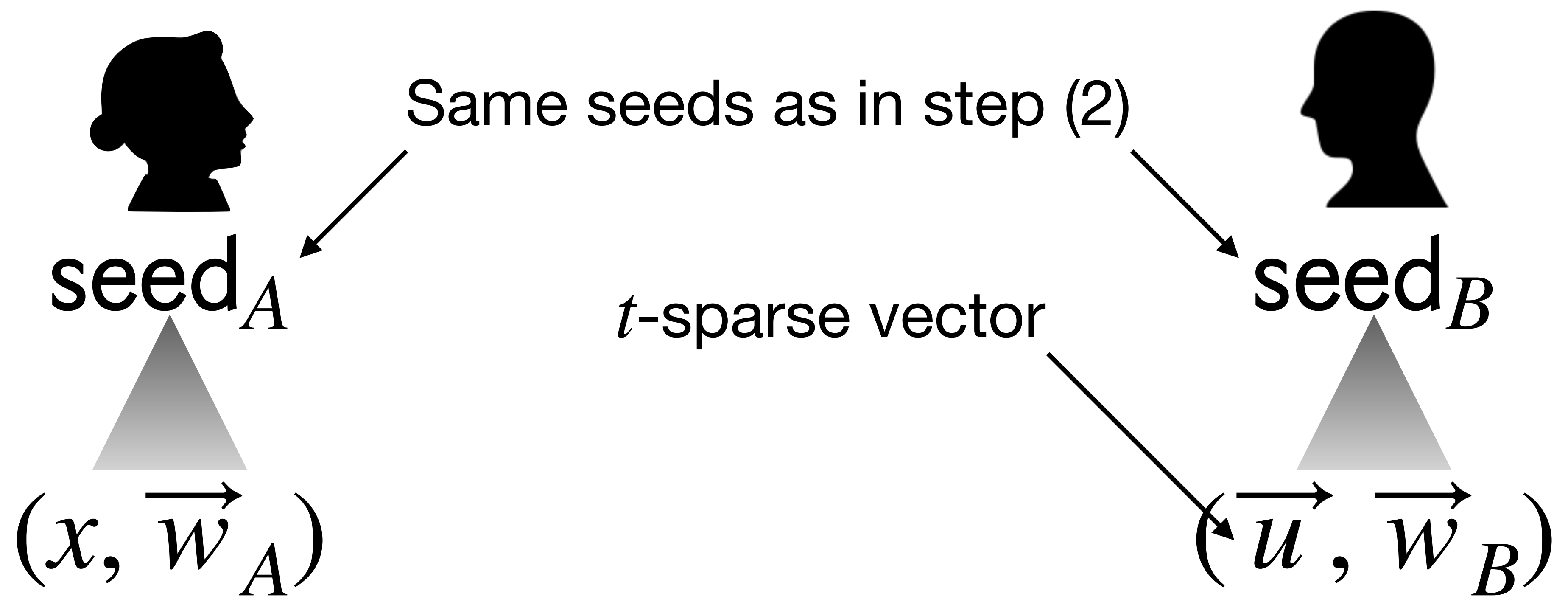
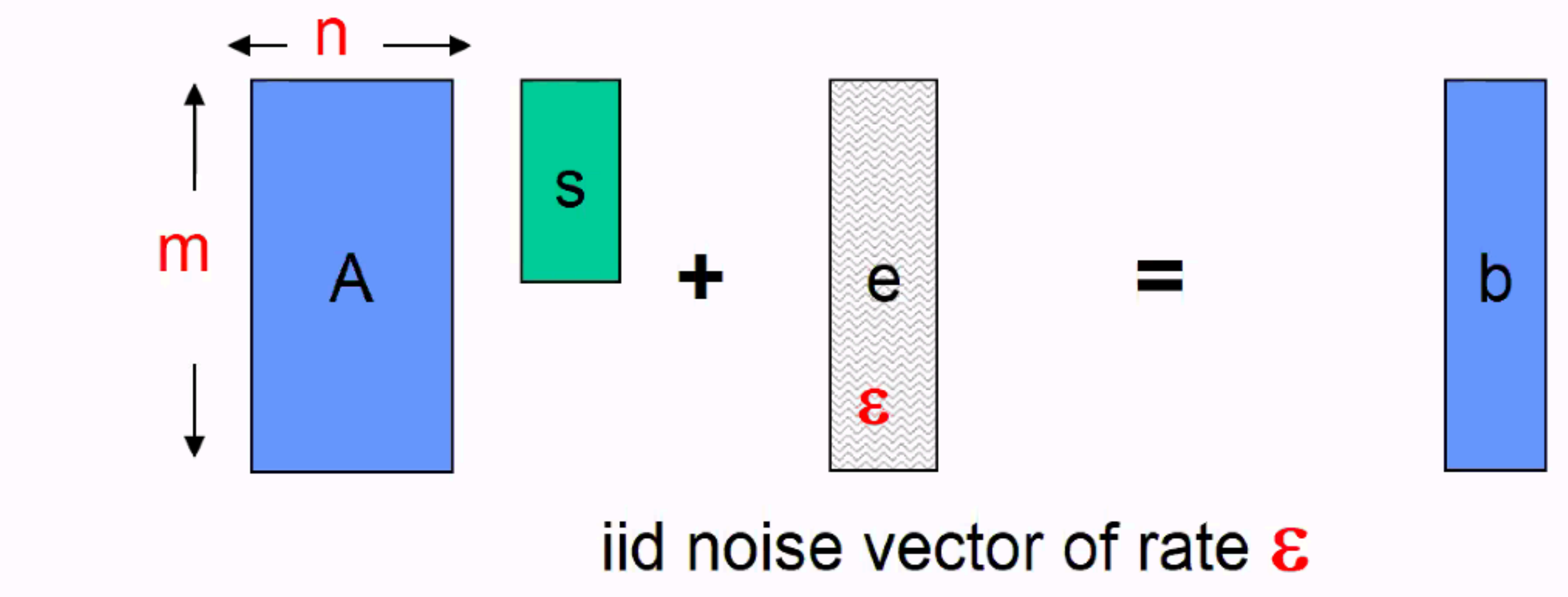


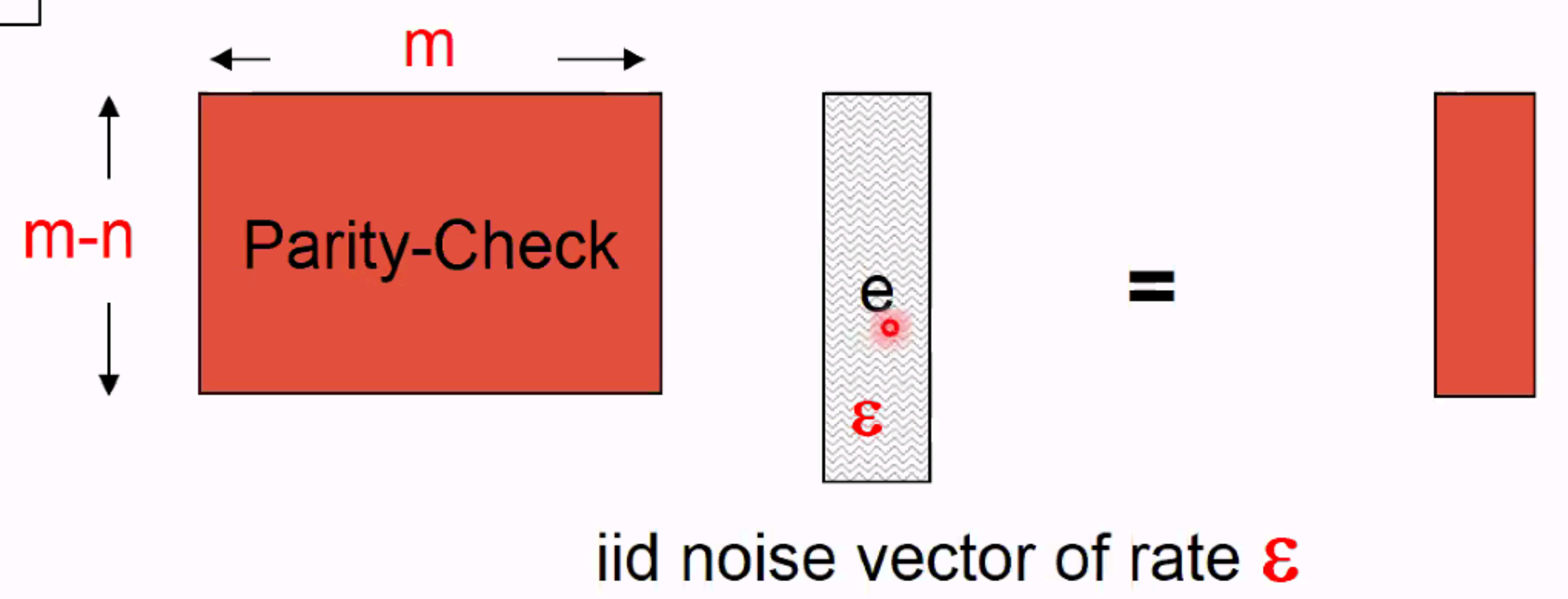
The LPN assumption - dual

Dual Version: Syndrome Decoding

Problem: find s



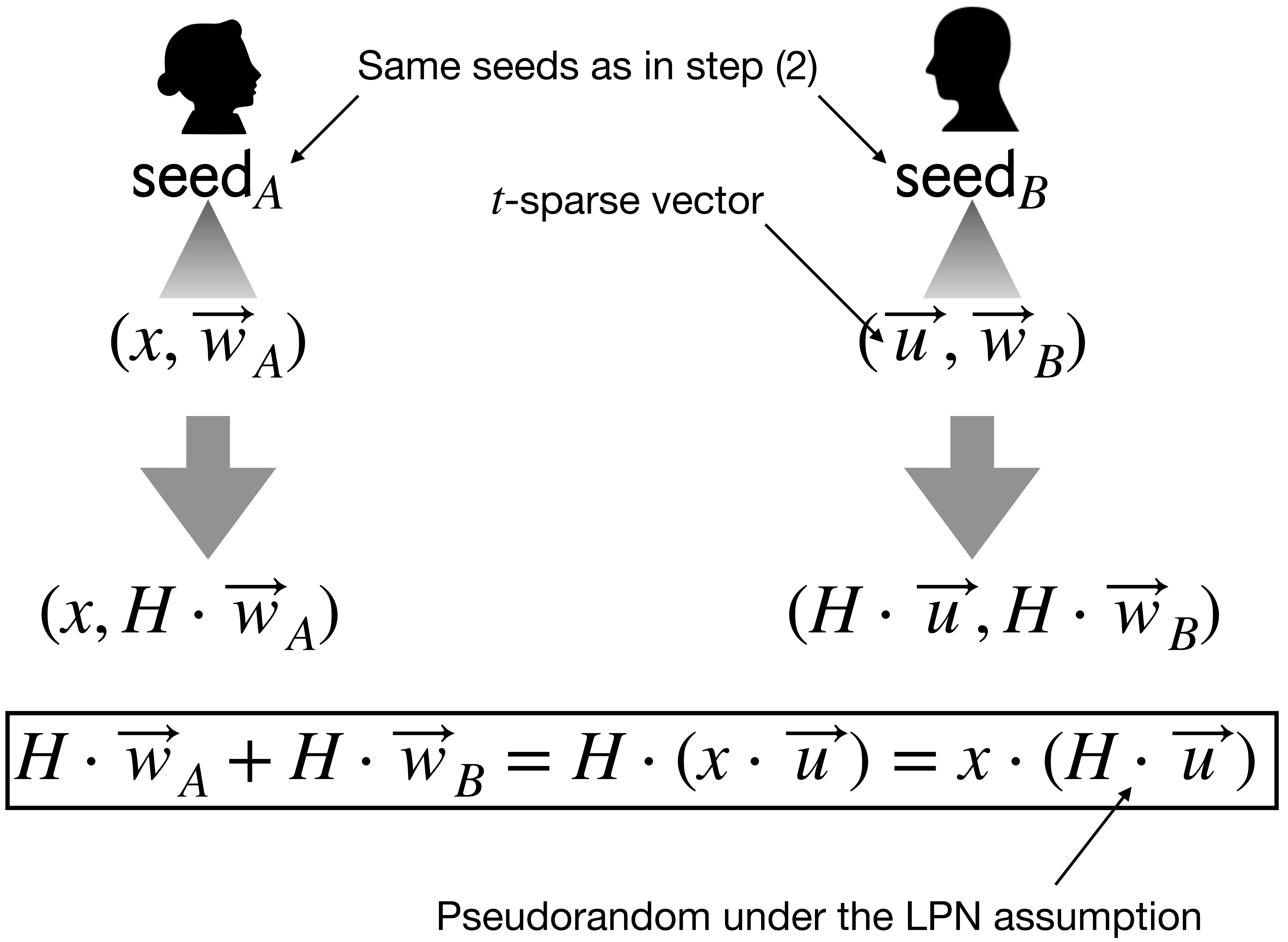
Problem: find e



Remember, from Benny's talk on Monday

Pseudorandom Correlation Generators - Walkthrough

③ Construction for a pseudorandom vector $\vec{u}$
using dual-LPN



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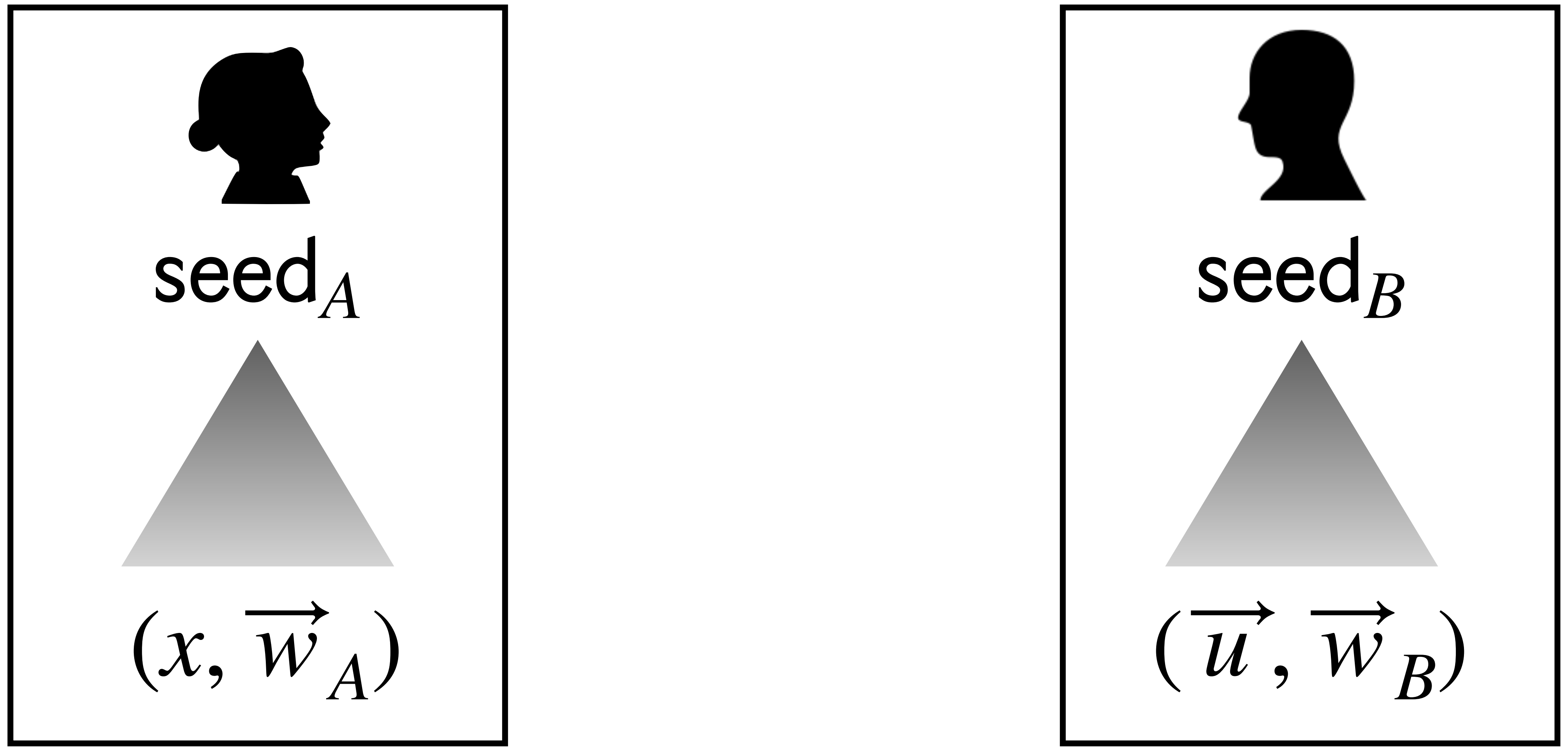
Pseudorandom Correlation Generators - Efficiently?

Wrapping-up

$$\overrightarrow{w}_A + \overrightarrow{w}_B = x \cdot \overrightarrow{u}$$

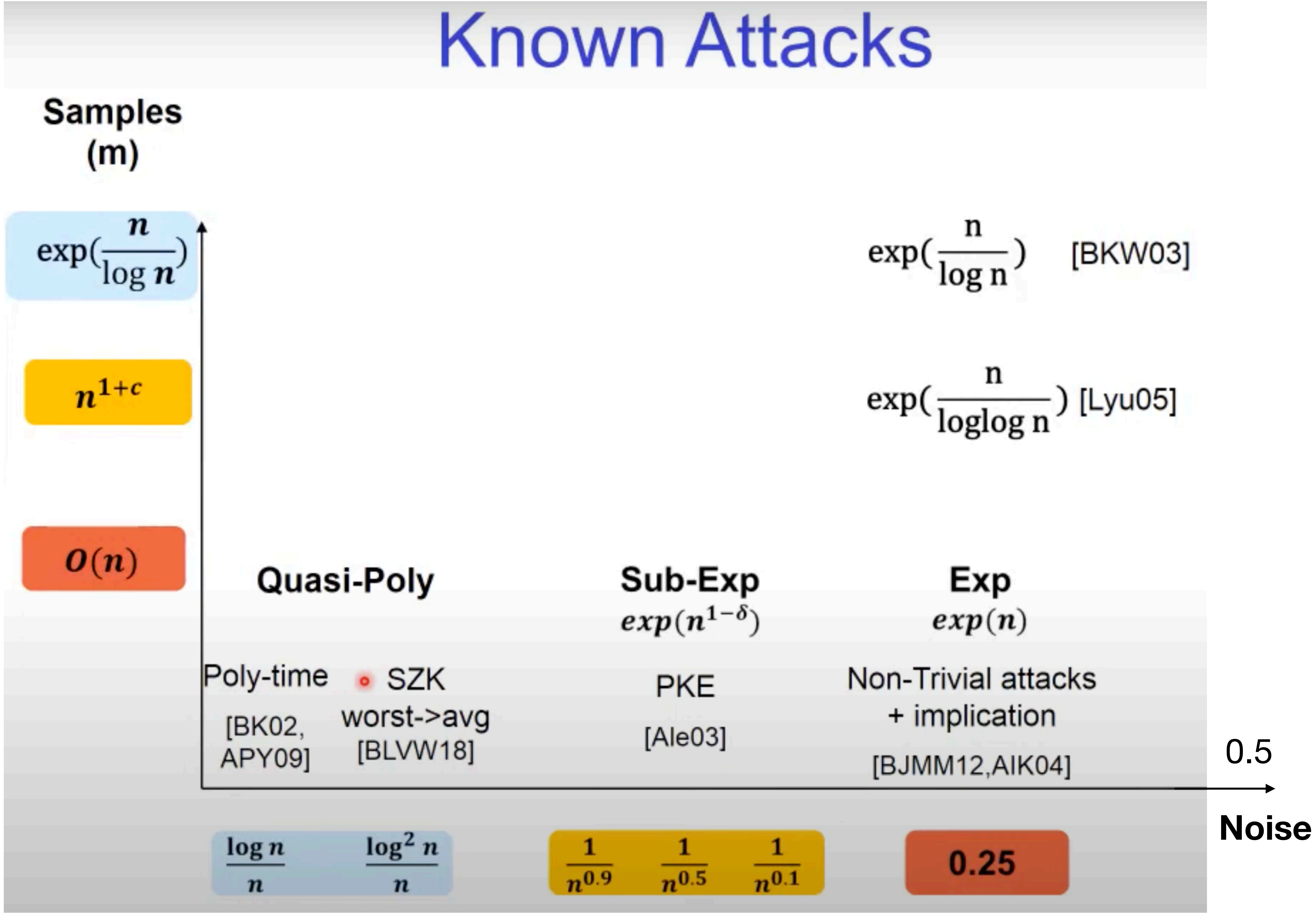


$|\mathsf{seed}_A| \approx \lambda \cdot t$ $\qquad$ $|\mathsf{seed}_B| \approx \lambda \cdot t \cdot \log n$

- λ is a security parameter, t is an LPN noise parameter, n is the vector length.
- Converted to n pseudorandom OTs via a correlation-robust hash function.

Placing the Assumption in the LPN Landscape

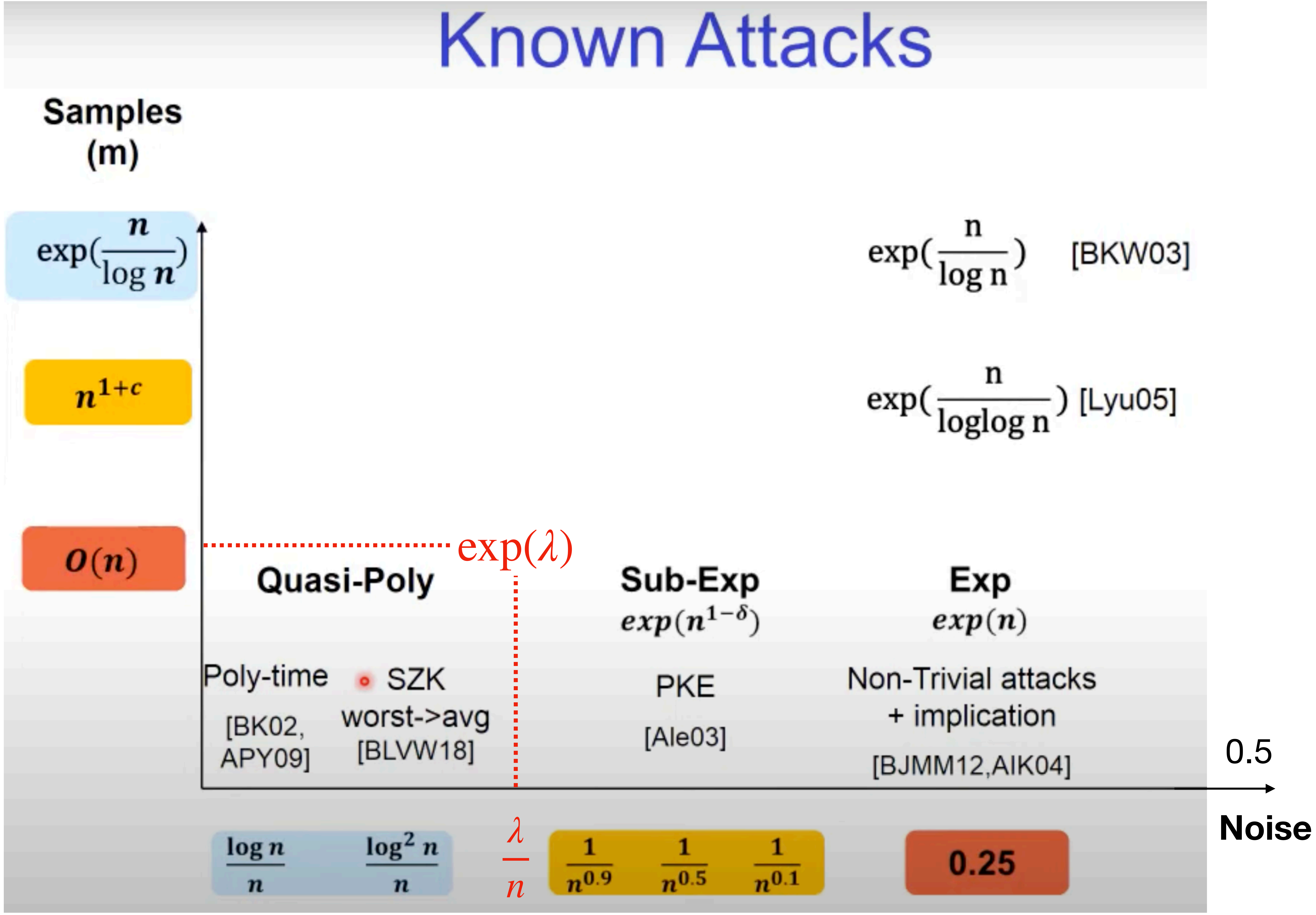
Remember this slide, shamefully stolen from Benny's talk on Monday?



Placing the Assumption in the LPN Landscape

Remember this slide, shamefully stolen from Benny's talk on Monday?

We use $O(n)$ samples and noise λ/n. Therefore, we have exp. security *in* λ, and we do not view n as the security parameter anymore (since it is our target number of OTs)



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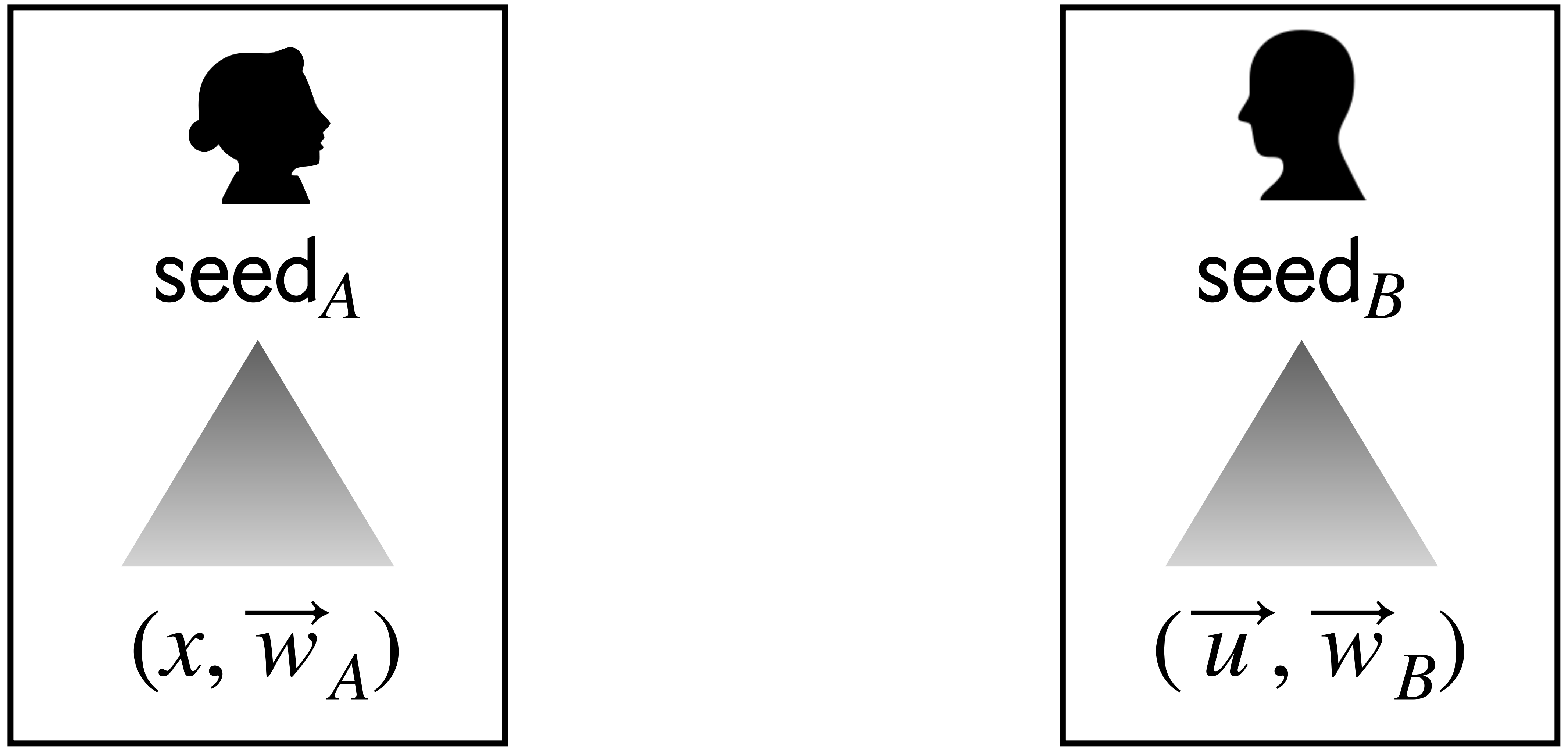

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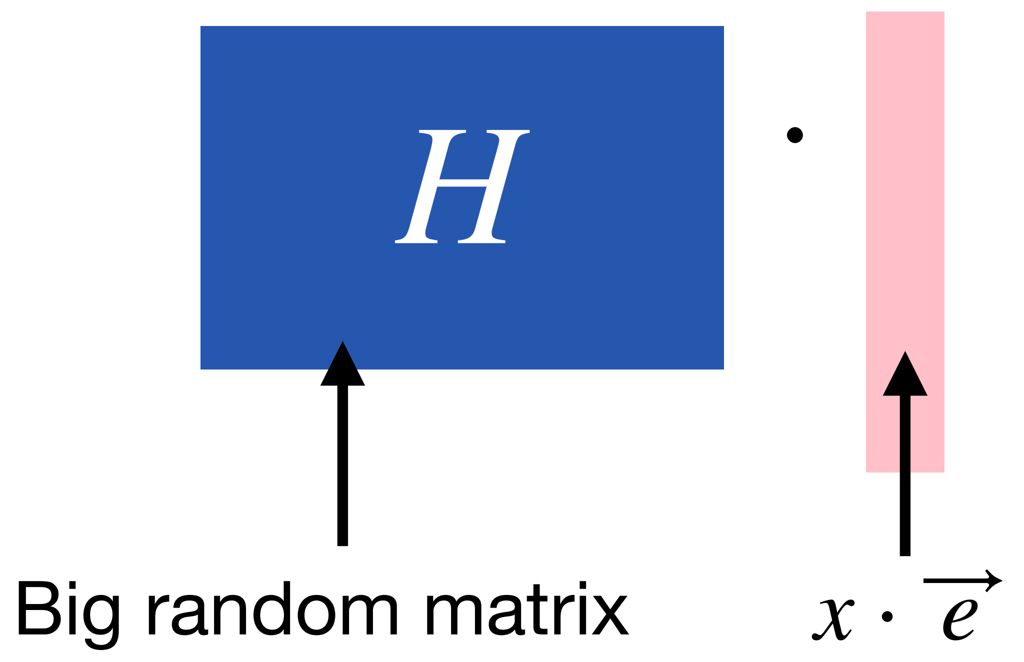
Is this really efficient?

The expansion of the PCG boils down to the computation of



Where $\vec{e}$ is a very sparse vector, and (the shares of) the entries of $x \cdot \vec{e}$ can be computed individually in log-time.

Pseudorandom Correlation Generators - Efficiently?

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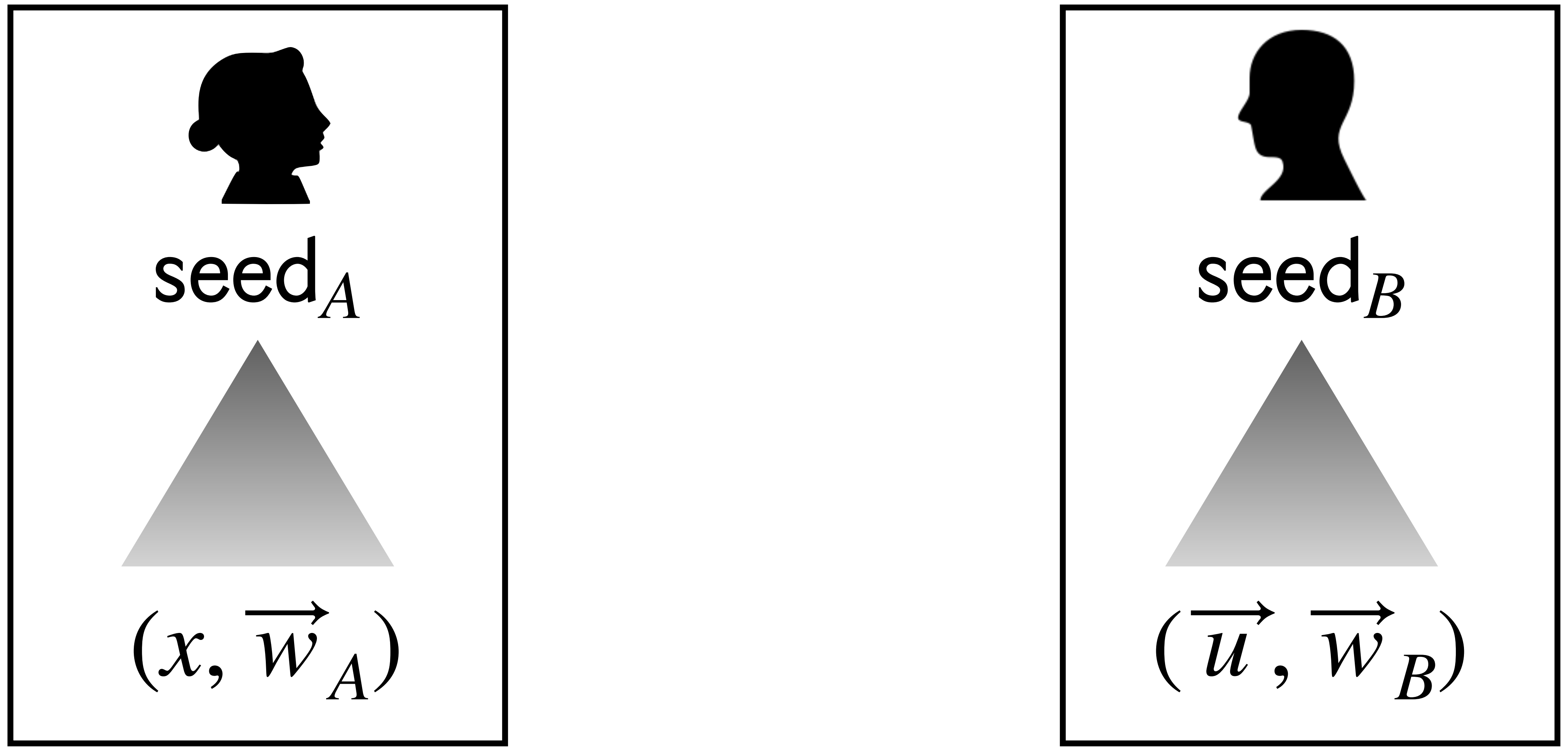

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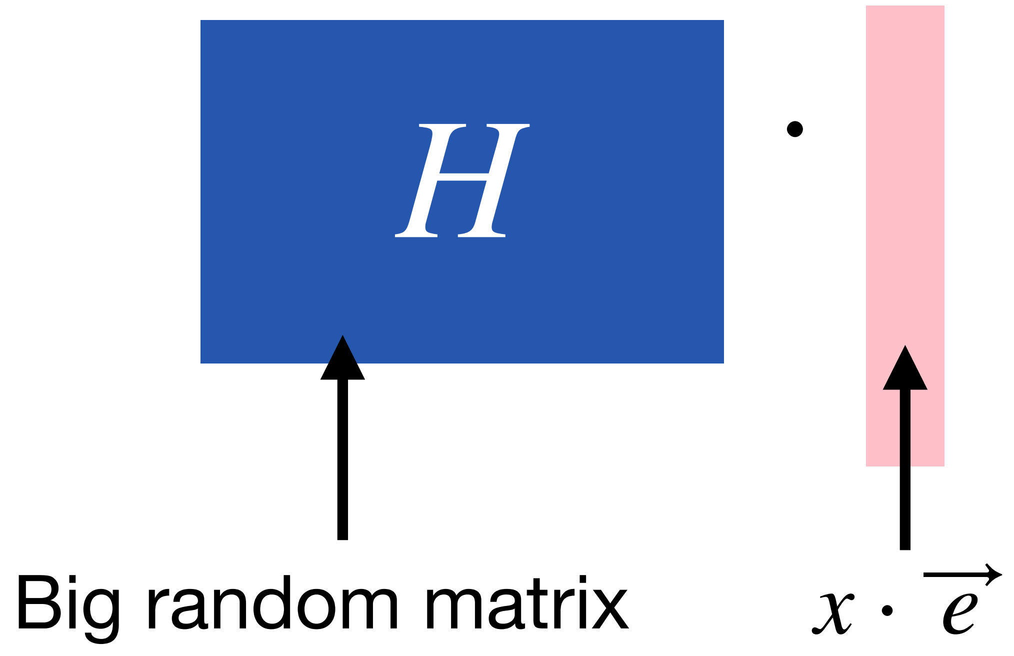


Where $\vec{e}$ is a very sparse vector, and (the shares of) the entries of $x \cdot \vec{e}$ can be computed individually in log-time.

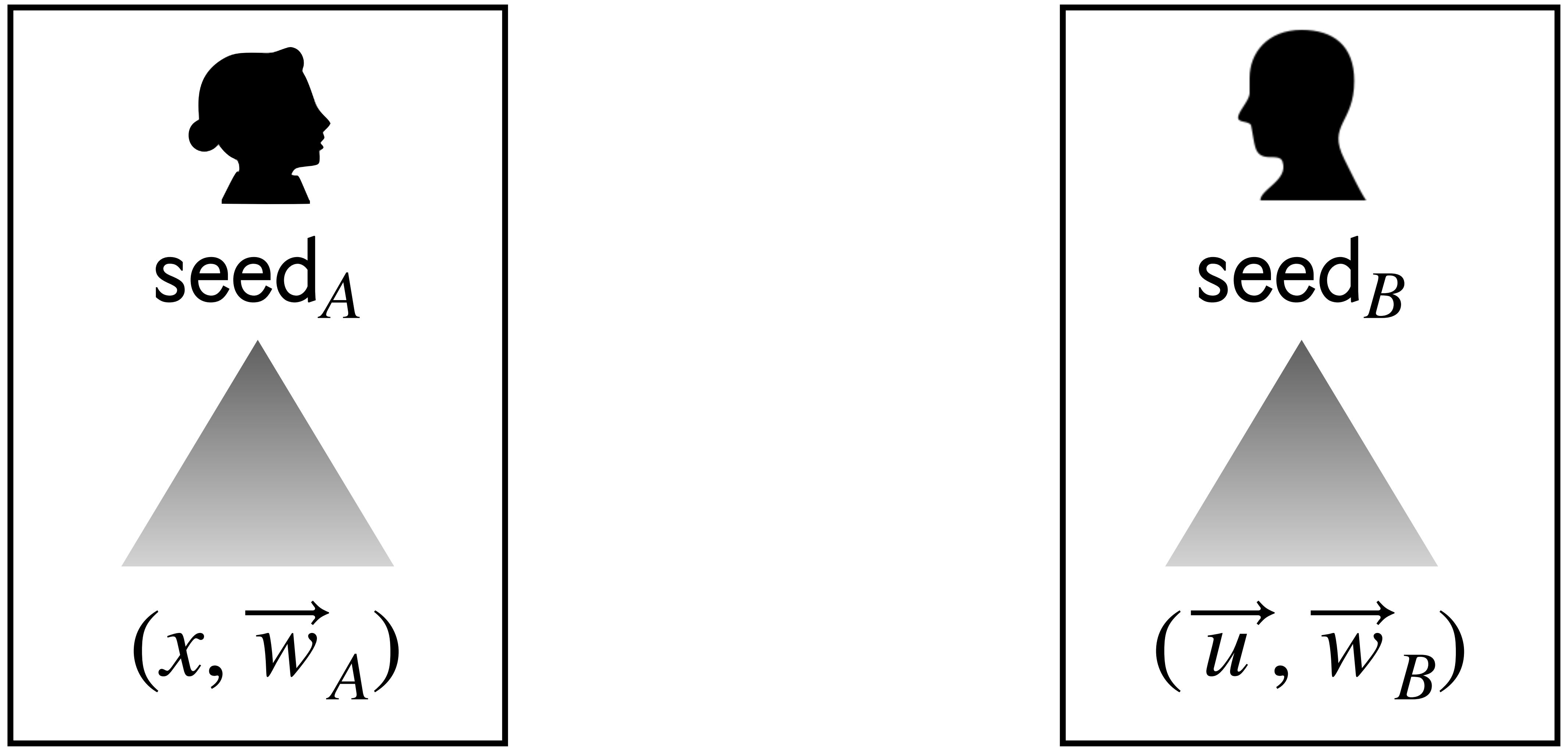
Computing $H \cdot \langle x \cdot \vec{e} \rangle$ takes a time *quadratic* in n…
But remember that n is the number of OTs we want: it's easily in the millions or billions.

This is nowhere near practical!

Pseudorandom Correlation Generators - Efficiently?

Wrapping-up

$$\overrightarrow{w}_A + \overrightarrow{w}_B = x \cdot \overrightarrow{u}$$

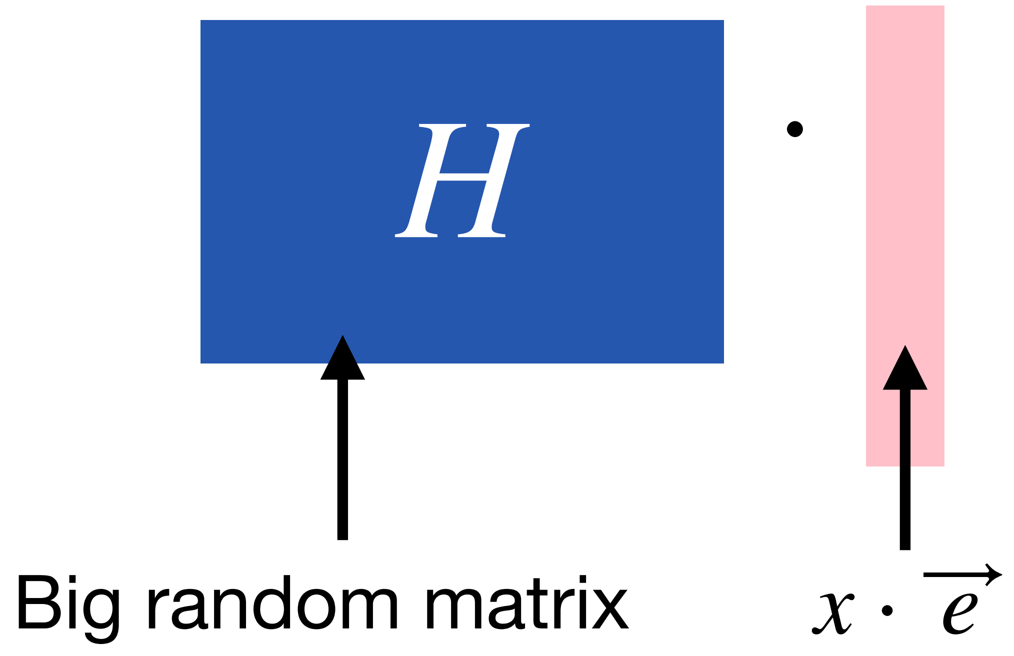


$|\mathsf{seed}_A| \approx \lambda \cdot t$ $\quad$ $|\mathsf{seed}_B| \approx \lambda \cdot t \cdot \log n$

- λ is a security parameter, t is an LPN noise parameter, n is the vector length.
- Converted to n pseudorandom OTs via a correlation-robust hash function.

Is this really efficient?

The expansion of the PCG boils down to the computation of



Where $\overrightarrow{e}$ is a very sparse vector, and (the shares of) the entries of $x \cdot \overrightarrow{e}$ can be computed individually in log-time.

Computing $H \cdot \langle x \cdot \overrightarrow{e} \rangle$ takes a time *quadratic* in n…
But remember that n is the number of OTs we want: it's easily in the millions or billions.

This is nowhere near practical!

We need to use *variants* of LPN, where multiplication by H is (much) faster, ideally linear-time.

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We want: computing $H \cdot$ is *fast*, and the code generated by G is LPN-friendly

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- **Resistant against LPN attacks:** highly plausible ✓ (was used in the design of several NIST proposals, e.g. BIKE, HQC, and LEDA, and are considered well studied)
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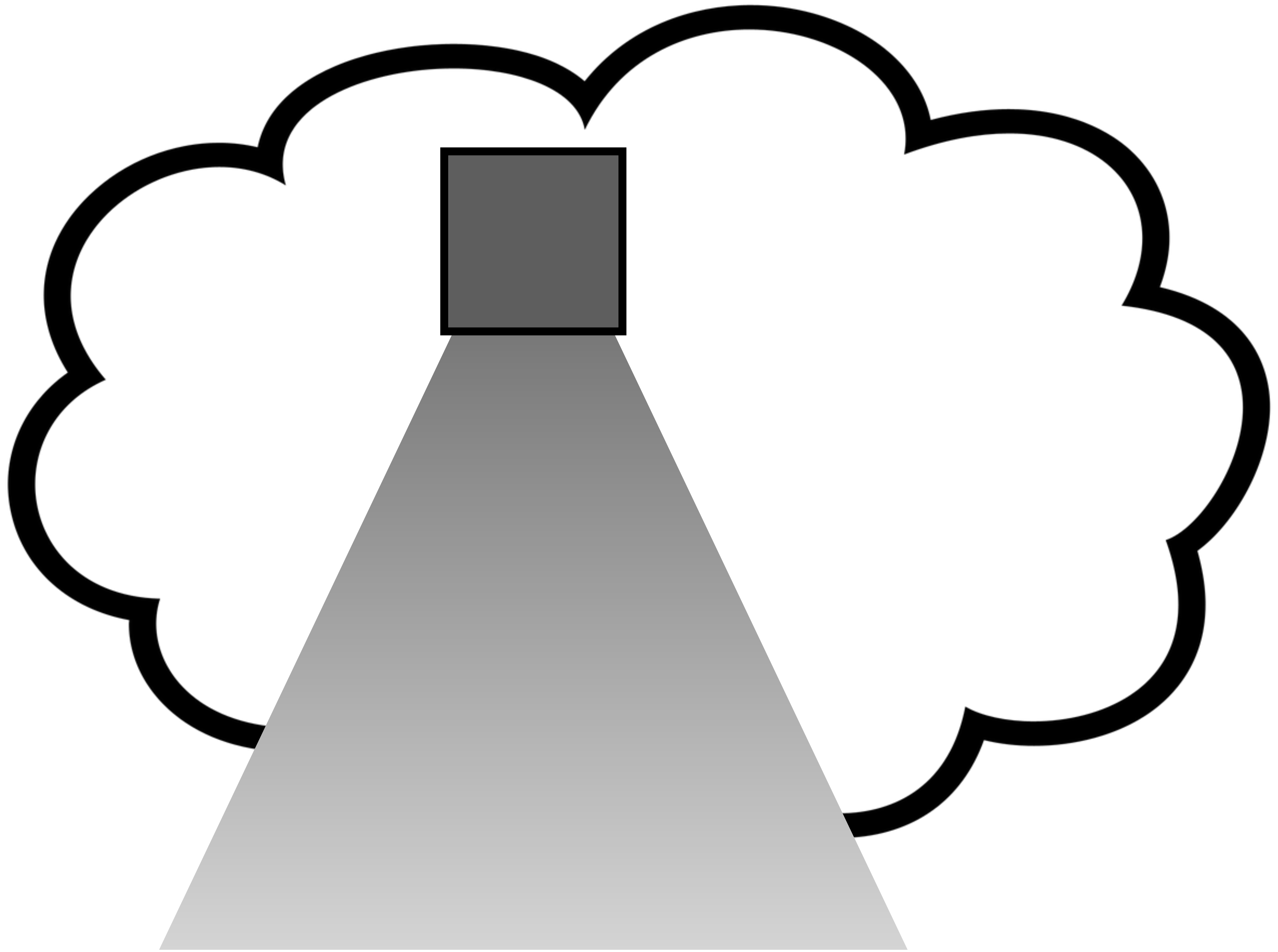
$O(n \cdot \log n)$ is not too bad, but when n is huge, as in our scenario, it still gives a significant slowdown… Unfortunately, no existing well-understood 'LPN-friendly' candidate has linear time multiplication by H. So… What do we do?

We try to understand what makes a code 'LPN-friendly', and we craft our own!

Security of (variants of) LPN - Linear Tests

A tremendous number of attacks on LPN have been published…



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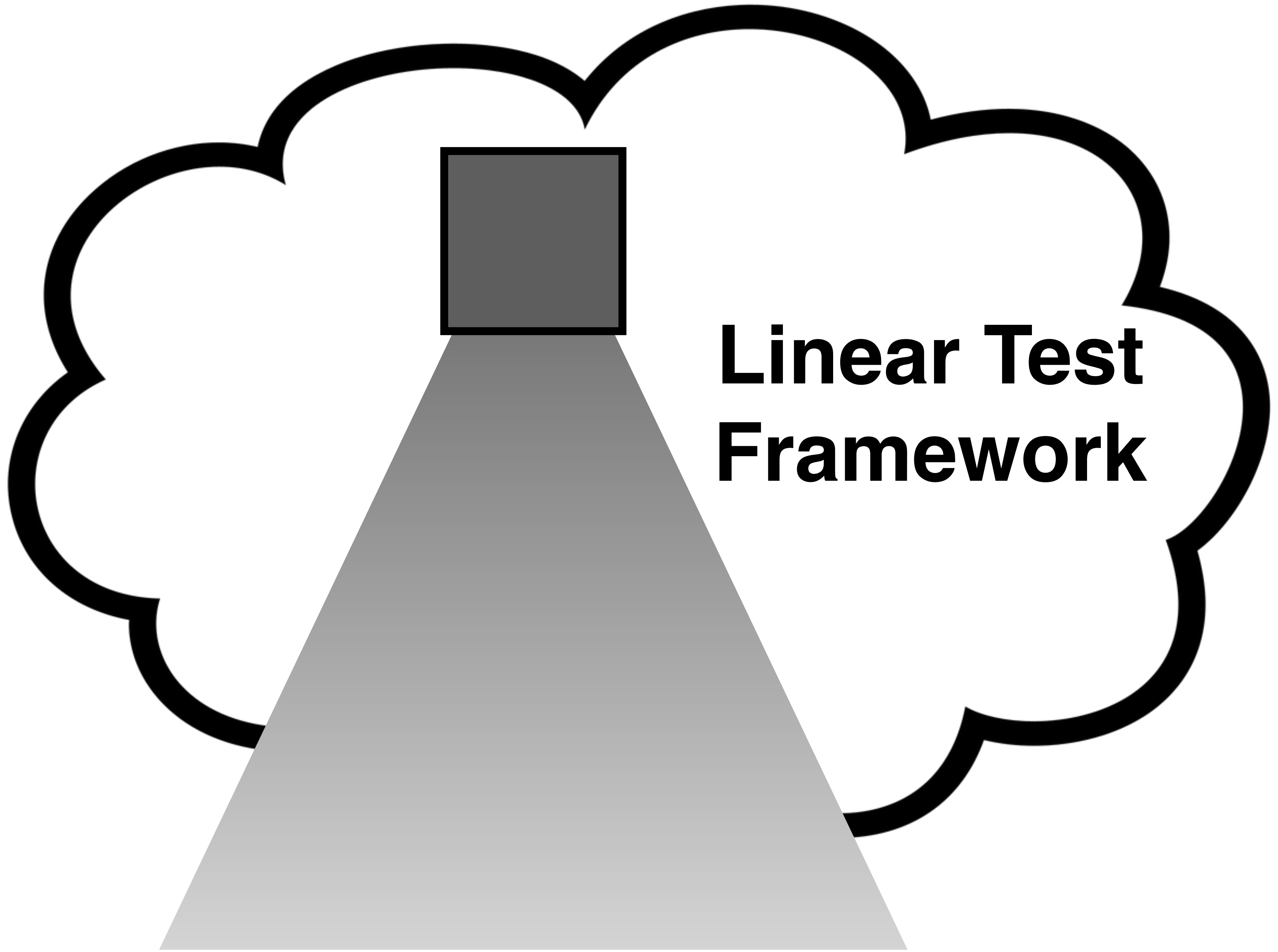

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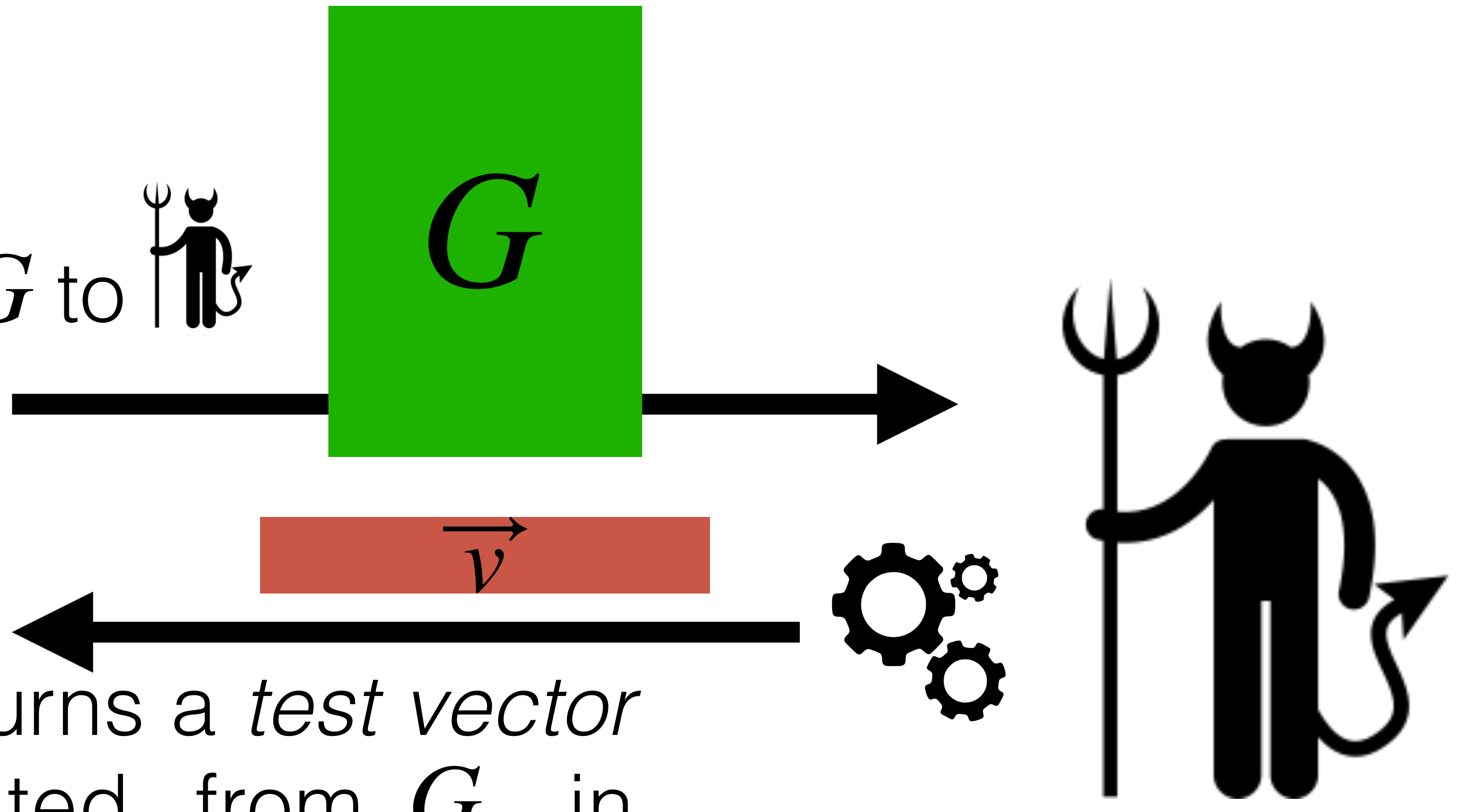

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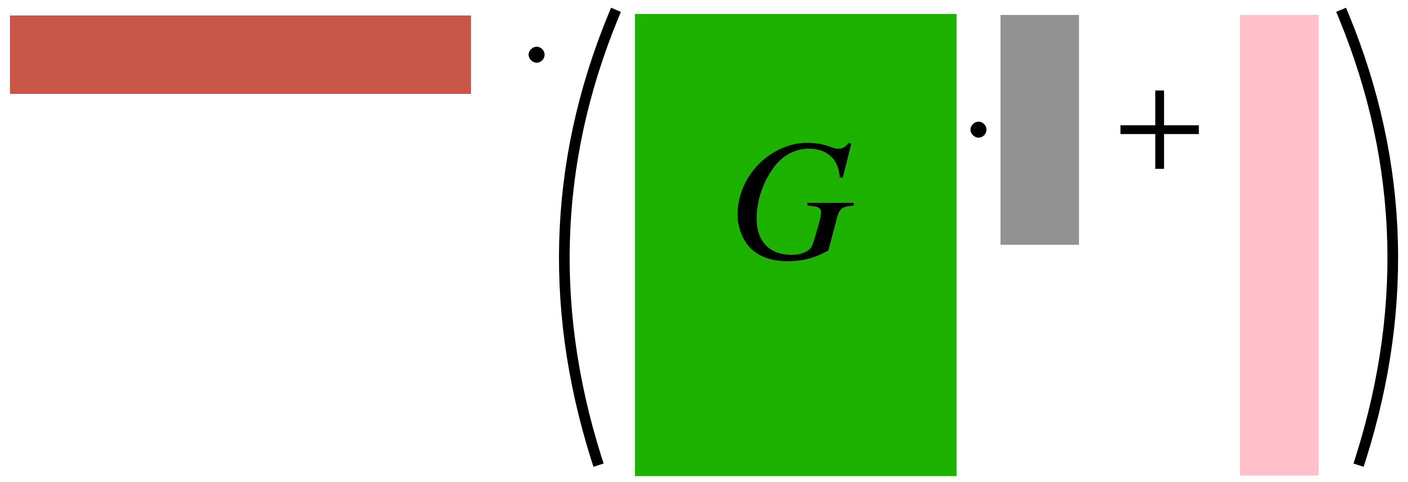

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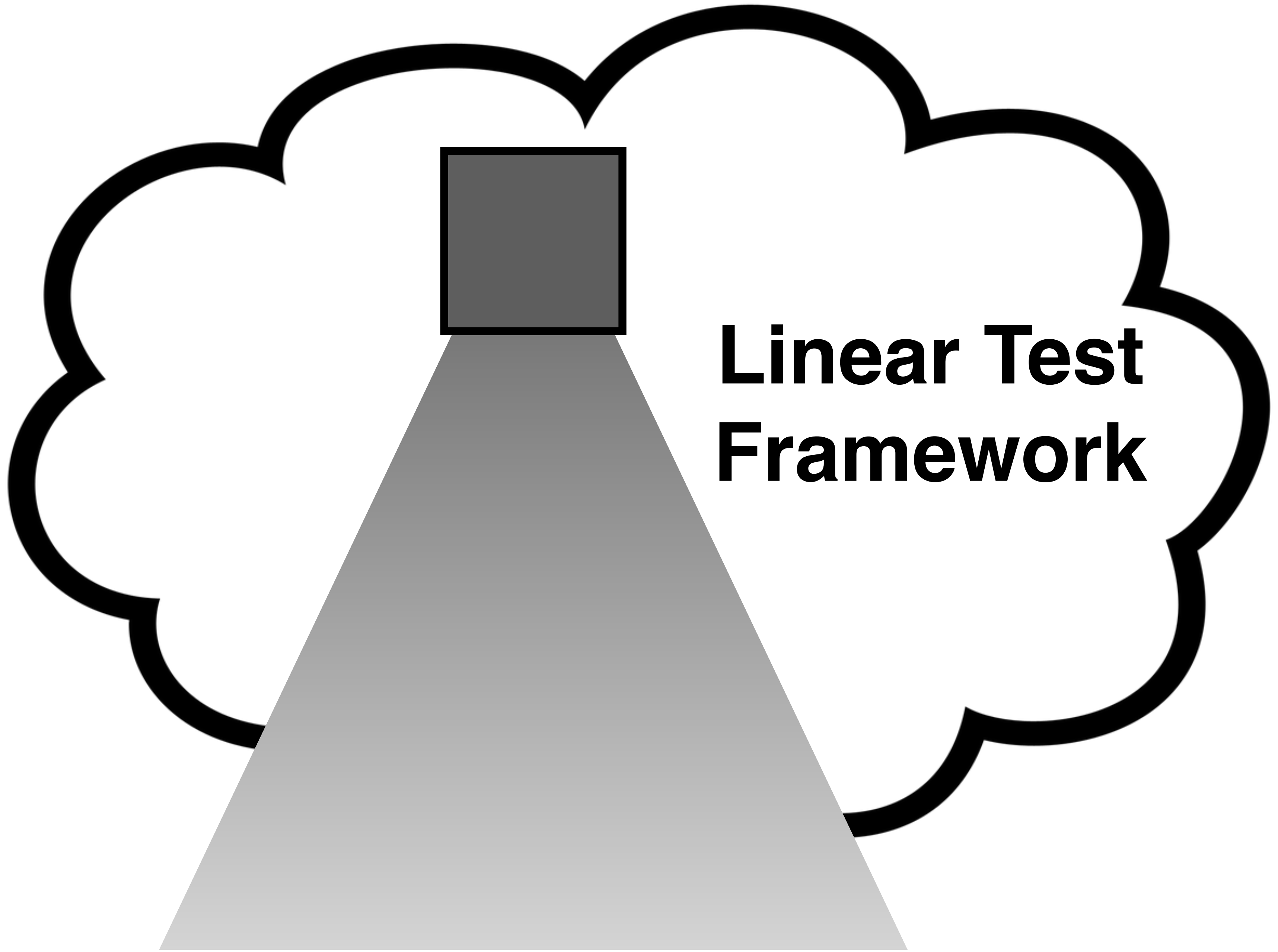

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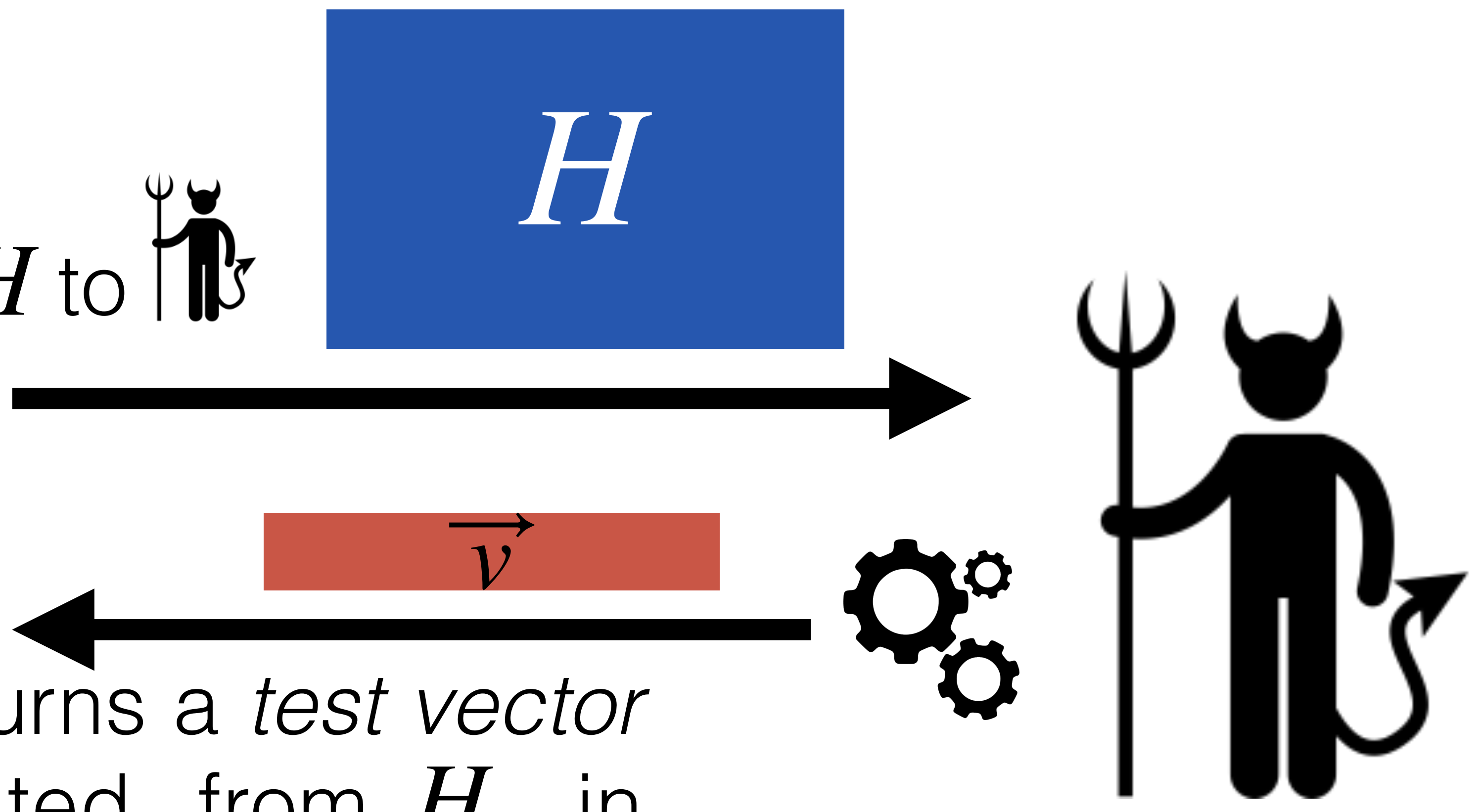

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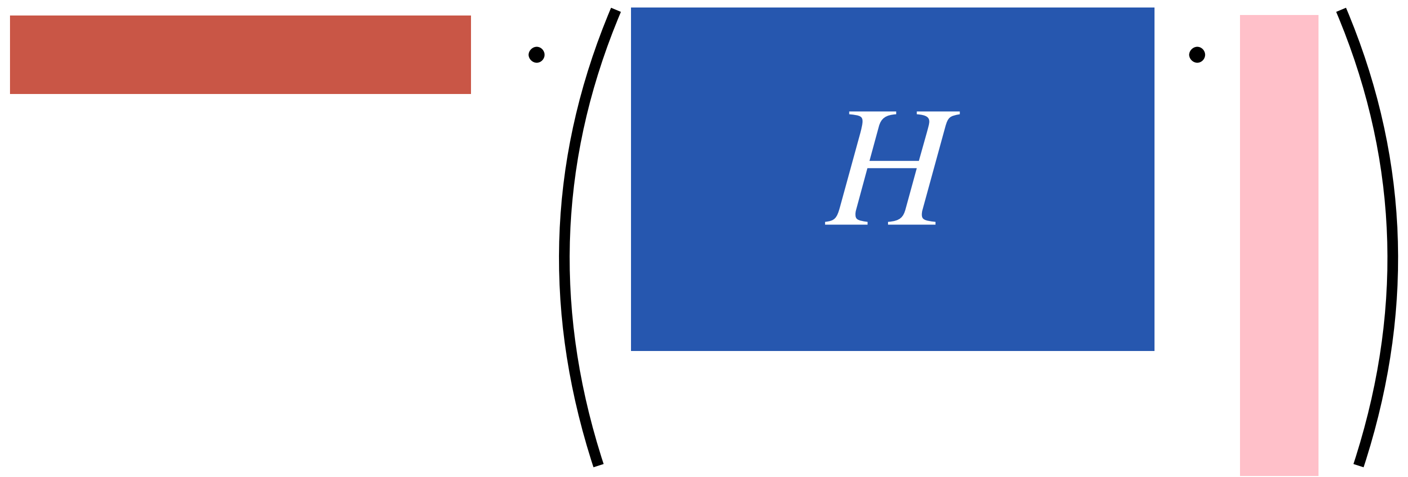

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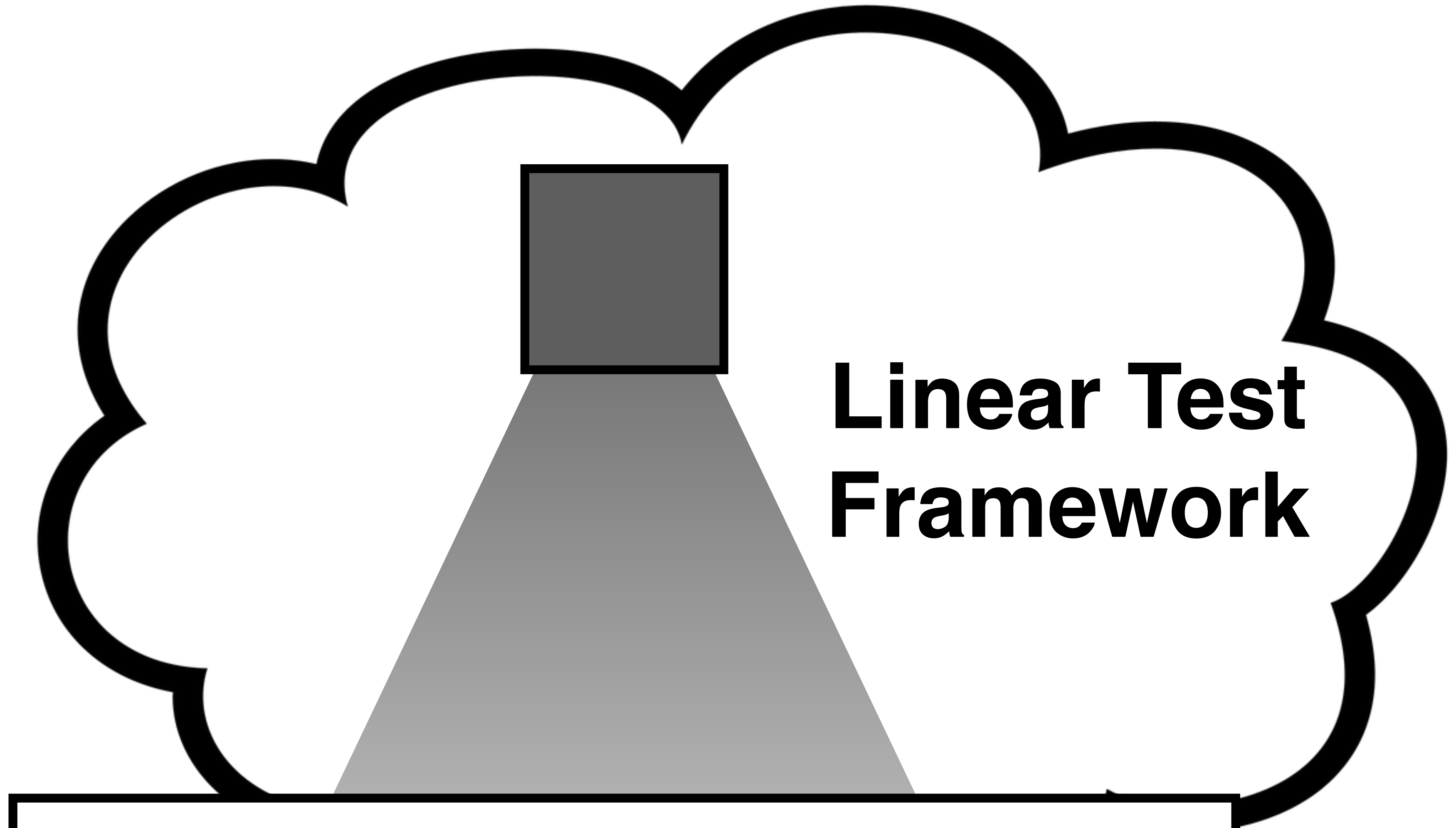

(*): highly structured algebraic codes (e.g. Reed-Solomon) are a different beast

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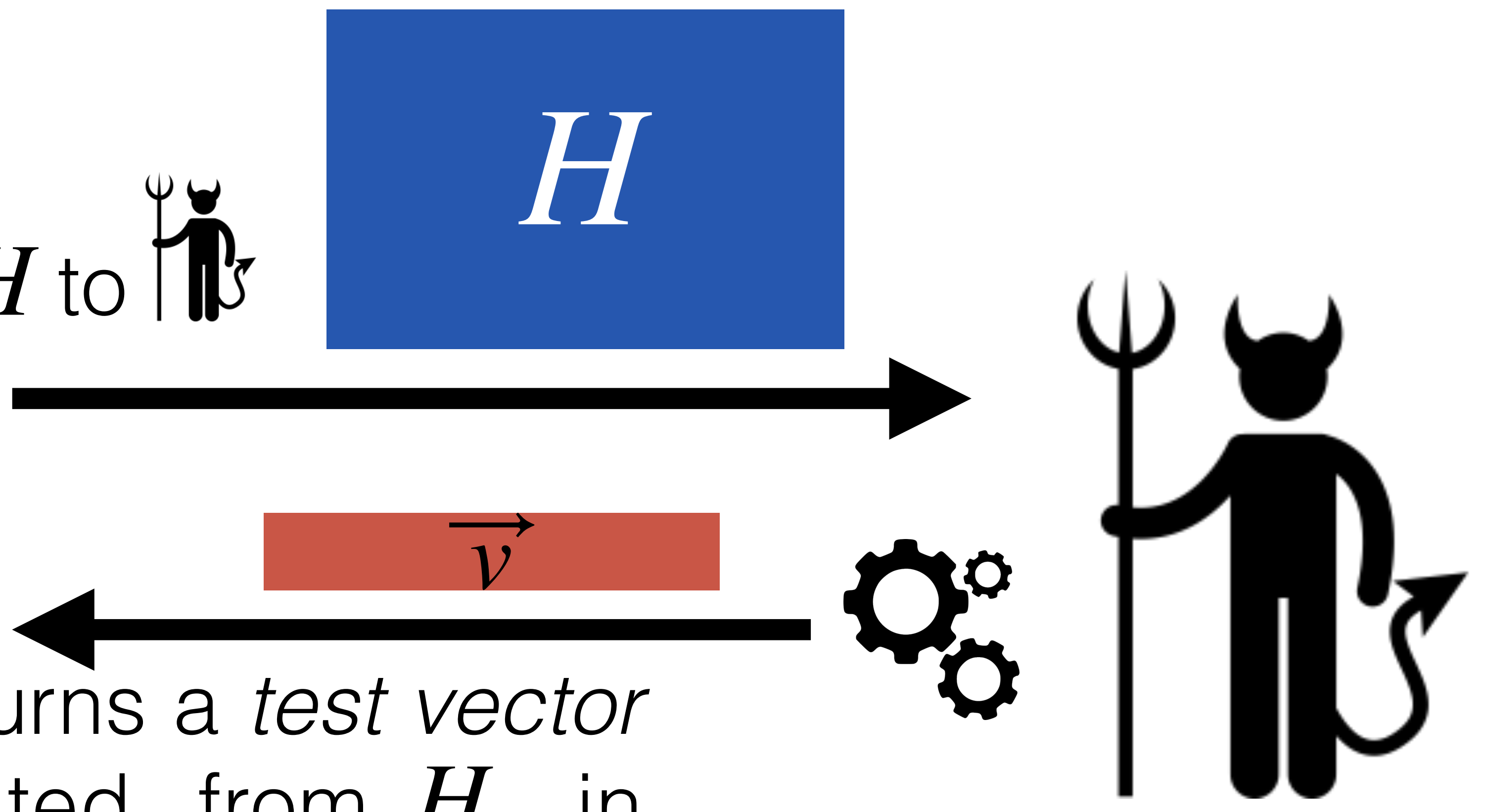

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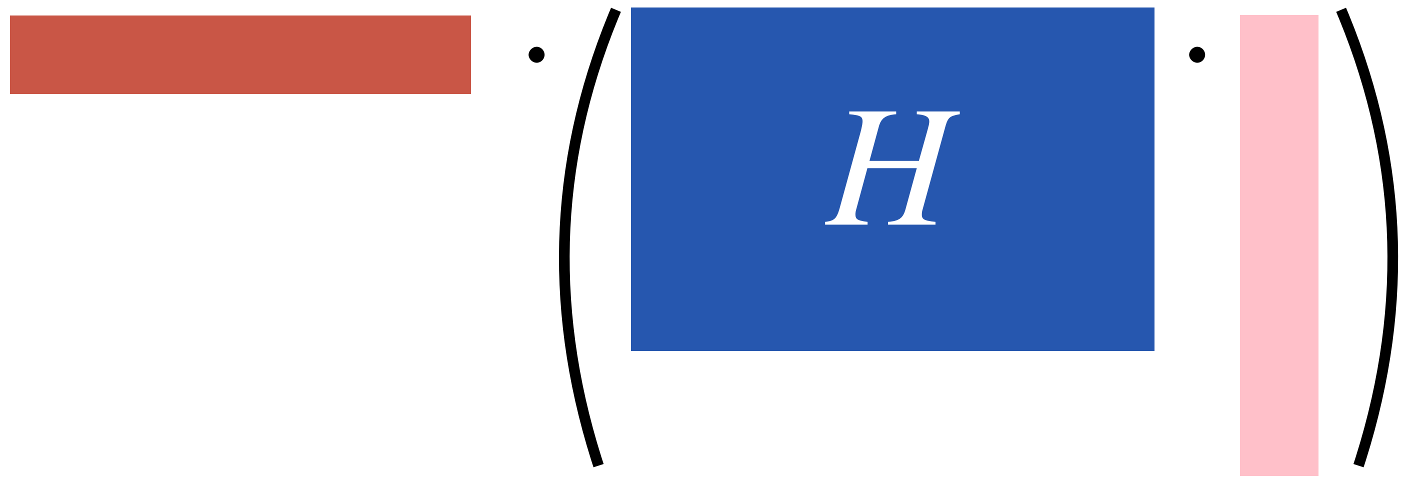

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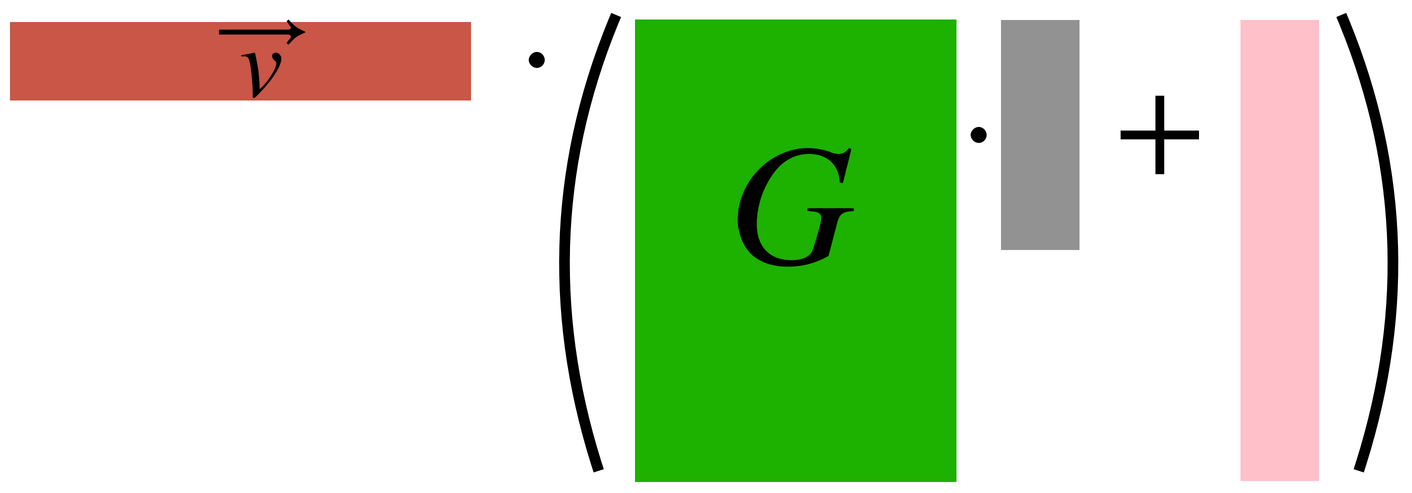

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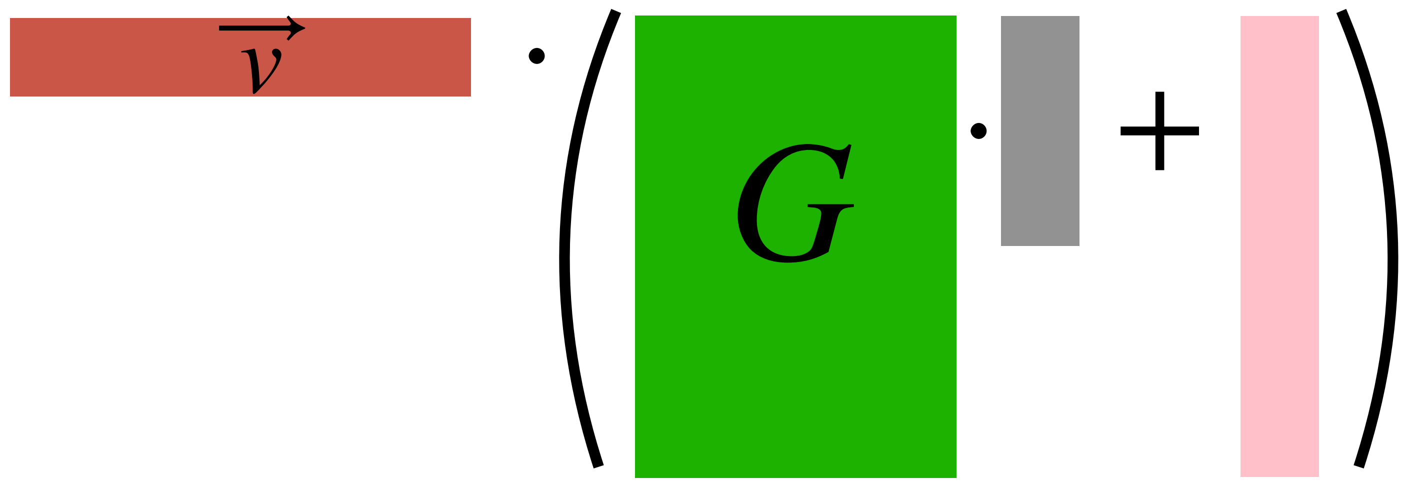

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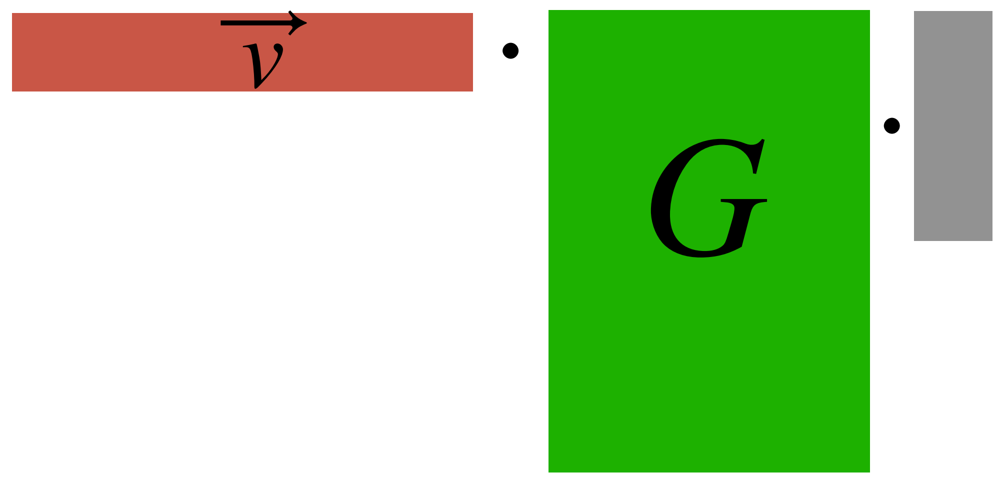
Induced by the *codeword*



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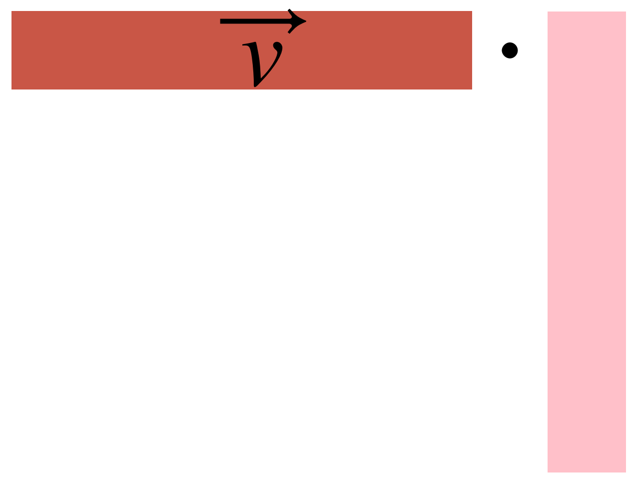

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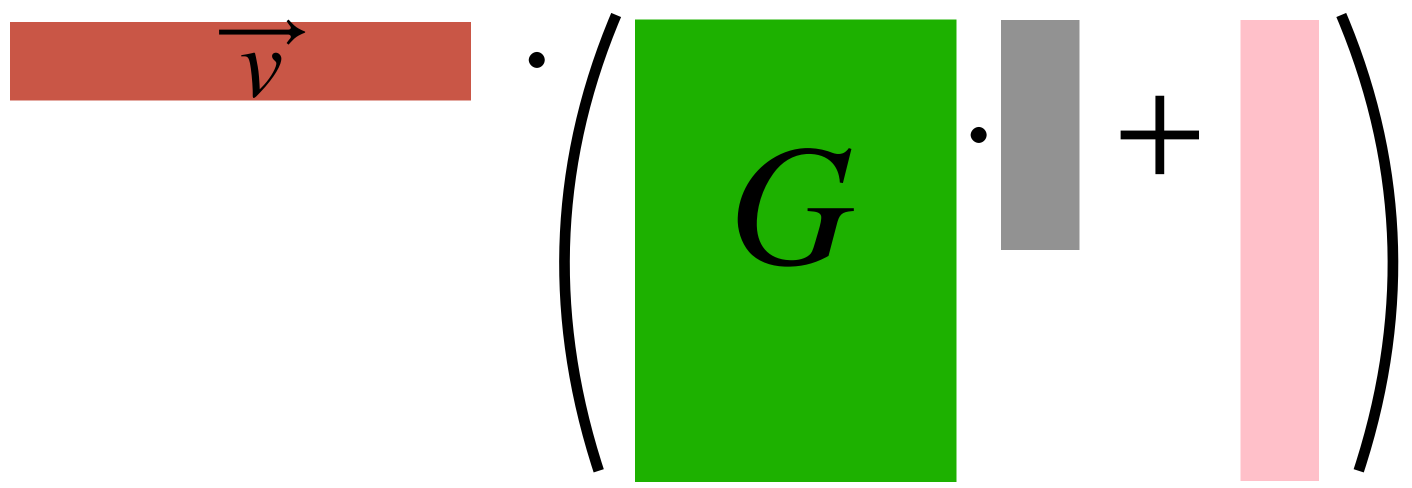

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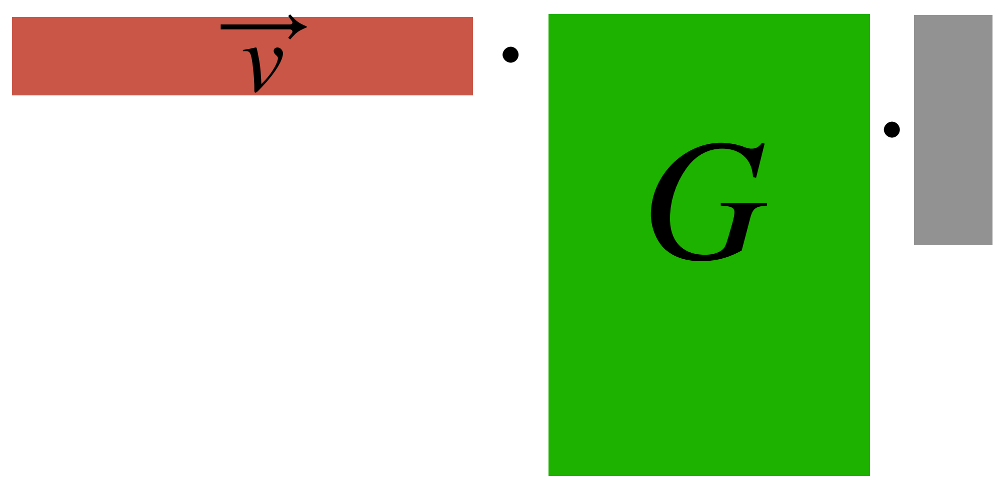

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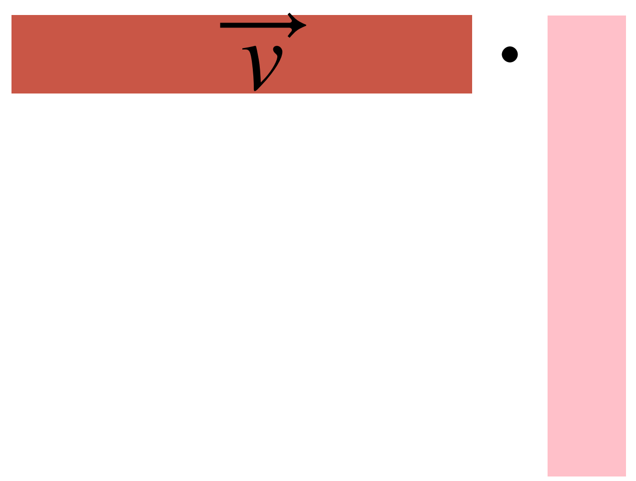

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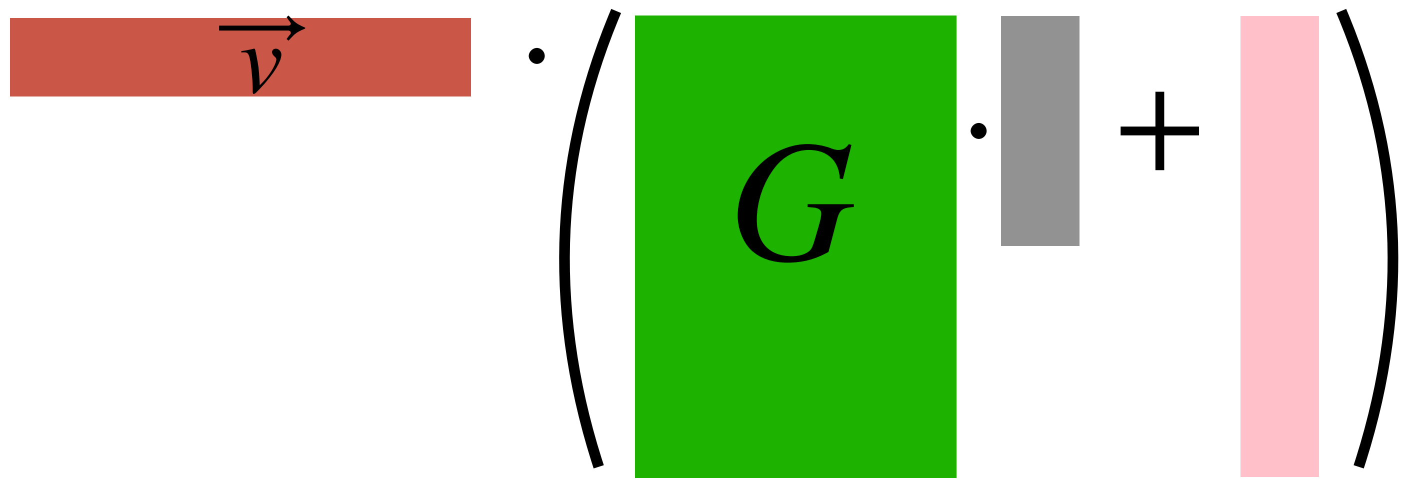

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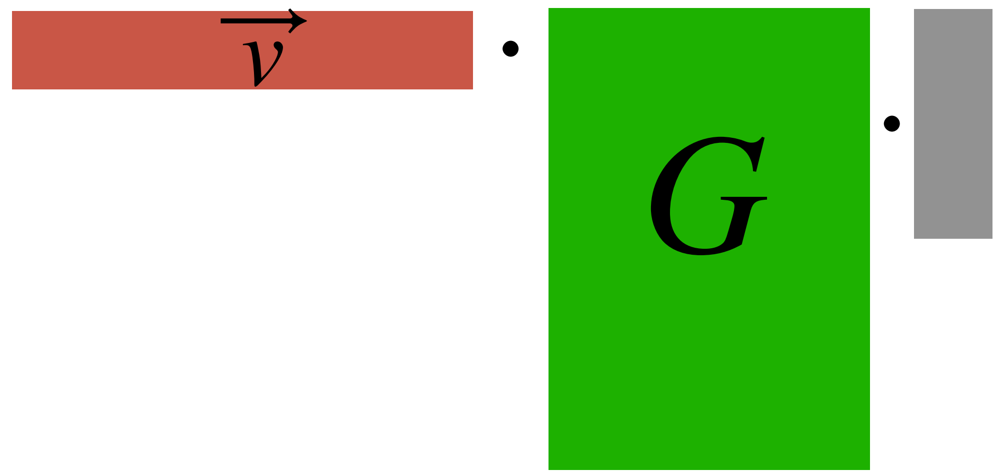

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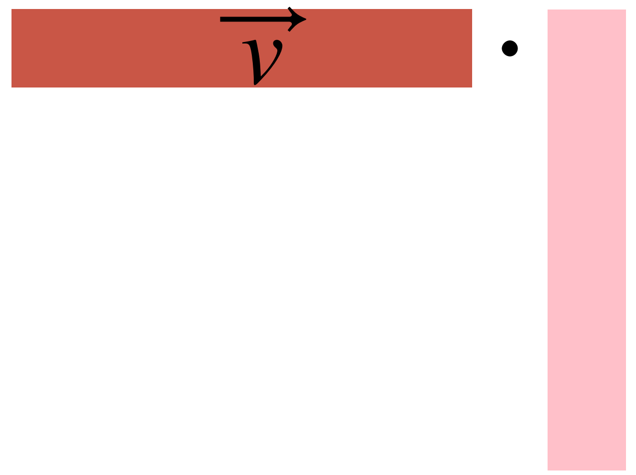

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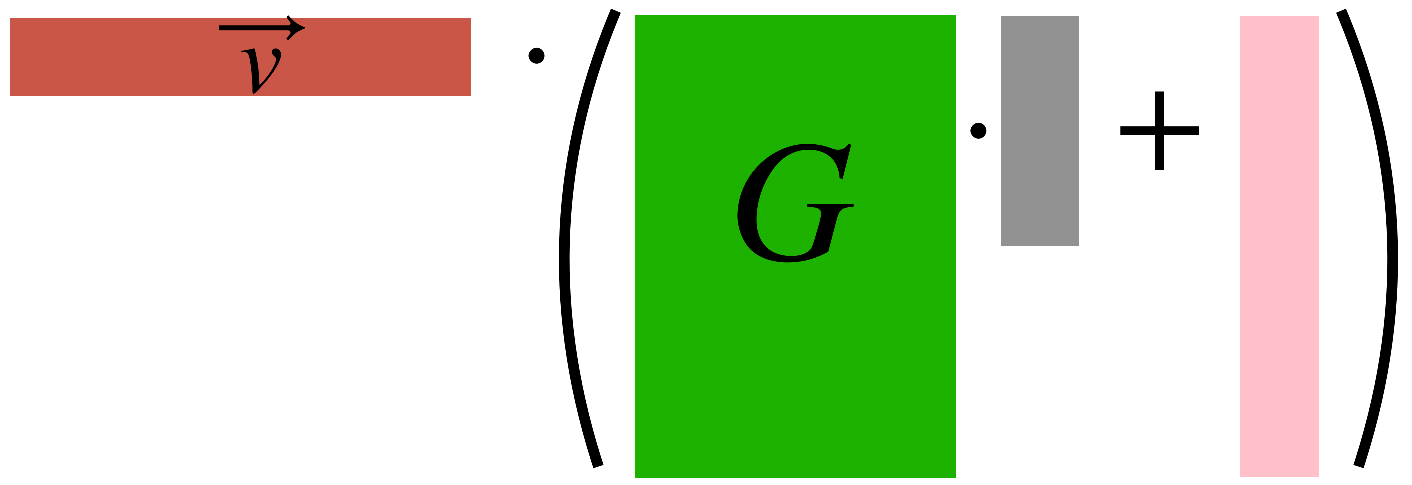

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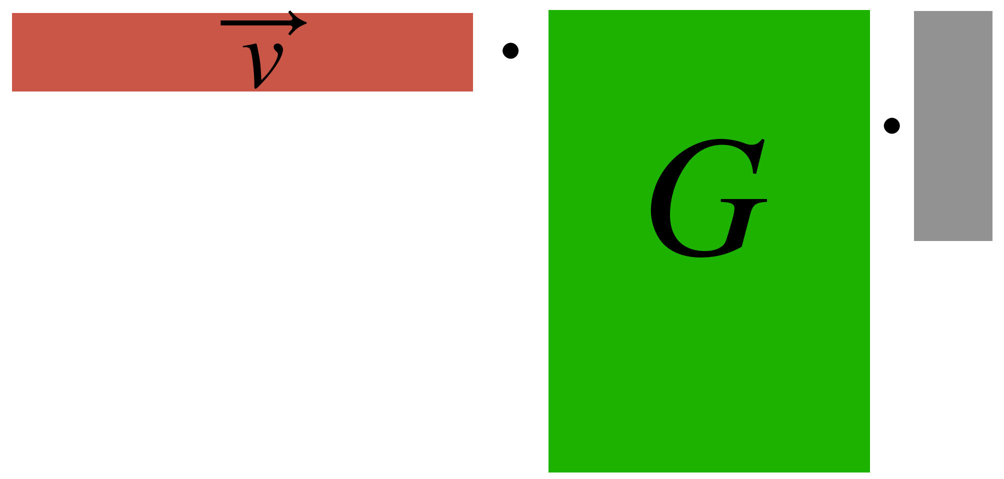

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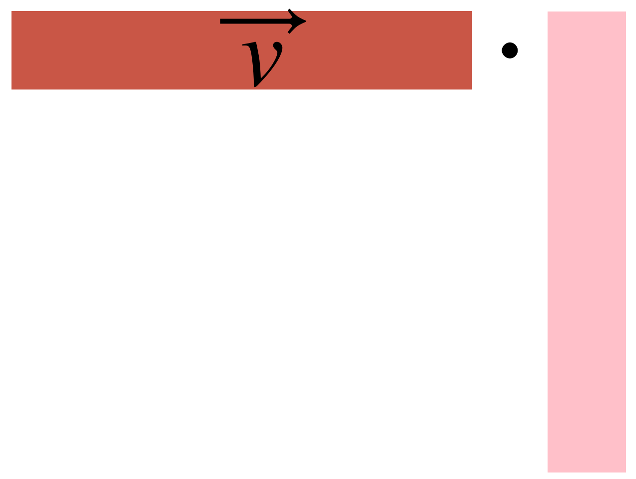

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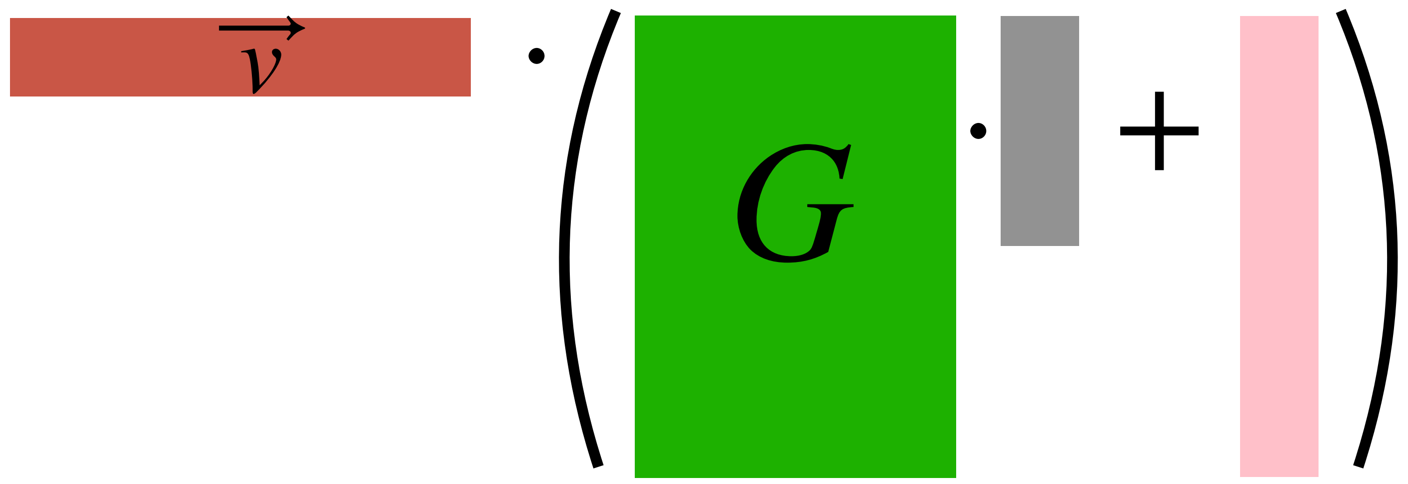

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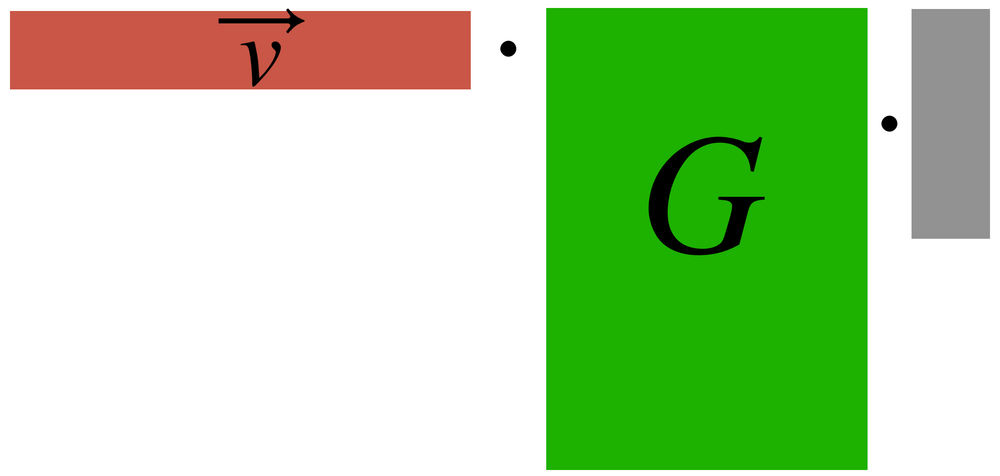

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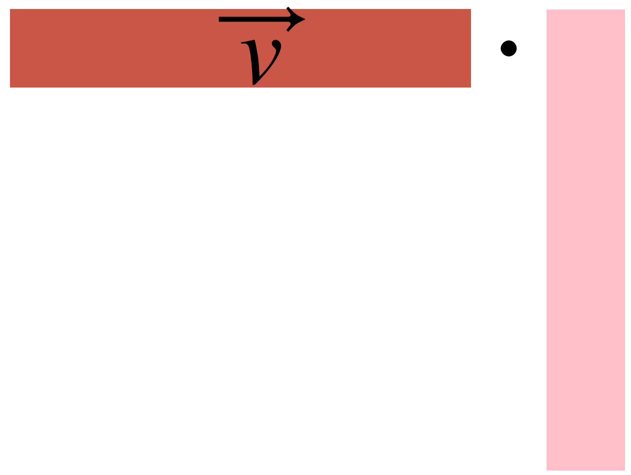

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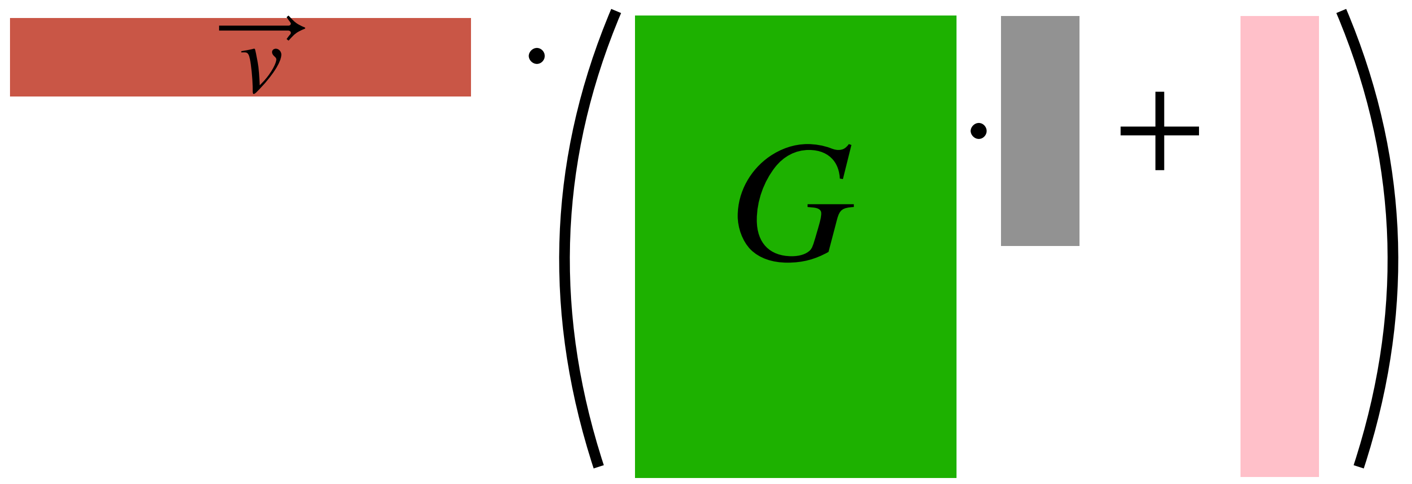

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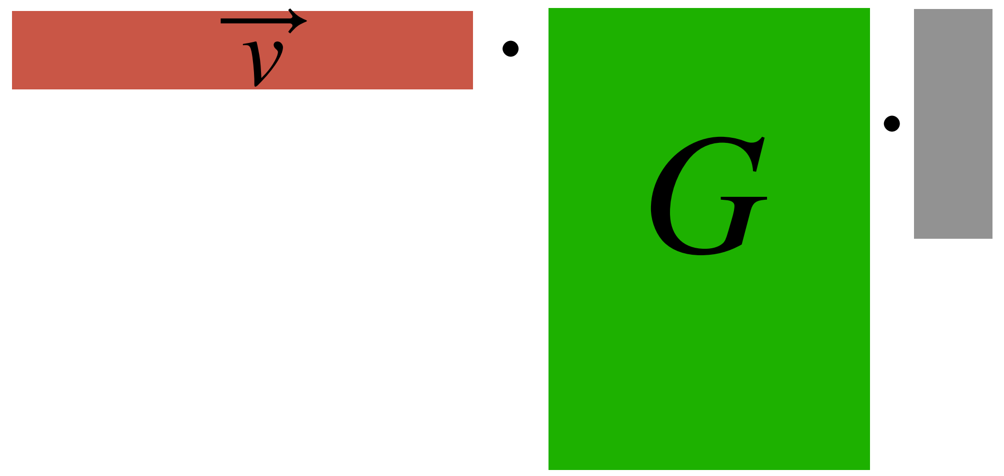

Protects against *light* linear tests

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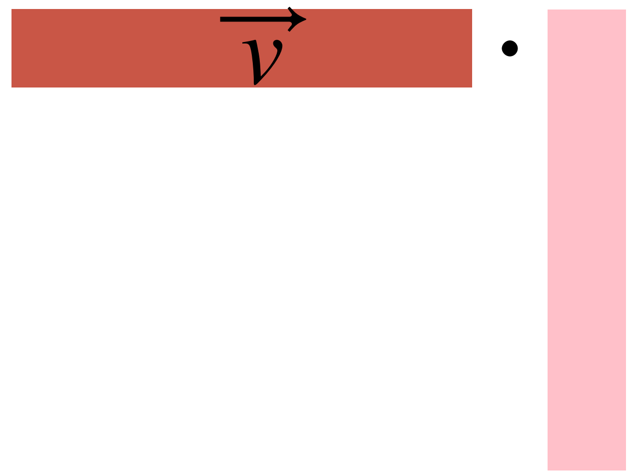

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Every subset of $O(n)$ rows of G is linearly independent
$\Longleftrightarrow$ the left-kernel of G does not contain nonzero vector of weight less than $O(n)$
$\Longleftrightarrow$ the *dual code* of G, i.e., the code generated by *the transpose of its parity check matrix* H, has linear minimum distance

‘**Provable’ candidates:** recursive codes such as GDP, Spielman, Druk-Ishai (lack concrete efficiency).
Heuristic / experimental candidates: [CRYPTO:**C**RS21] (based on Tillick-Zémor LDPC codes)
More to come in the future

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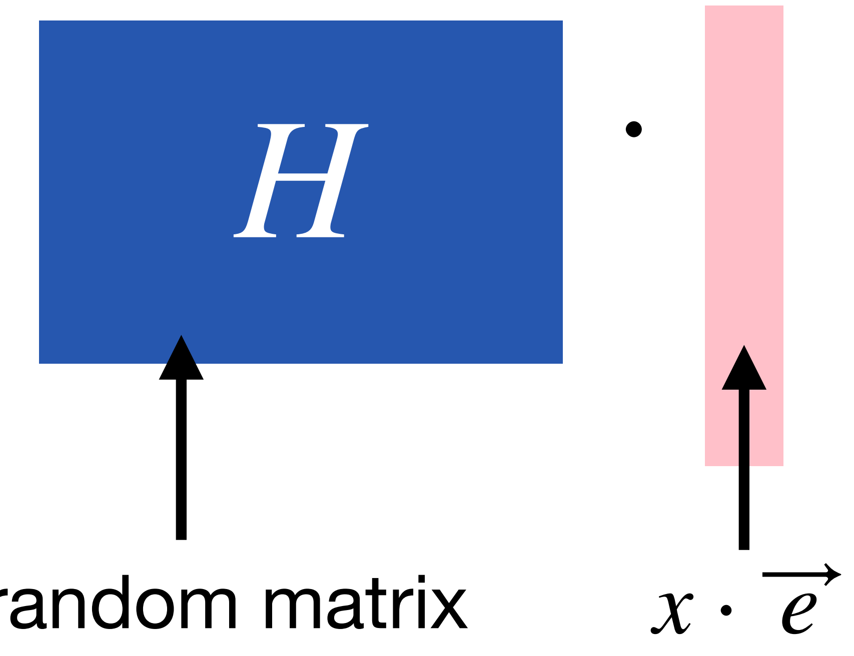

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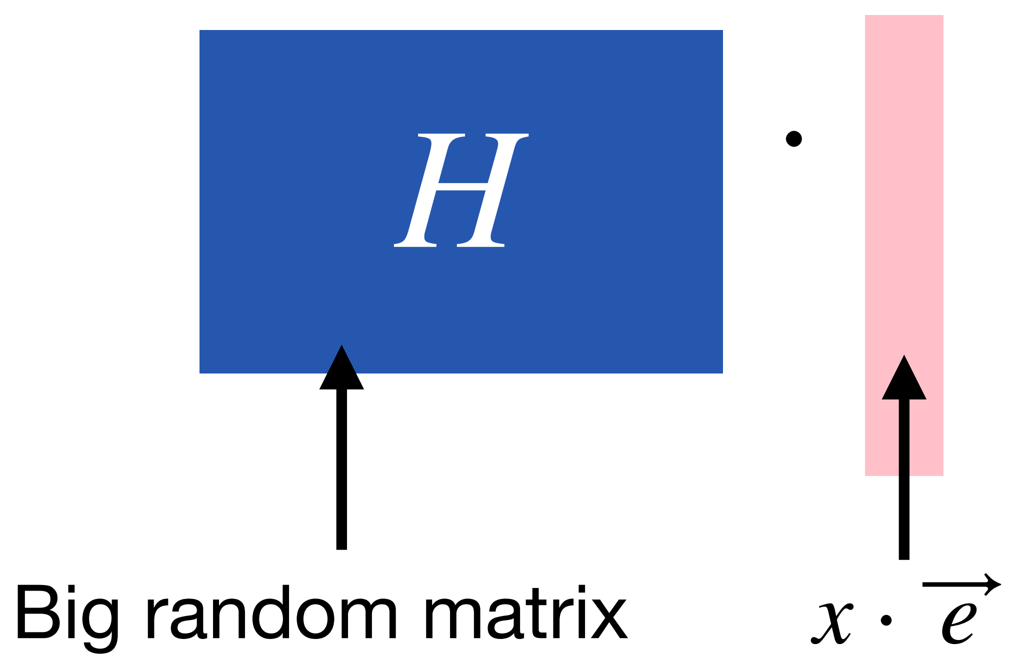
Every subset of $O(n)$ rows of G is linearly independent
$\iff$ the left-kernel of G does not contain nonzero vector of weight less than $O(n)$
$\iff$ the *dual code* of G, i.e., the code generated by *the transpose of its parity check matrix* H, has linear minimum distance

The expansion of the PCG boils down to the computation of



Where $\overrightarrow{e}$ is a very sparse vector, and (the shares of) the entries of $x \cdot \overrightarrow{e}$ can be computed individually in log-time.

We want to find a matrix $M = H^\intercal$ such that:

- The code generated by M is a good code
- Computing $M^\intercal \cdot \overrightarrow{v}$ takes time $O(n)$ for any $\overrightarrow{v}$

‘**Provable’ candidates:** recursive codes such as GDP, Spielman, Druk-Ishai (lack concrete efficiency).
Heuristic / experimental candidates: [CRYPTO:**C**RS21] (based on Tillick-Zémor LDPC codes)
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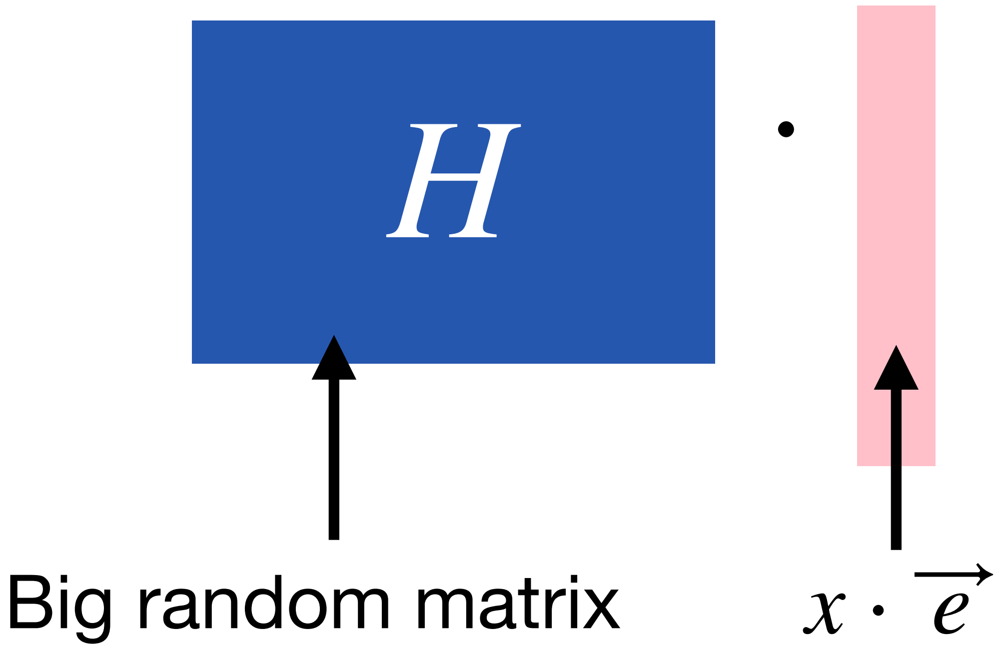

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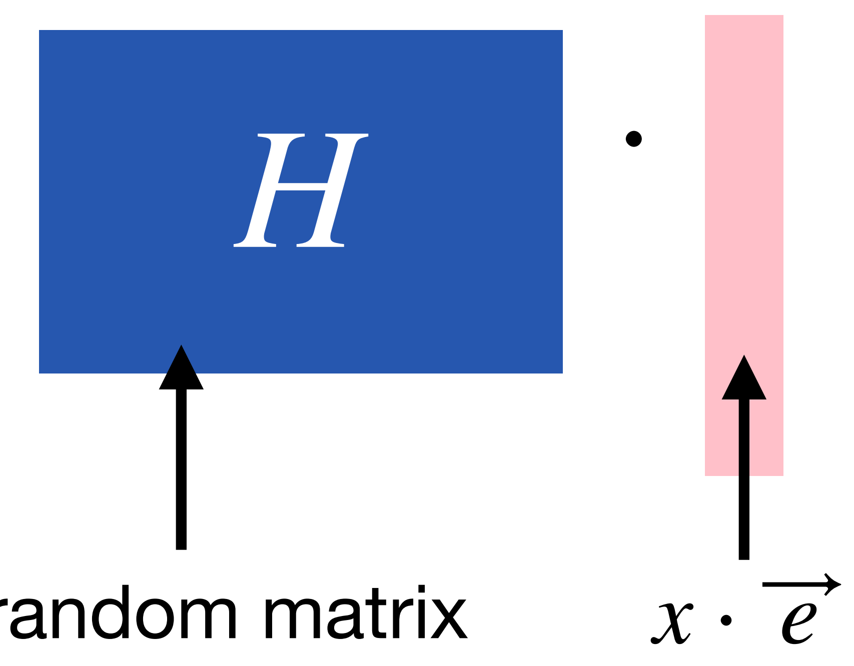

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$\Longrightarrow$ We need to find a *good* and *linear-time encodable* code. And we want it concretely efficient!

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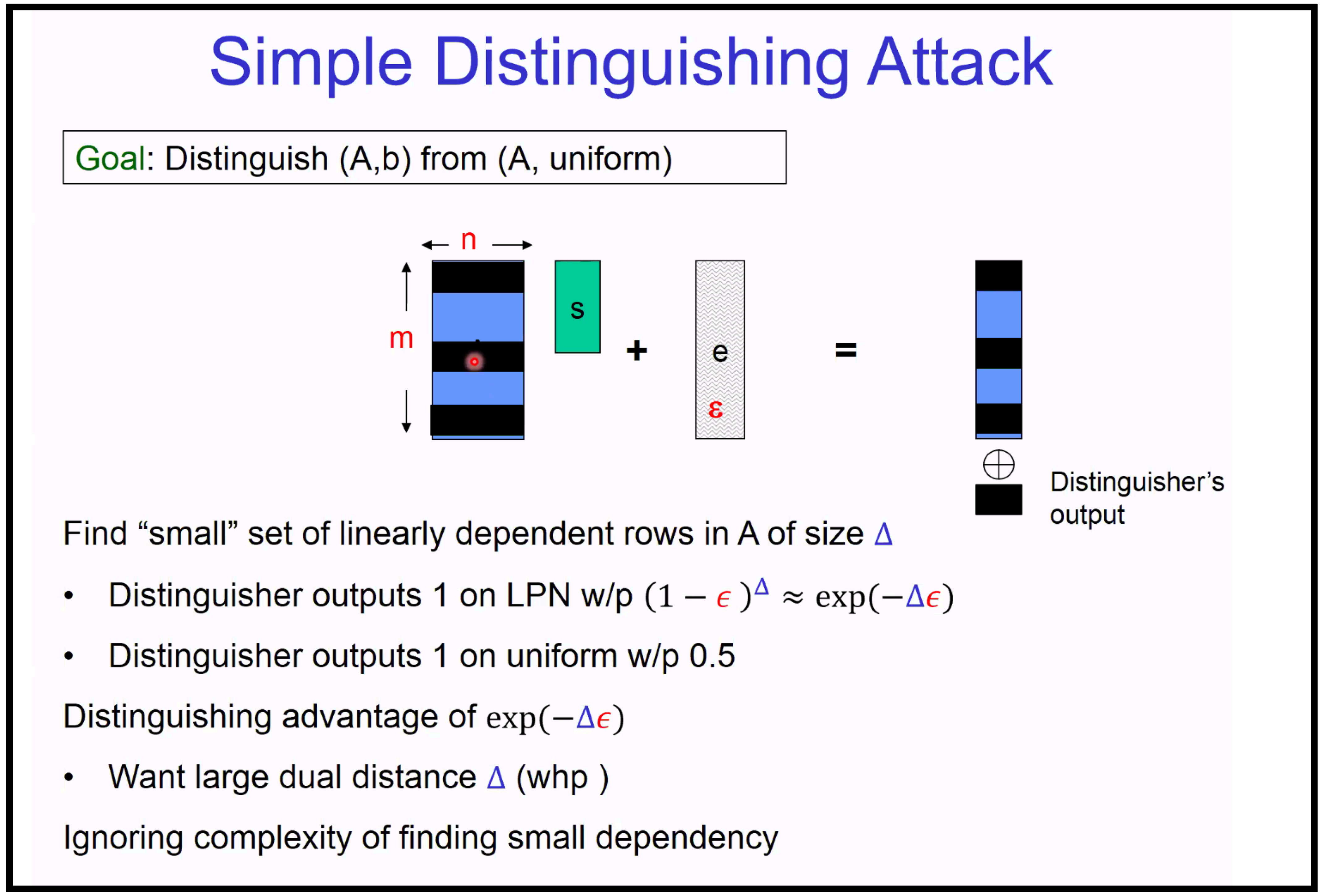
In a sense, this is a (very partial) converse to the result, described by Benny last Monday, that this condition is also a *necessary* condition.

(Benny's slide)



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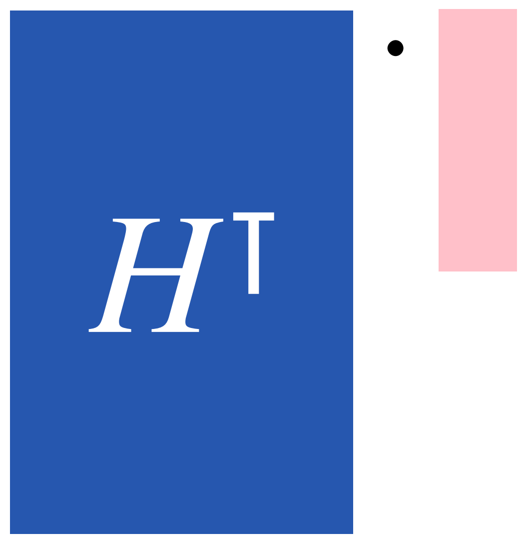
$\iff$ computing 

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Core idea: use a *sparse* M which can be brought in *approximate lower triangular form:*

- We have fast encoder for such parity-check matrices
- We have good insights on the minimum distance of the associated code, e.g. Tillich-Zémor, ISIT'06

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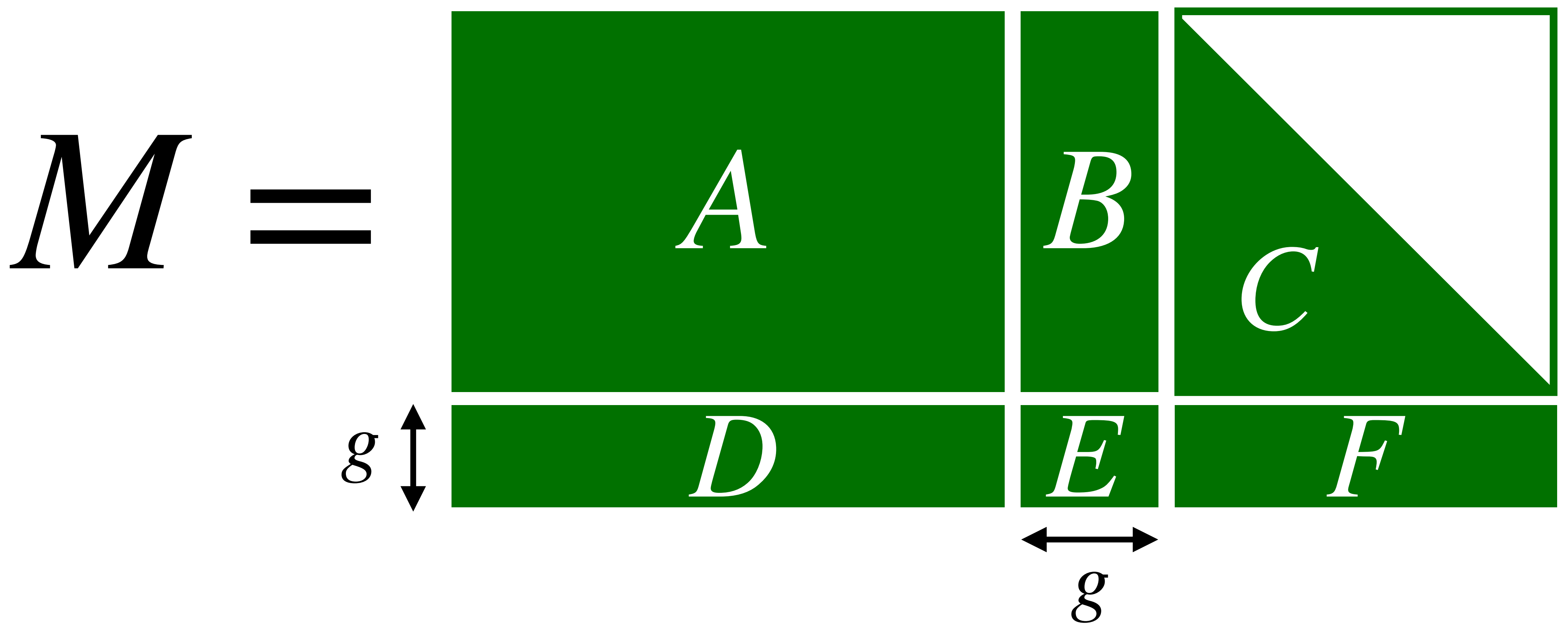

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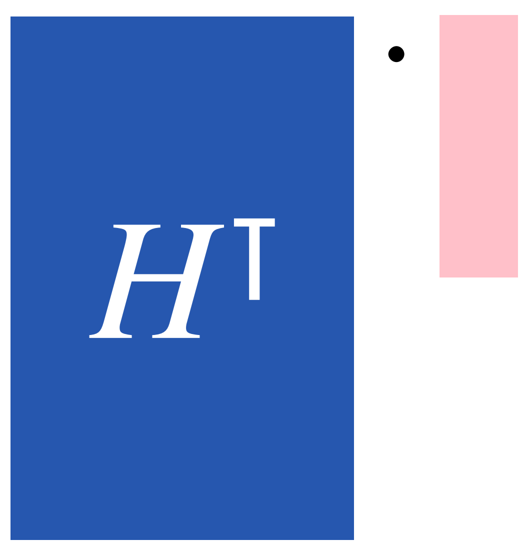
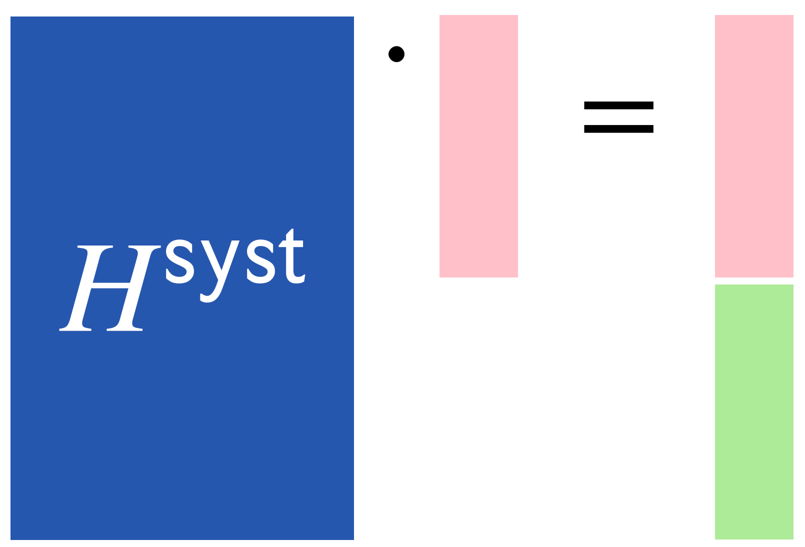
$\iff$ computing 

$\iff$ computing 

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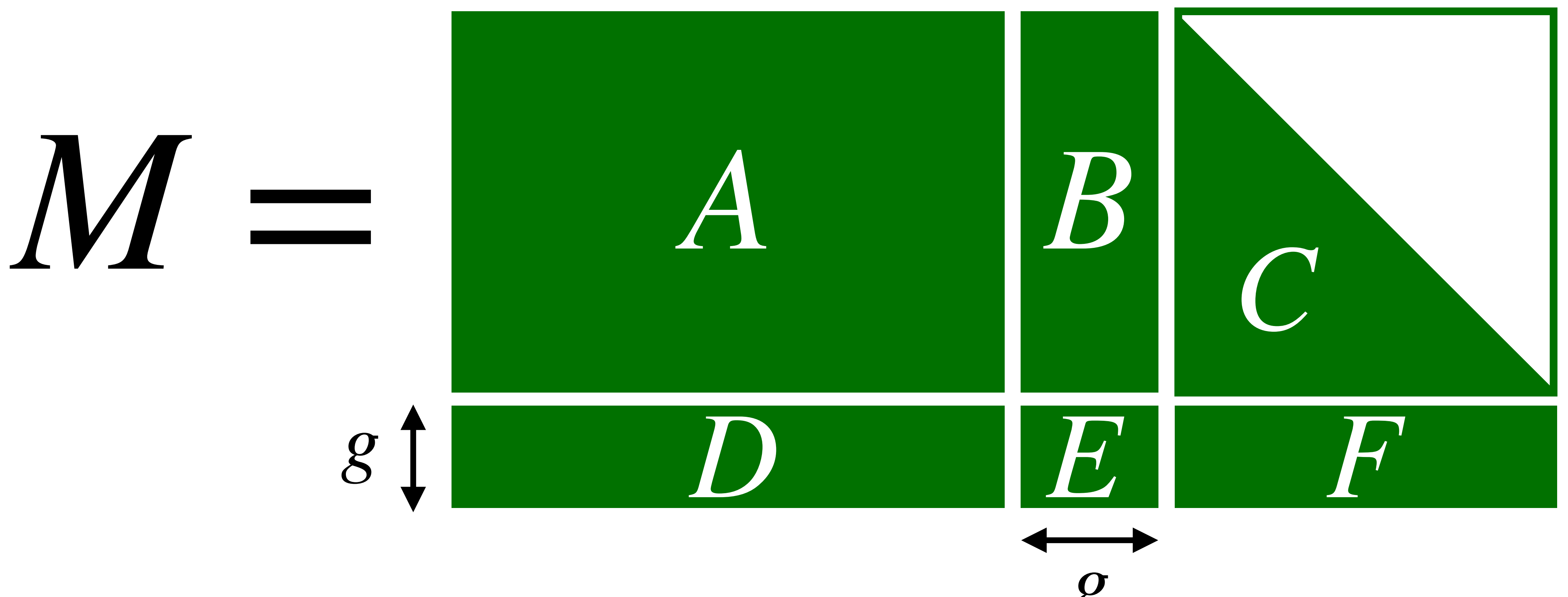
$\implies$ encode in time $O(n + g^2)$, linear if $g < \sqrt{n}$.



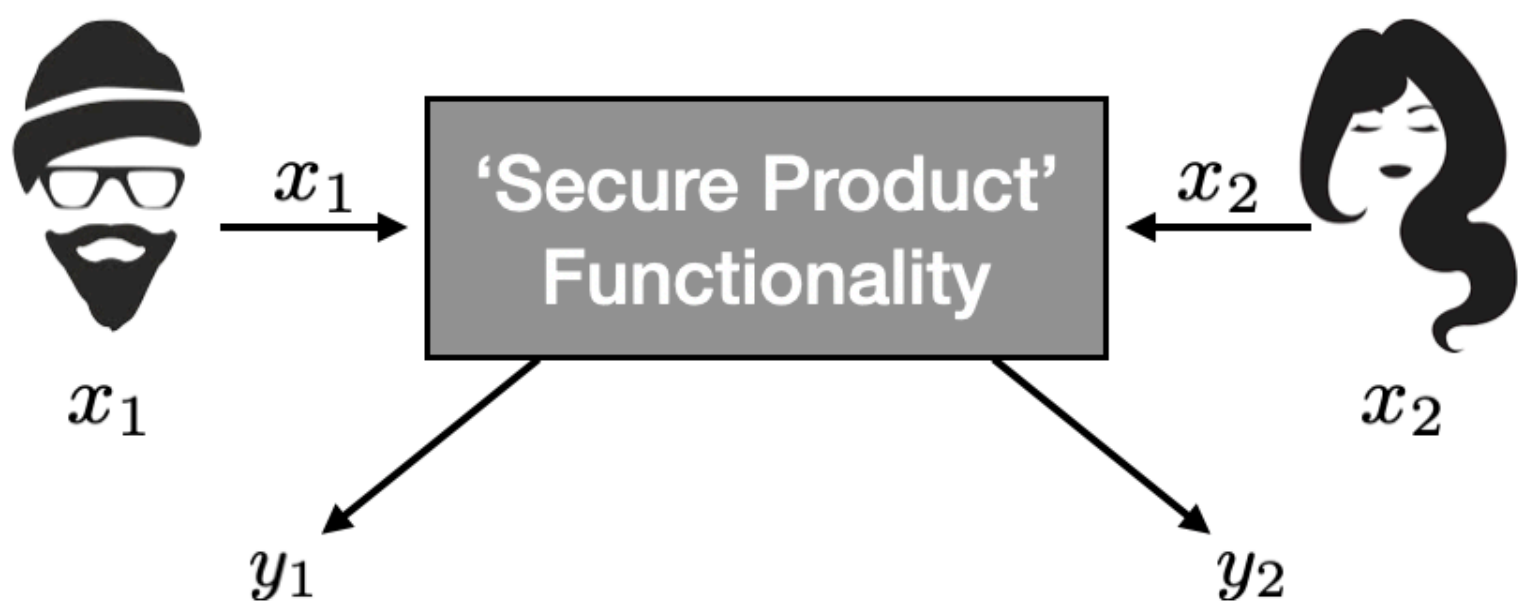
Thank You for Your Attention!

Questions?

Backup Slides

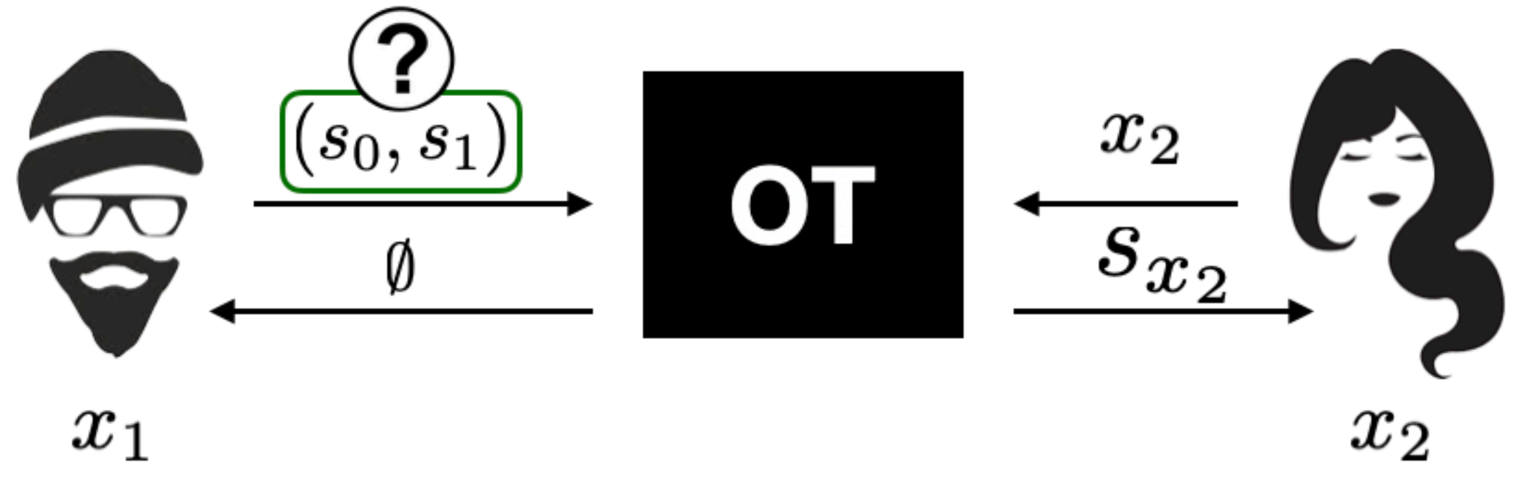
Secure Computation from Oblivious Transfer

Warm-up I: 2-Party Product Sharing



(y_1, y_2) random conditioned on $y_1 \oplus y_2 = x_1 x_2$

Step-by Step Solution



- We use an OT functionality where Alice is the receiver, and her *selection bit* is her input x_2
- What should be Bob's input? Let's work out the equation:

$$\begin{aligned} s_{x_2} &= x_2 \cdot s_1 + (1 - x_2) \cdot s_0 \\ &= x_2 \cdot s_1 \oplus (1 \oplus x_2) \cdot s_0 \\ &= s_0 \oplus (s_0 \oplus s_1) \cdot x_2 \end{aligned}$$

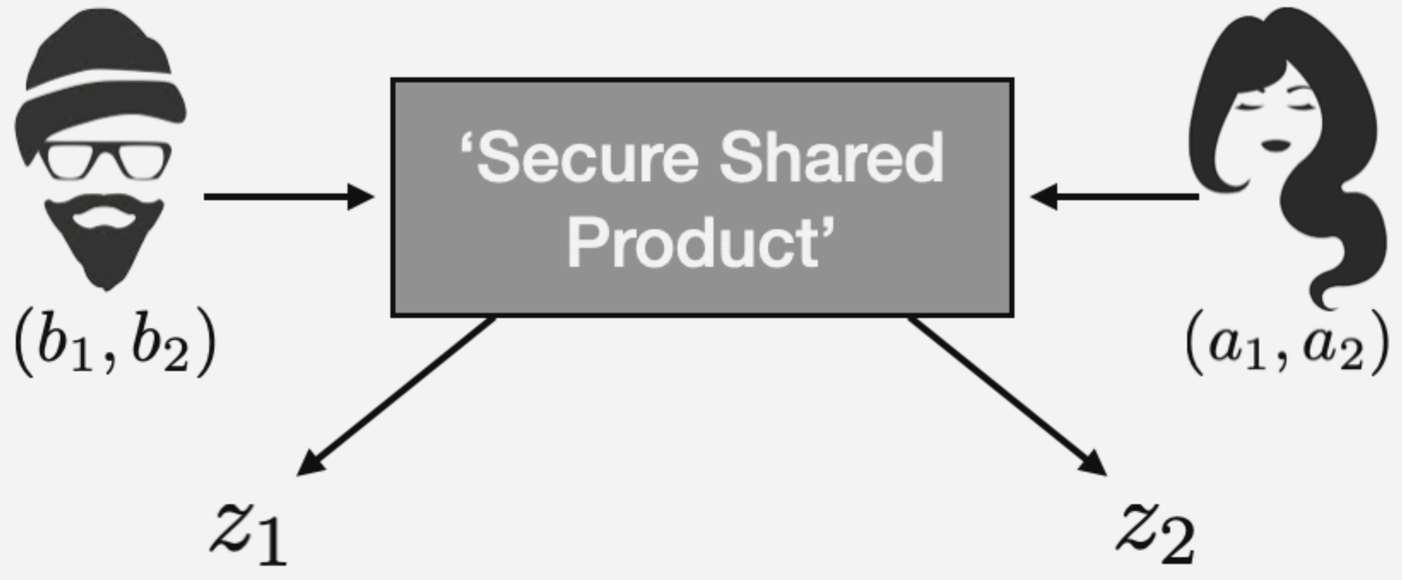
$$\implies \boxed{s_0} \oplus s_{x_2} = \boxed{(s_0 \oplus s_1)} \cdot x_2$$

Share of Bob — This should be x_1

$\implies (s_0, s_1)$ are (2,2)-shares of x_1.

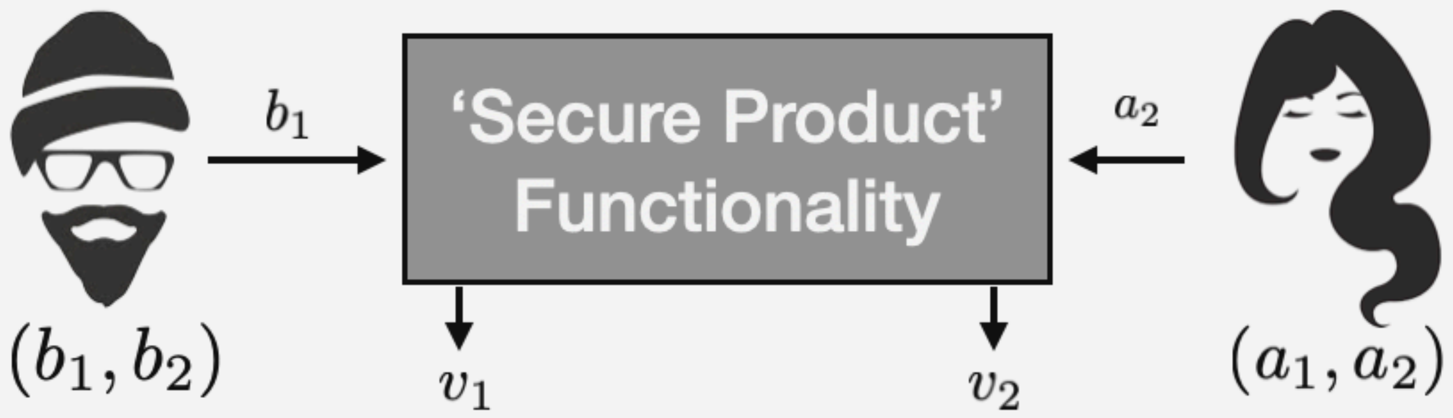
Warm-up II: Variant

This time, Alice and Bob start with *shares* of values (x,y), and want to compute shares of the product x.y



(a_1, b_1) are shares of x
(a_2, b_2) are shares of y
(z_1, z_2) are random shares of $z = x \cdot y$

Solution



$$\begin{aligned} x \cdot y &= (a_1 + b_1) \cdot (a_2 + b_2) \\ &= \boxed{a_1 \cdot a_2} + \boxed{a_1 \cdot b_2} + \boxed{a_2 \cdot b_1} + \boxed{b_1 \cdot b_2} \end{aligned}$$

Value known to Alice — Value known to Bob

Each of these values is the product of a value known to Alice and a value known to Bob