Secure Computation

Protecting the privacy of data used in distributed computation

FIC
Forum International de la Cybersécurité

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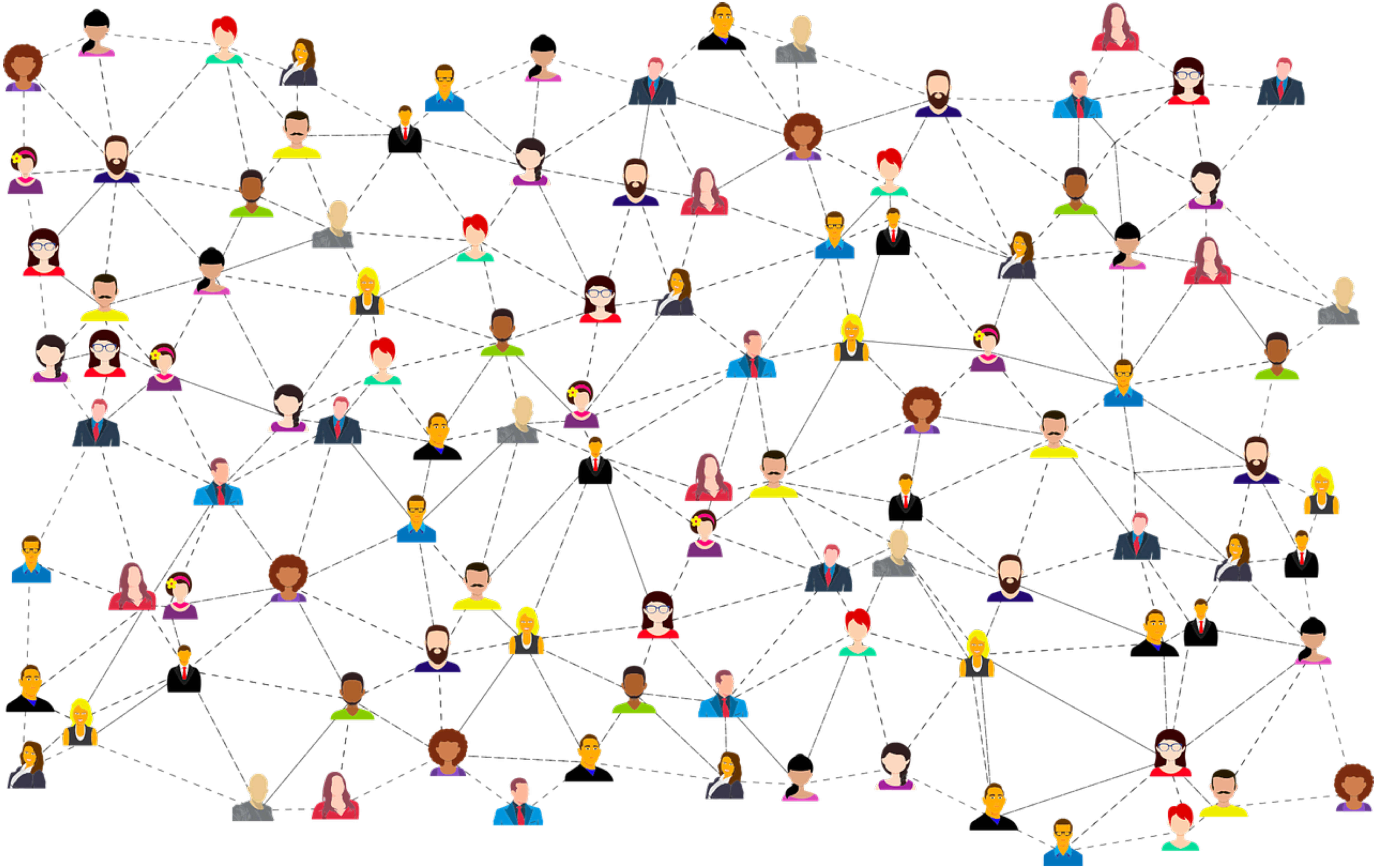
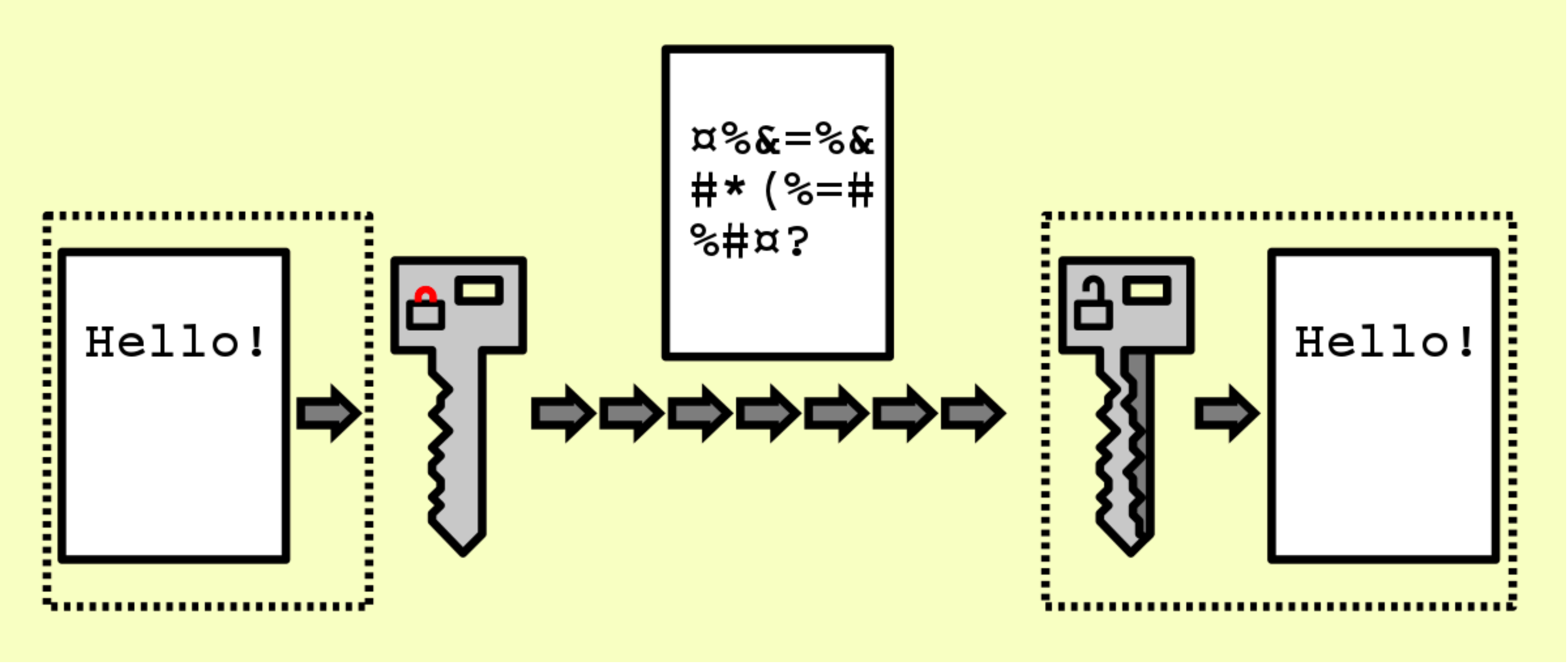
Are our Interactions over Large Networks Secure?



Are our Interactions over Large Networks Secure?

Our *Communications* are Mostly* Secure

Whenever we browse the web, use a website or an app, send a message, or make a call, we **communicate over a network**, and the content of our communication is private information. Most of the time*, this communication happens **securely**:



- Since 2020, **around 85% of the total internet traffic** is encrypted
- End-to-end encryption is becoming a standard on most messaging apps
- Cellular networks in France encrypt **all communications** by default

** not always!*

But our *Computations* are not!

Our use of networks has evolved: whenever we…

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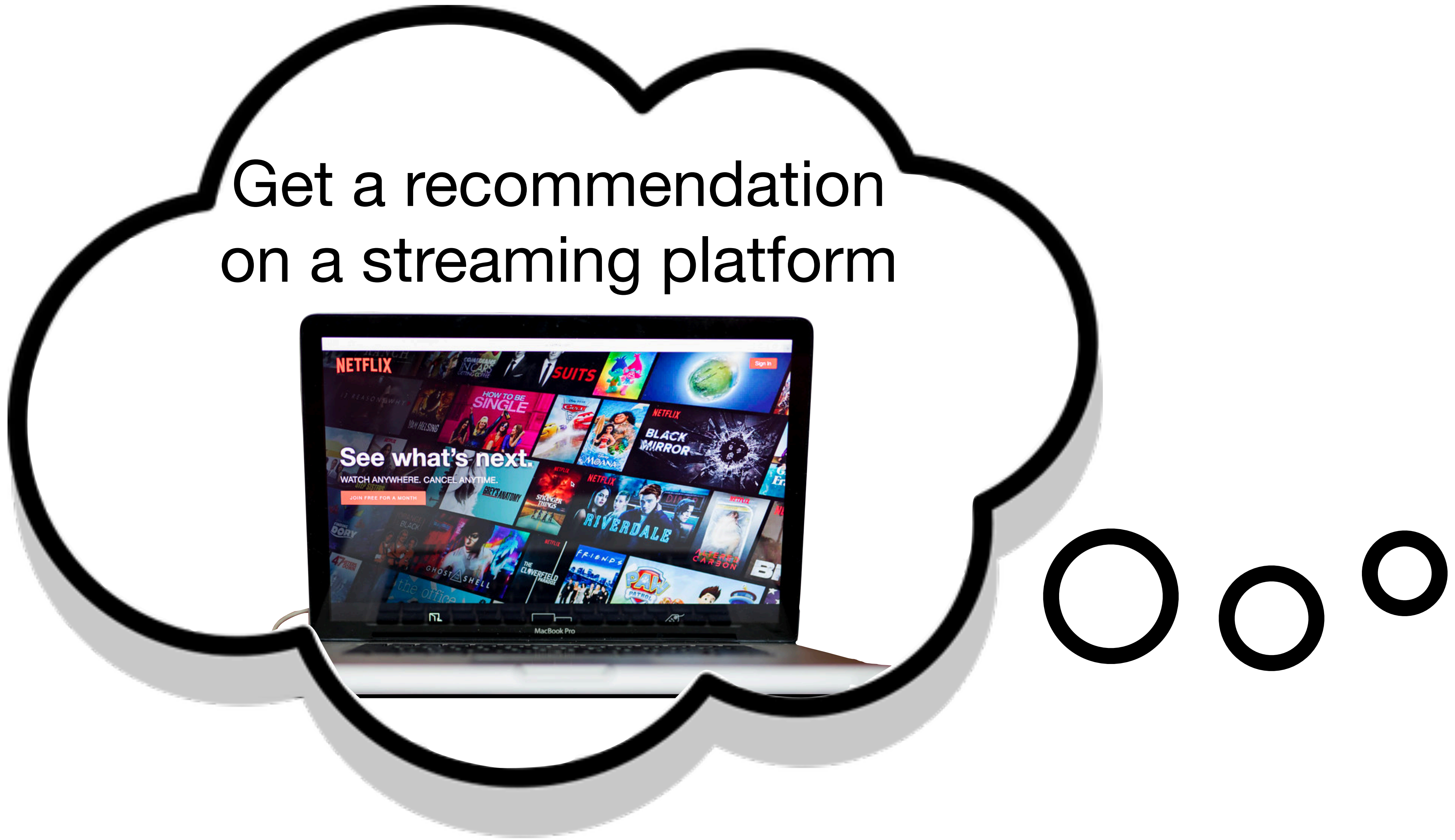

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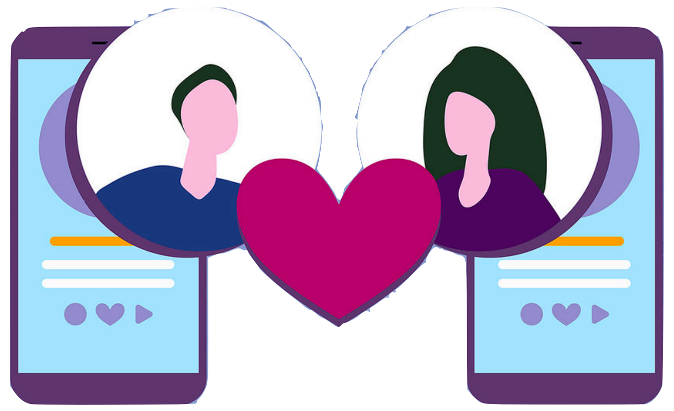

Get a recommendation on a streaming platform

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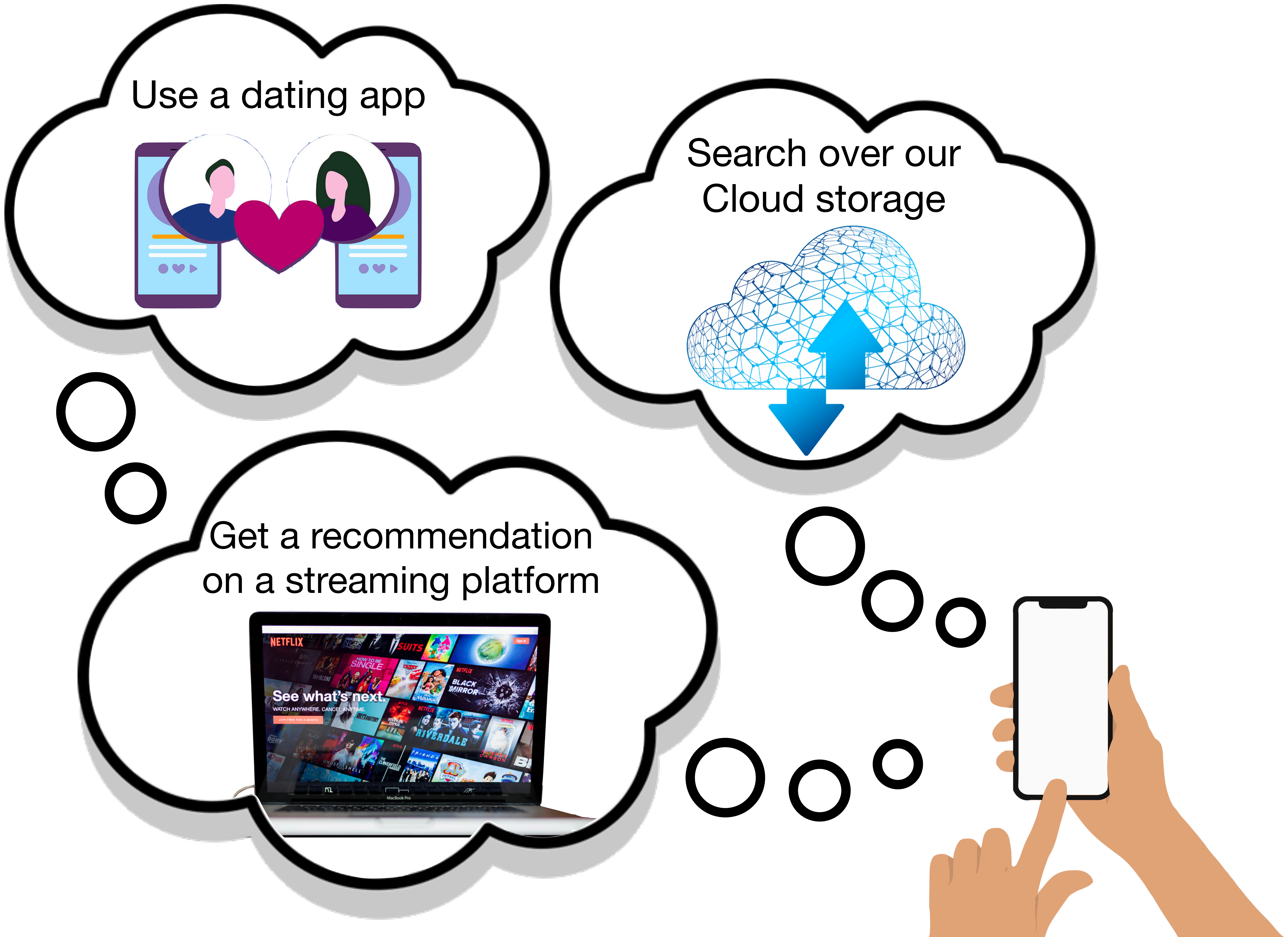

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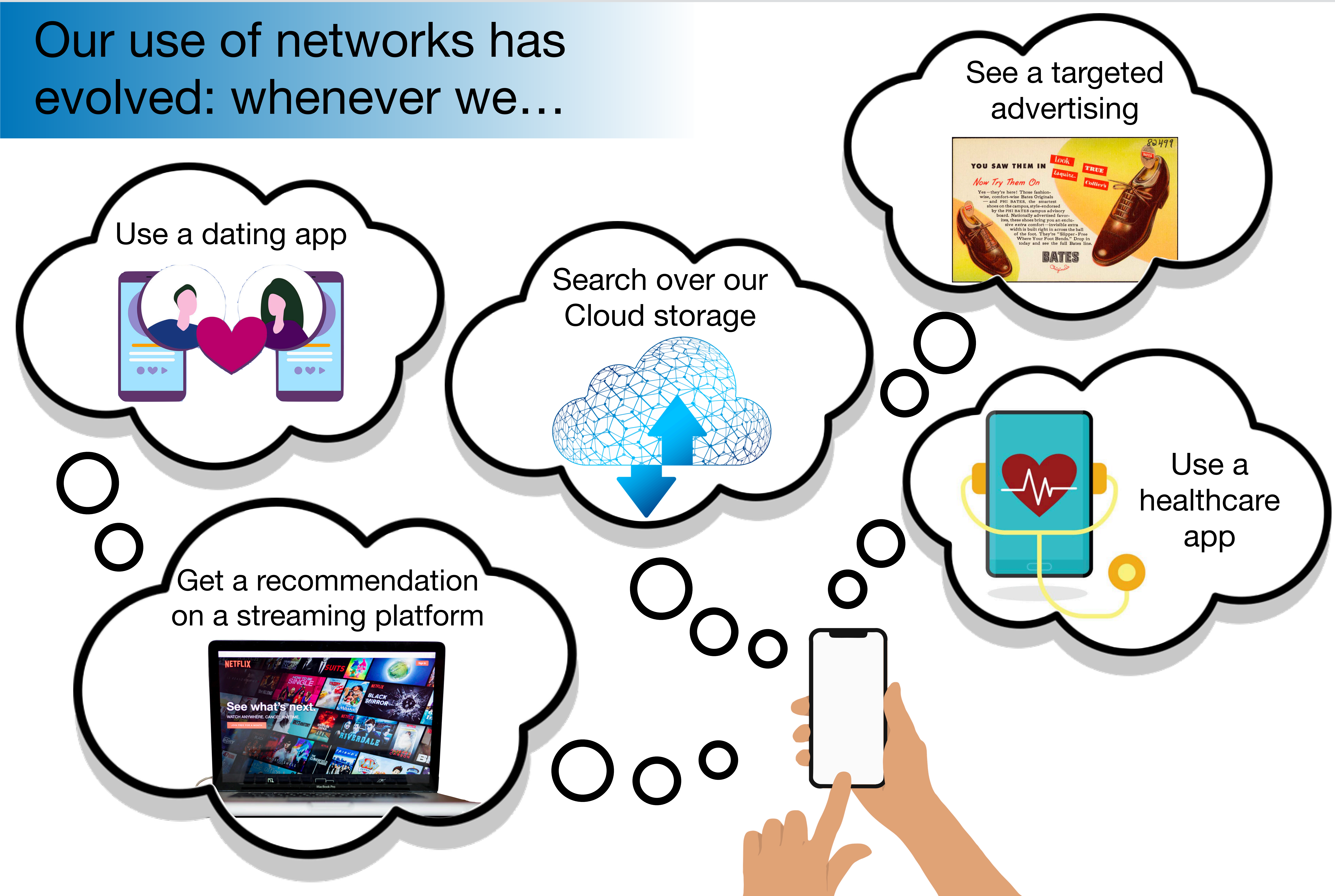

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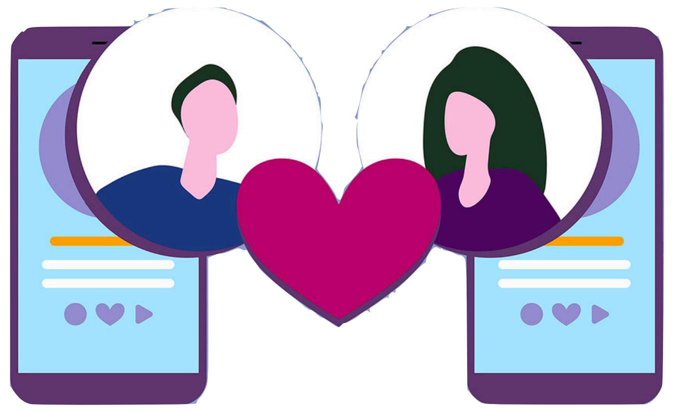
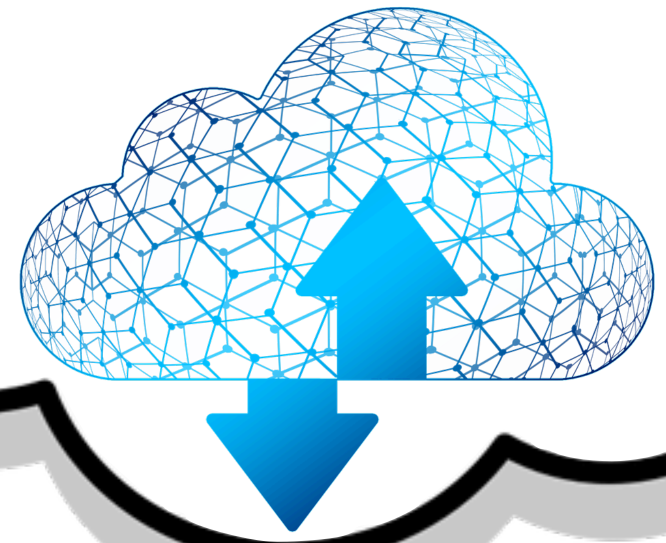
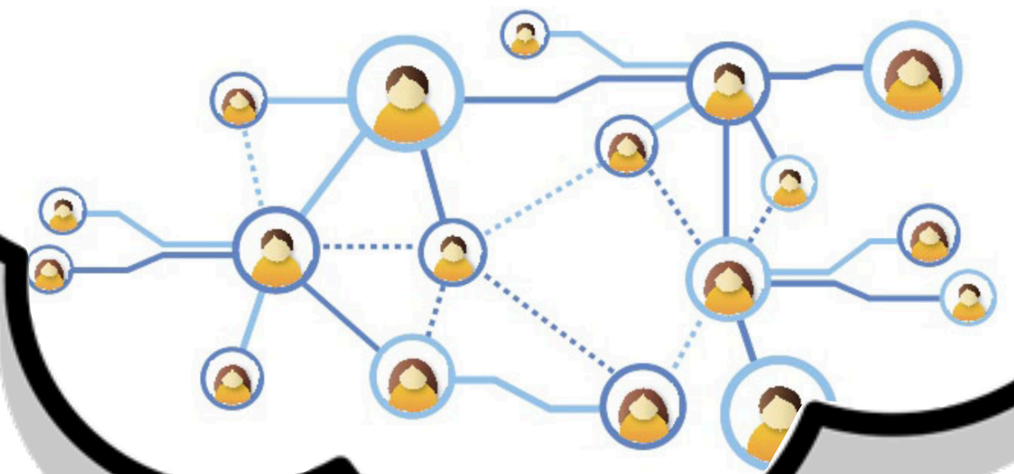


Search over our Cloud storage



See a targeted advertising

Use a social network



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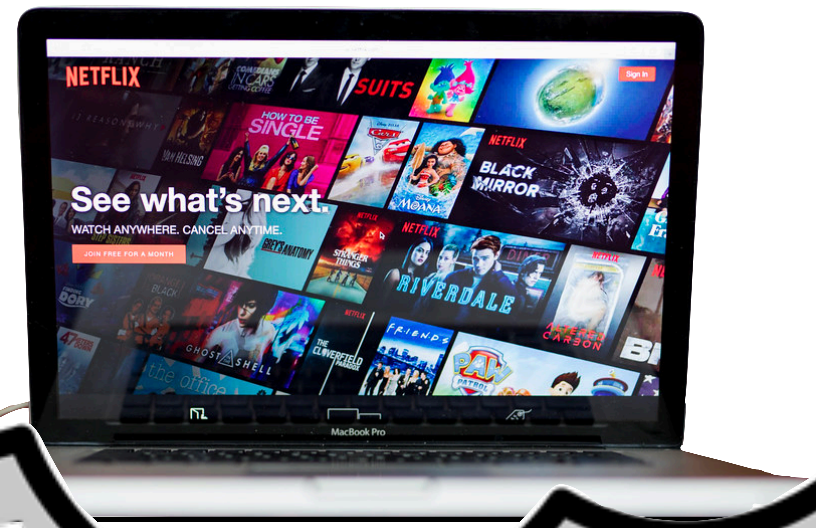

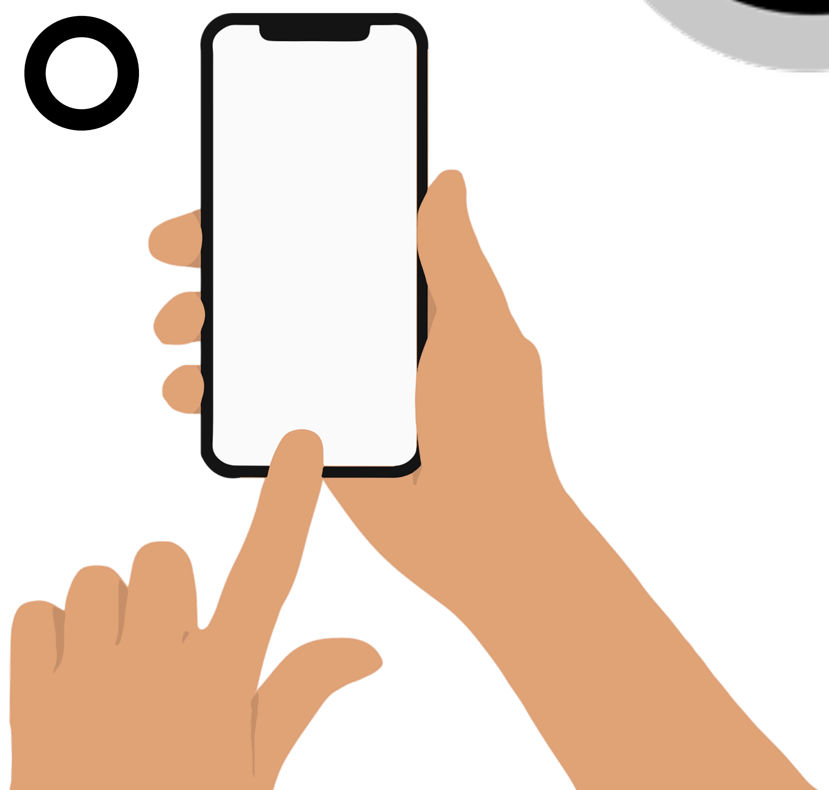

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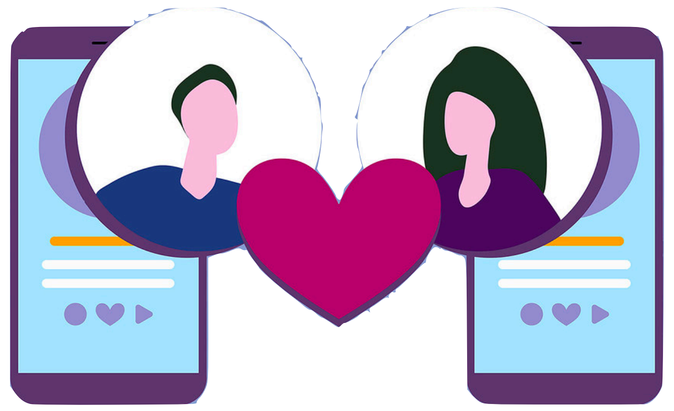

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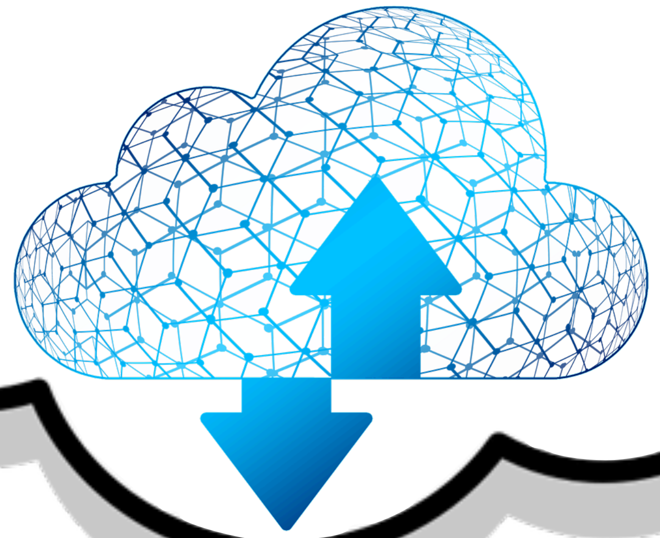

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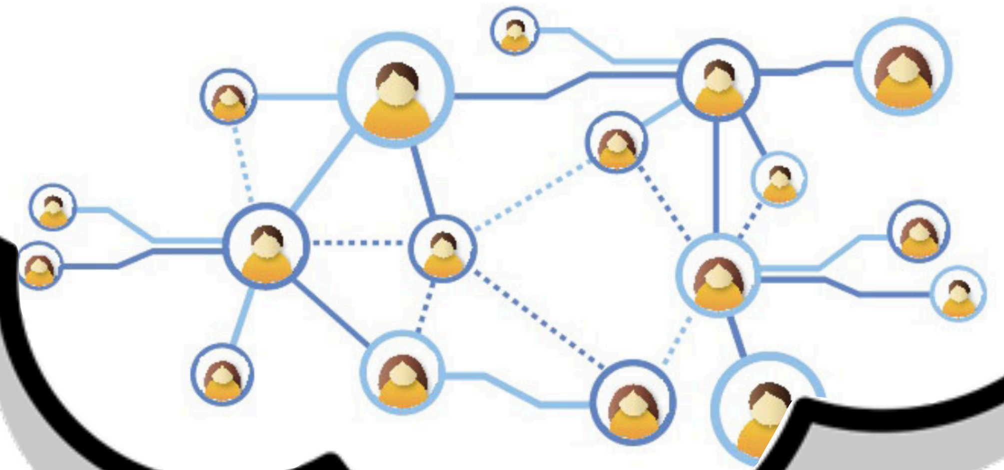

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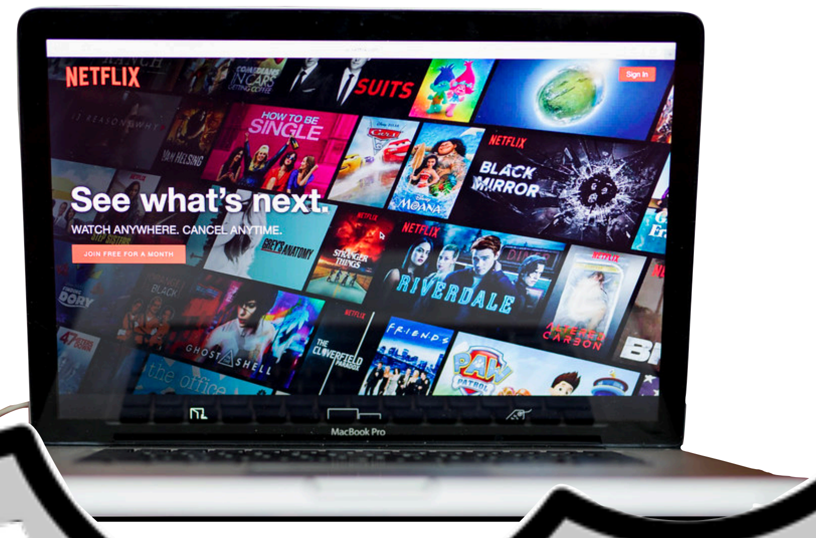

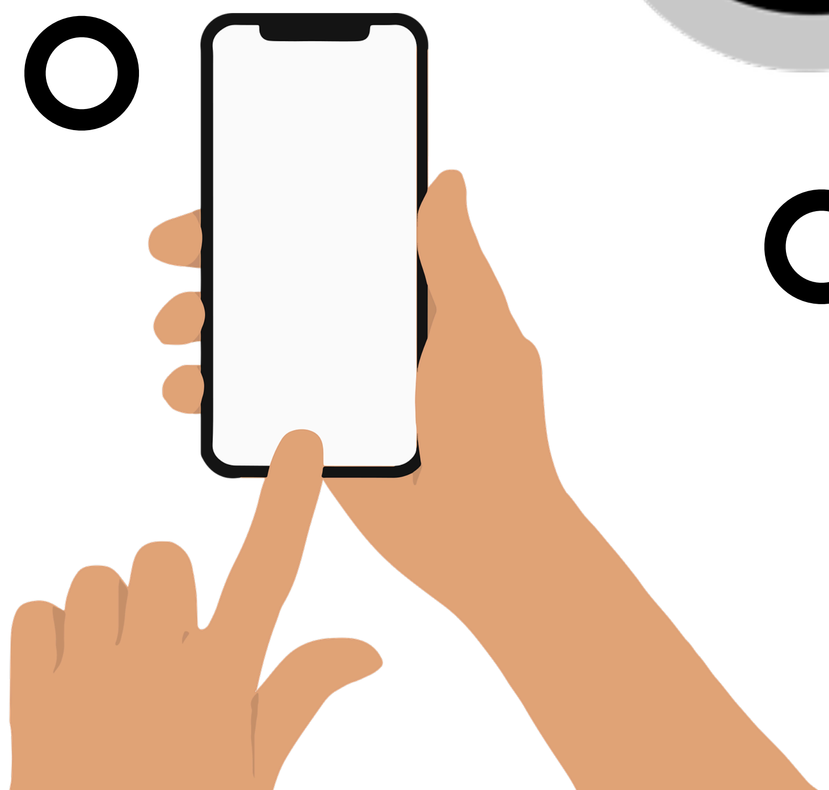

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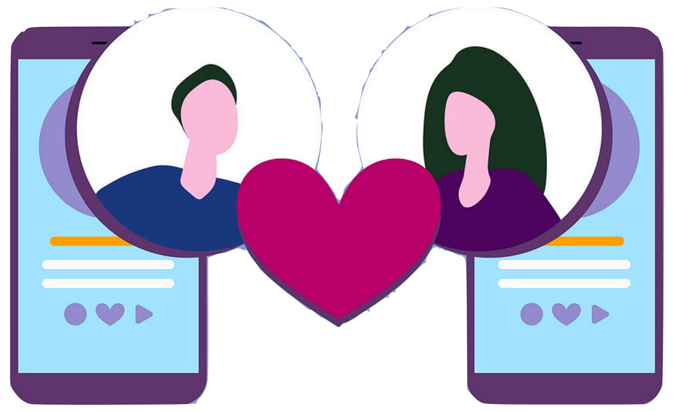

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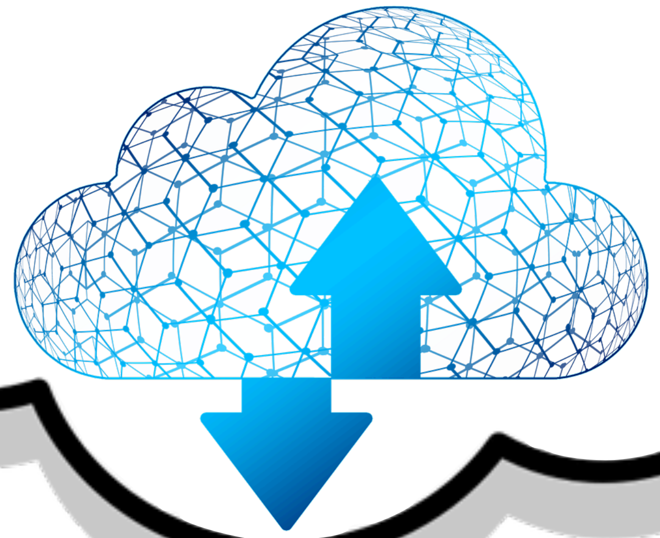

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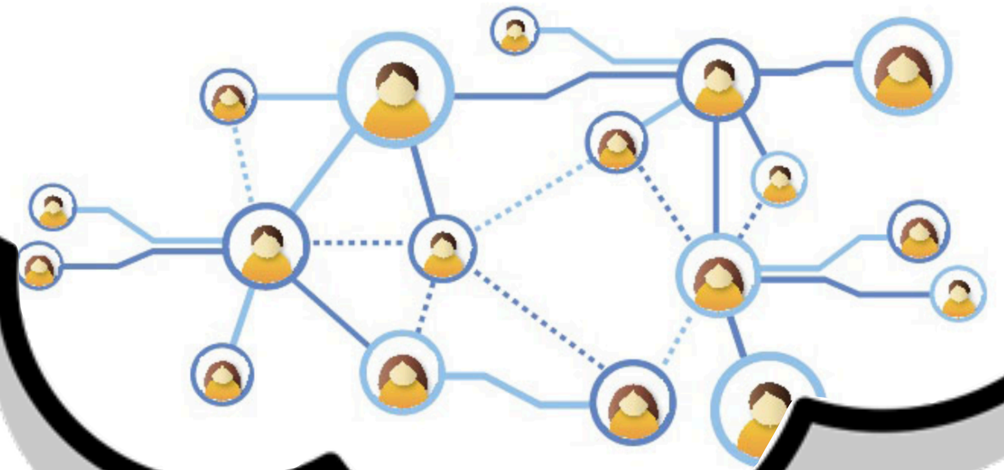

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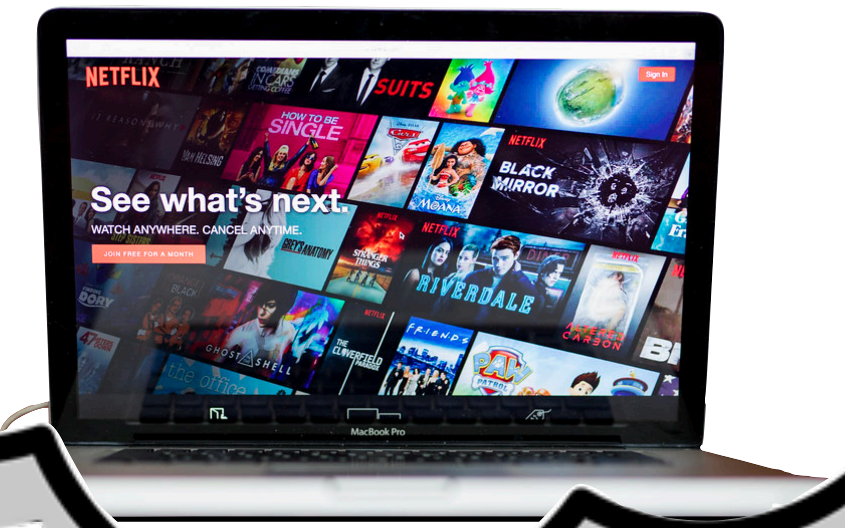

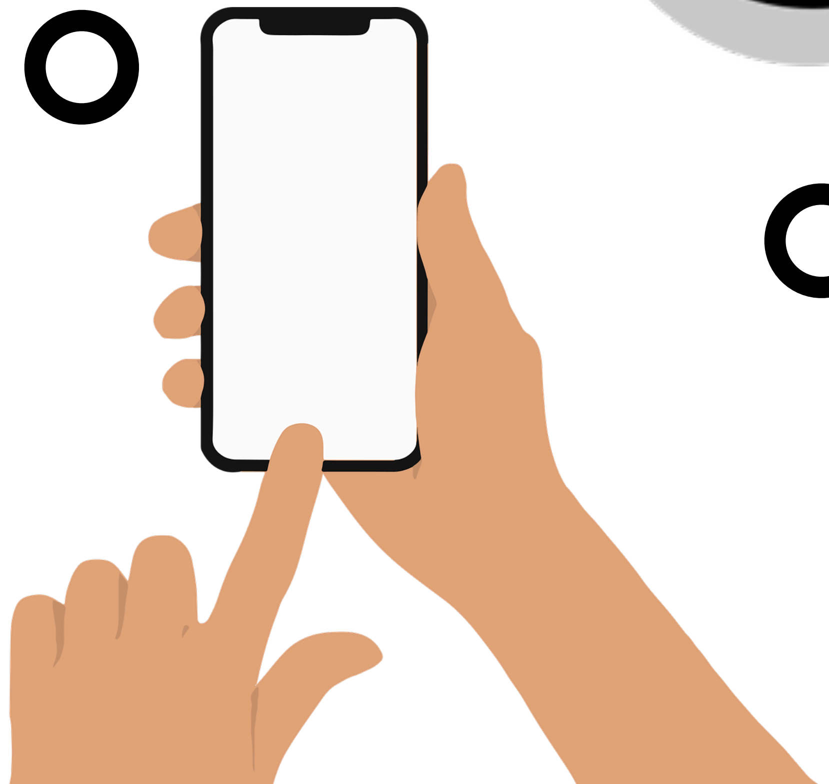

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Our private data is **used in computations**

A Paradoxical Situation

We become increasingly aware of the **need for privacy** in communications
- Over the web
- When using messaging apps

We are strongly **incentivized** to distribute our private data
- To benefit from AI-driven apps (photos, health apps…)
- To use social networks (friend recommendations, curated timelines…)

And our data is becoming **extremely valuable**
- For targeted advertising
- To train machine learning algorithms (e.g. to find new treatments)

As a result, we protect our privacy whenever we **communicate**, but give up on it whenever **computations** are required… Which happens on a daily basis.

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The solution is **not** to « tell users to be careful ». It is unrealistic:

- To hope that users will stop using apps and social networks, and
- To give up on societal benefits of computations on private data.

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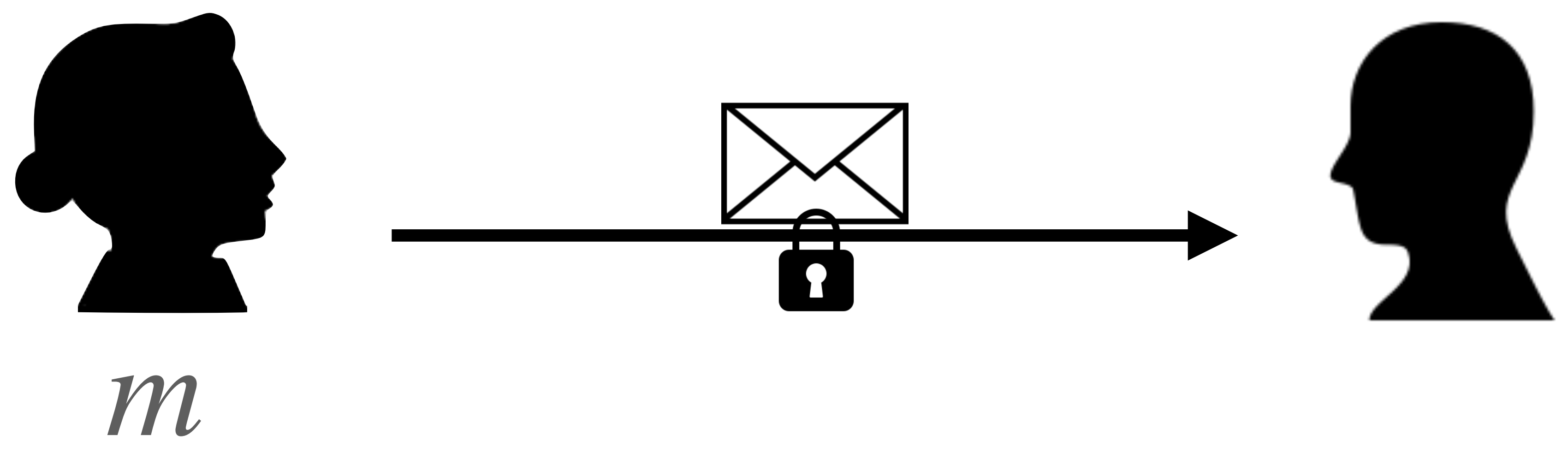
Secure computation aims to reconcile the (individual, societal) **benefits** of computations on data with the need to **protect its privacy**.

What is Secure Computation?

Protecting traditional uses of networks

Secure communication

Goal: *communicating* a secret message



Output: Bob learns m

Security: Eve learns nothing

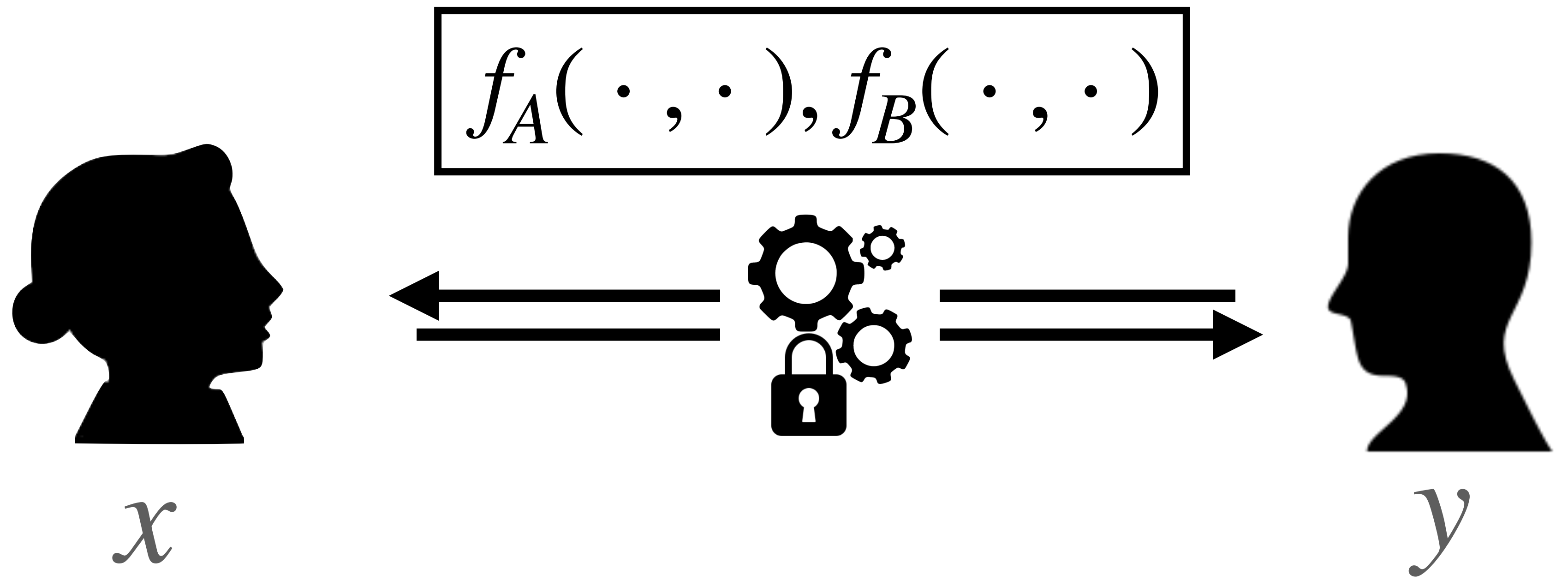
Solved by **encryption**

Locks the message in a digital « box »
Only the owner of the key can read it

Protecting modern uses of networks

Secure computation

Goal: *computing* (public) functions on secret inputs



Output: Alice learns $f_A(x,y)$ and Bob learn $f_B(x,y)$

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Encryption is « all or nothing »

It does not allow a *fine-grained* access to some *specific* information about the data

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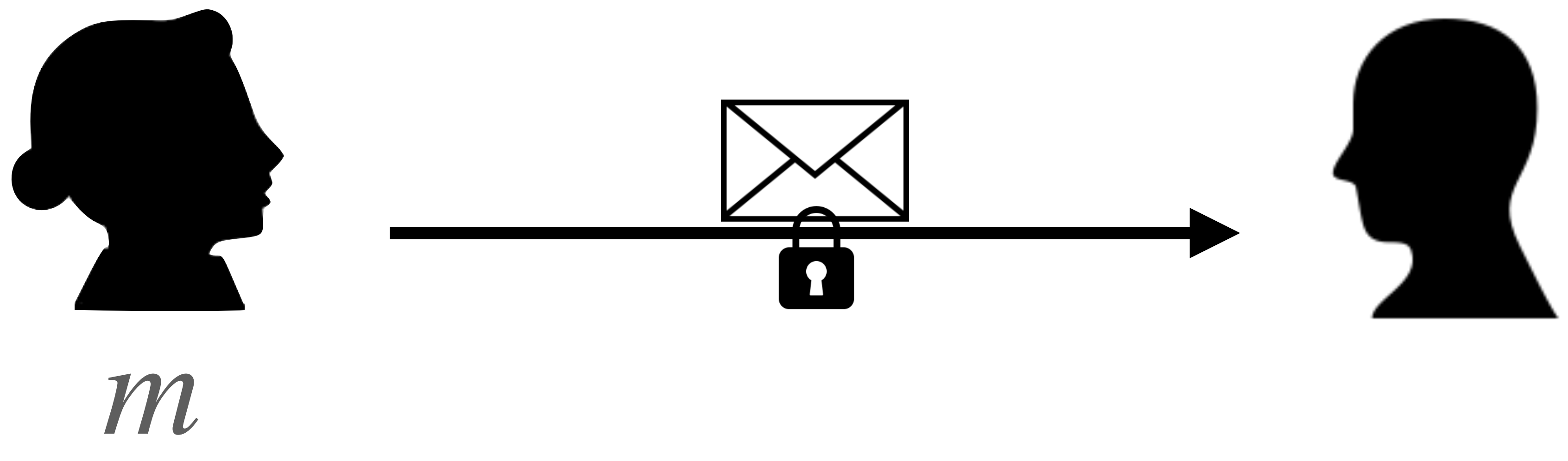

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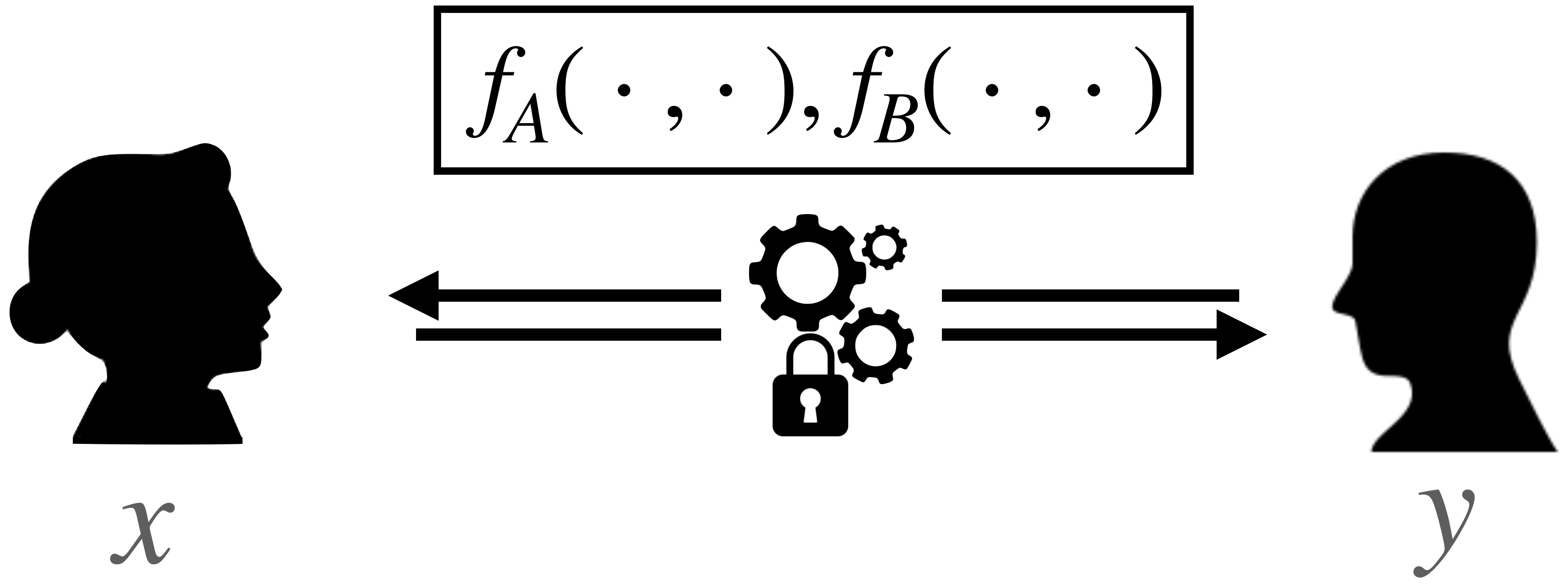

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Secure computation is the area of security that studies techniques and protocols to allow computing public functions on *private* inputs

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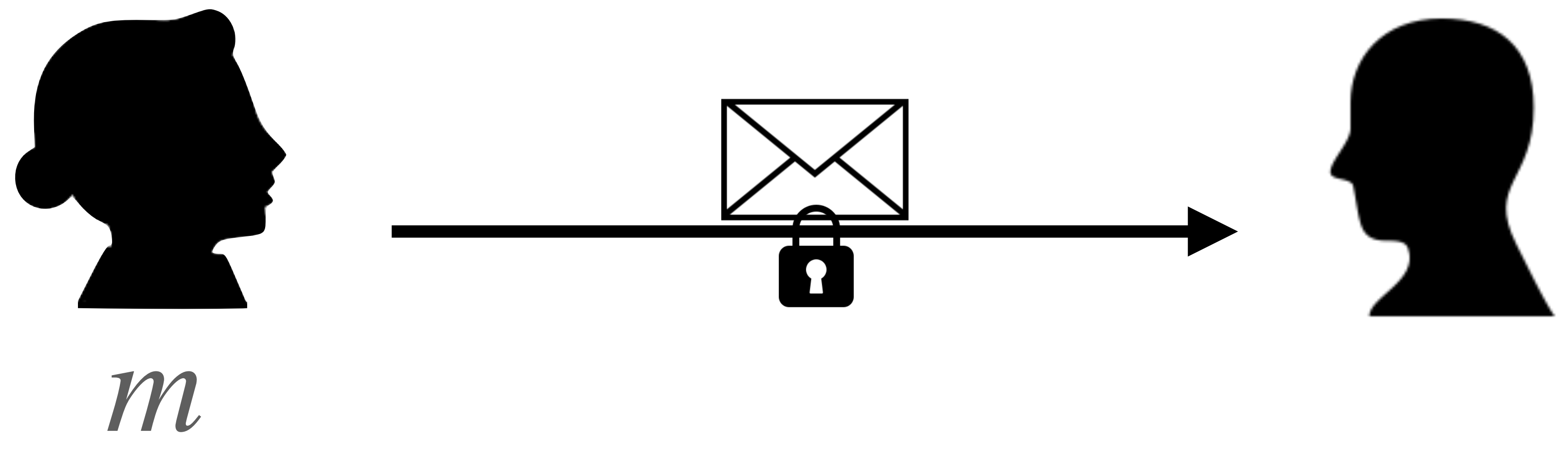

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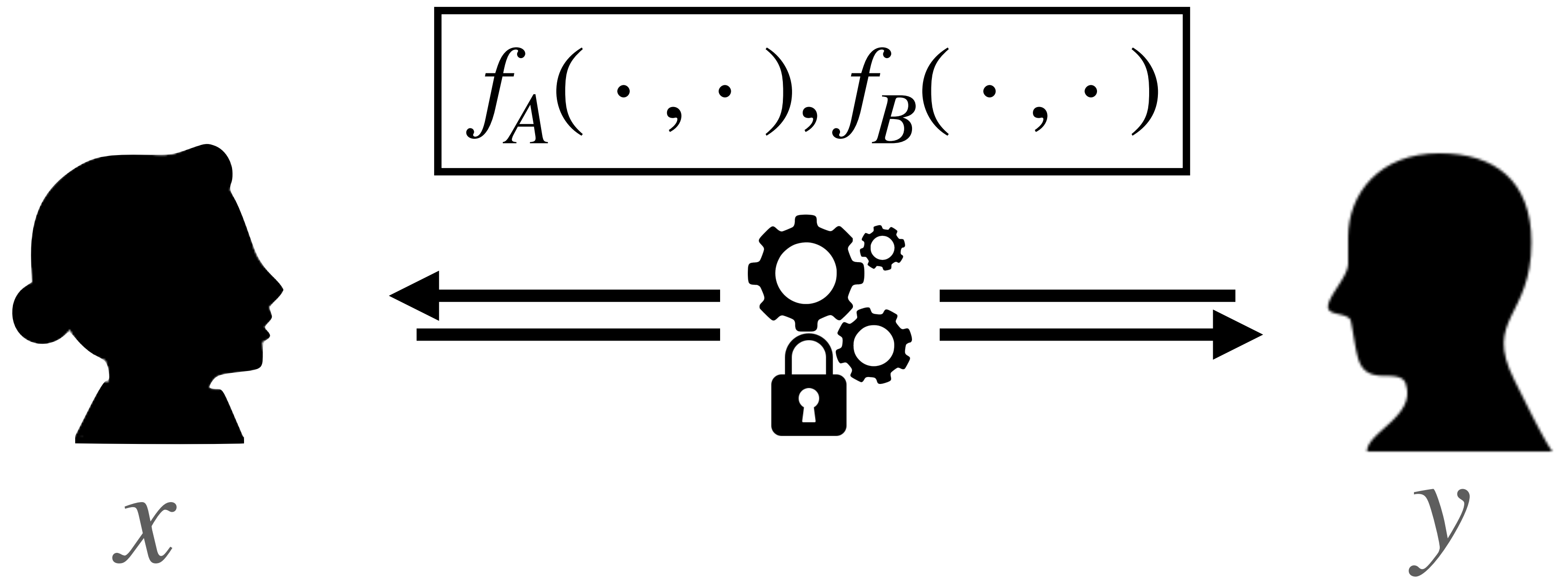

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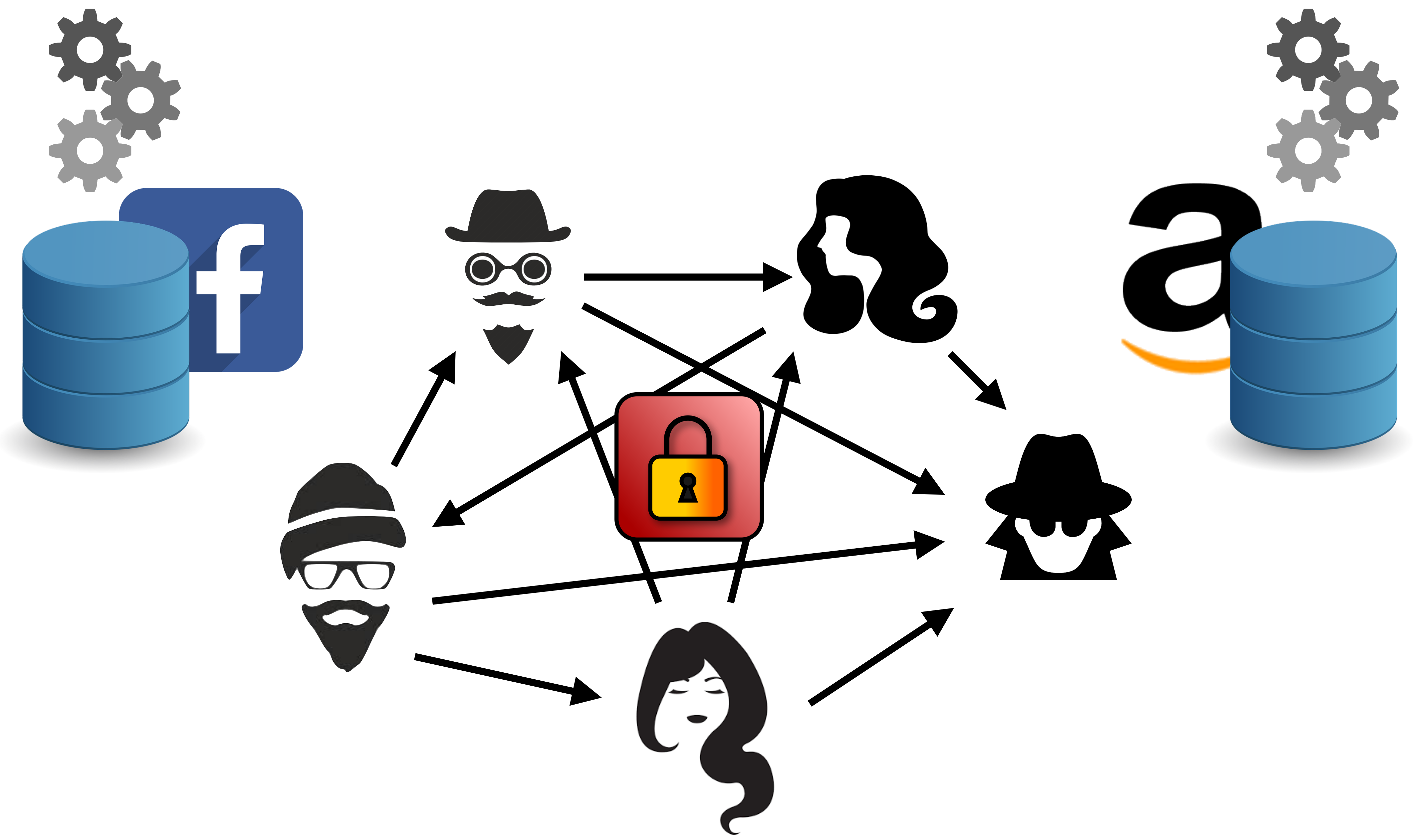
- Secure computation is a more *fine-grained* approach to security: the function controls precisely what is learned (secure communication is *all or nothing*)

- It is much more demanding: now the adversary is *internal* (Alice must be protected against Bob, and Bob against Alice), and can influence the protocol!

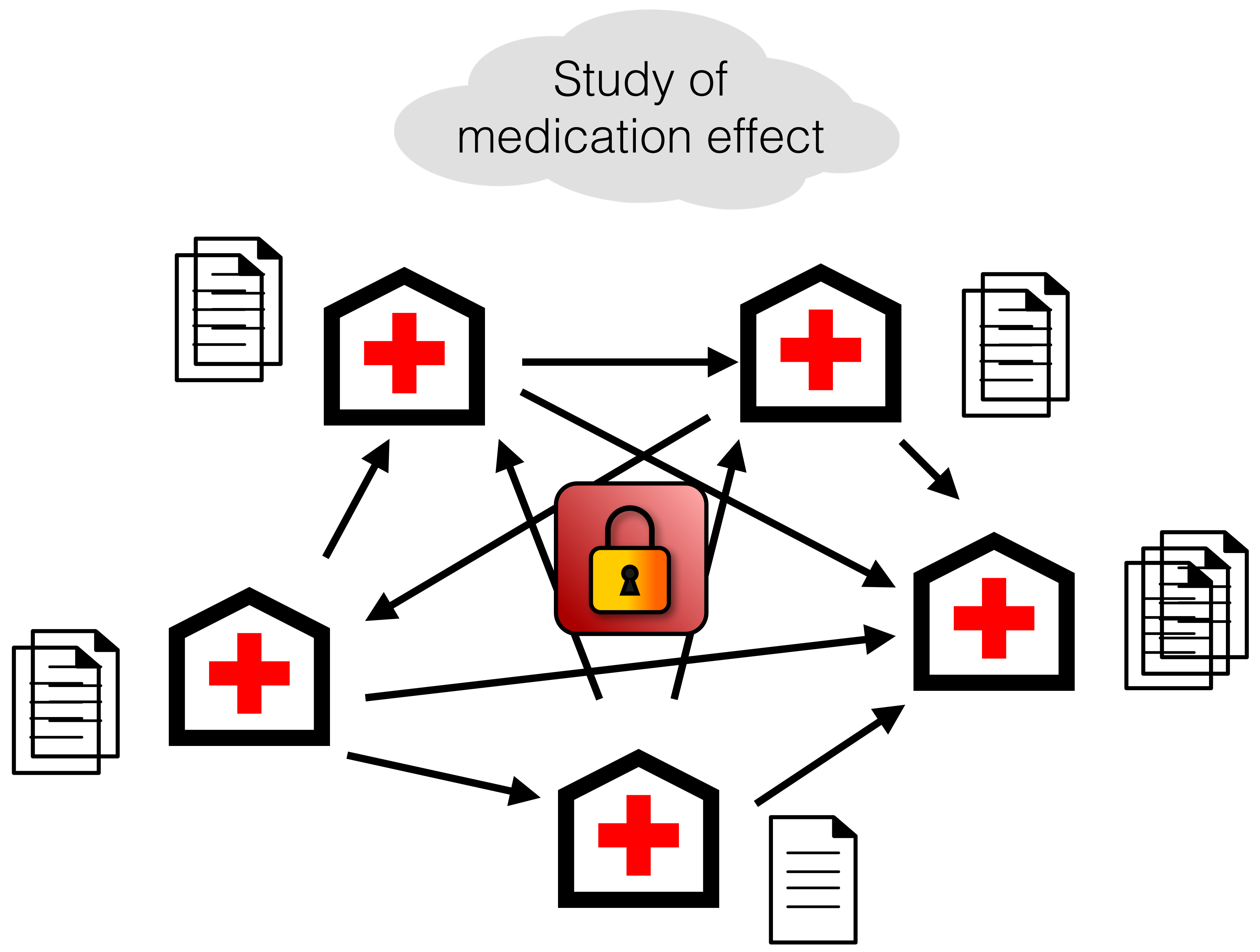
What is Secure Computation?



More generally, n participants $P_1, \cdots, P_n$ with private inputs $x_1, \cdots, x_n$ wish to distributively compute $(y_1, \cdots, y_n) \leftarrow f(x_1, \cdots, x_n)$ such that

- **Correctness:** at the end of the interaction, P_i learns y_i
- **Security:** no *coalition of parties* learns anything beyond their own inputs and outputs

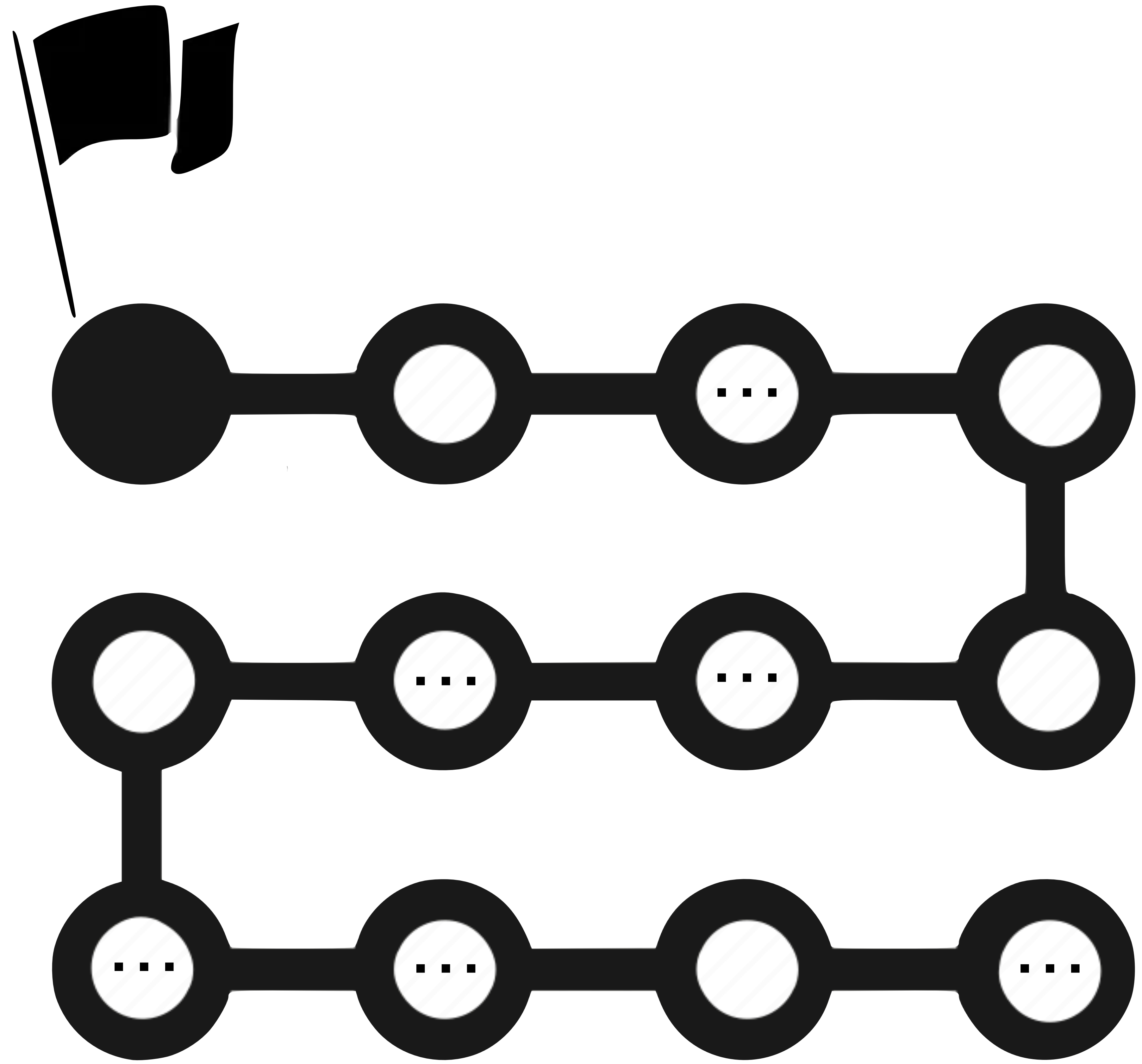
What is Secure Computation?



Example. n hospitals want to jointly perform statistical tests, or run ML algorithms, on the private data of their patients, to

- Uncover correlations between medical conditions and patient information
- Study the effect of medications
- Discover new treatments
- …

A Brief History of Secure Computation



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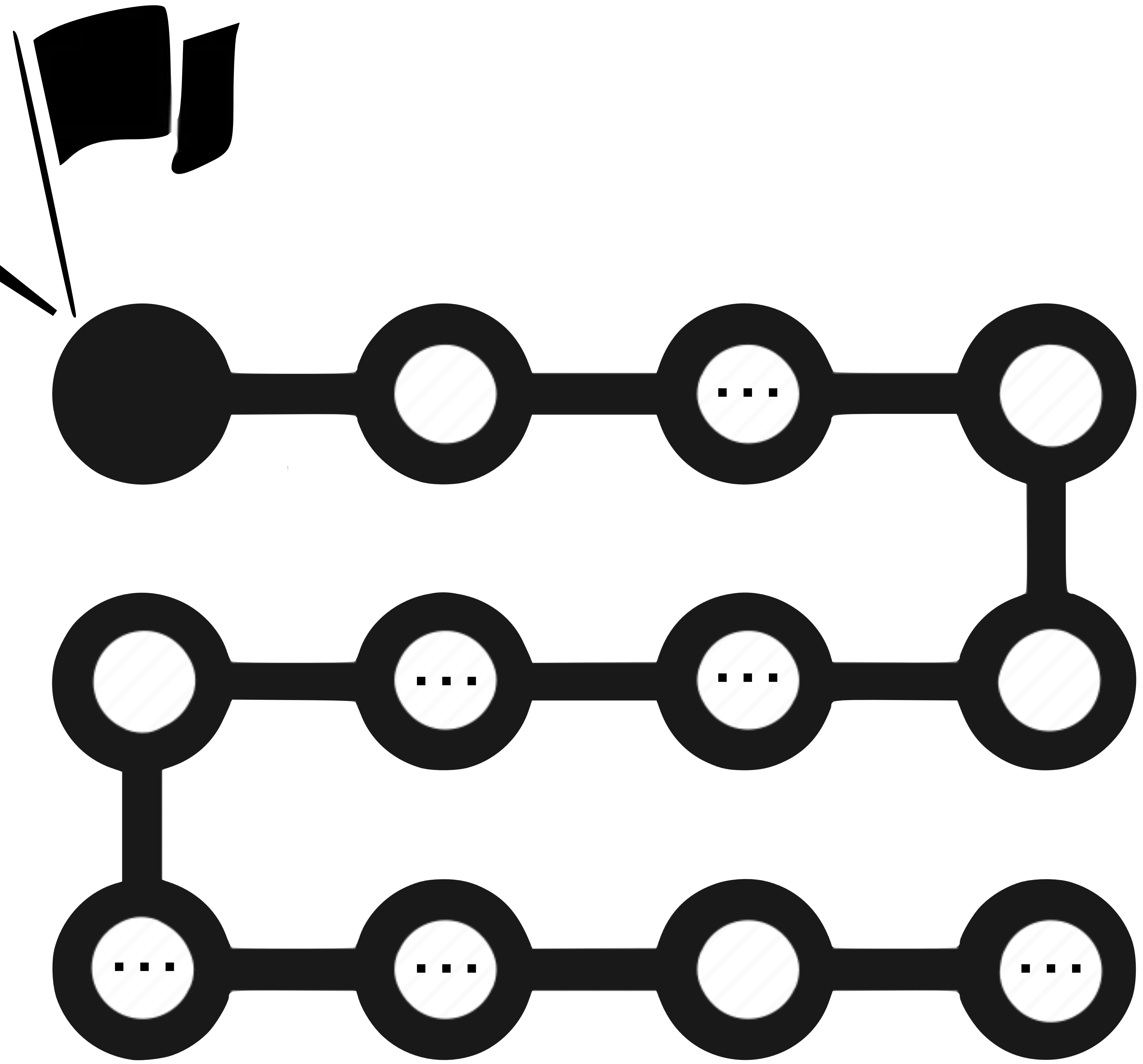
Yao, 1986 (two parties)
GMW, 1987 (n parties)

Secure computation is possible in theory

Very slow in practice: billions of expensive operations, TB of communication…



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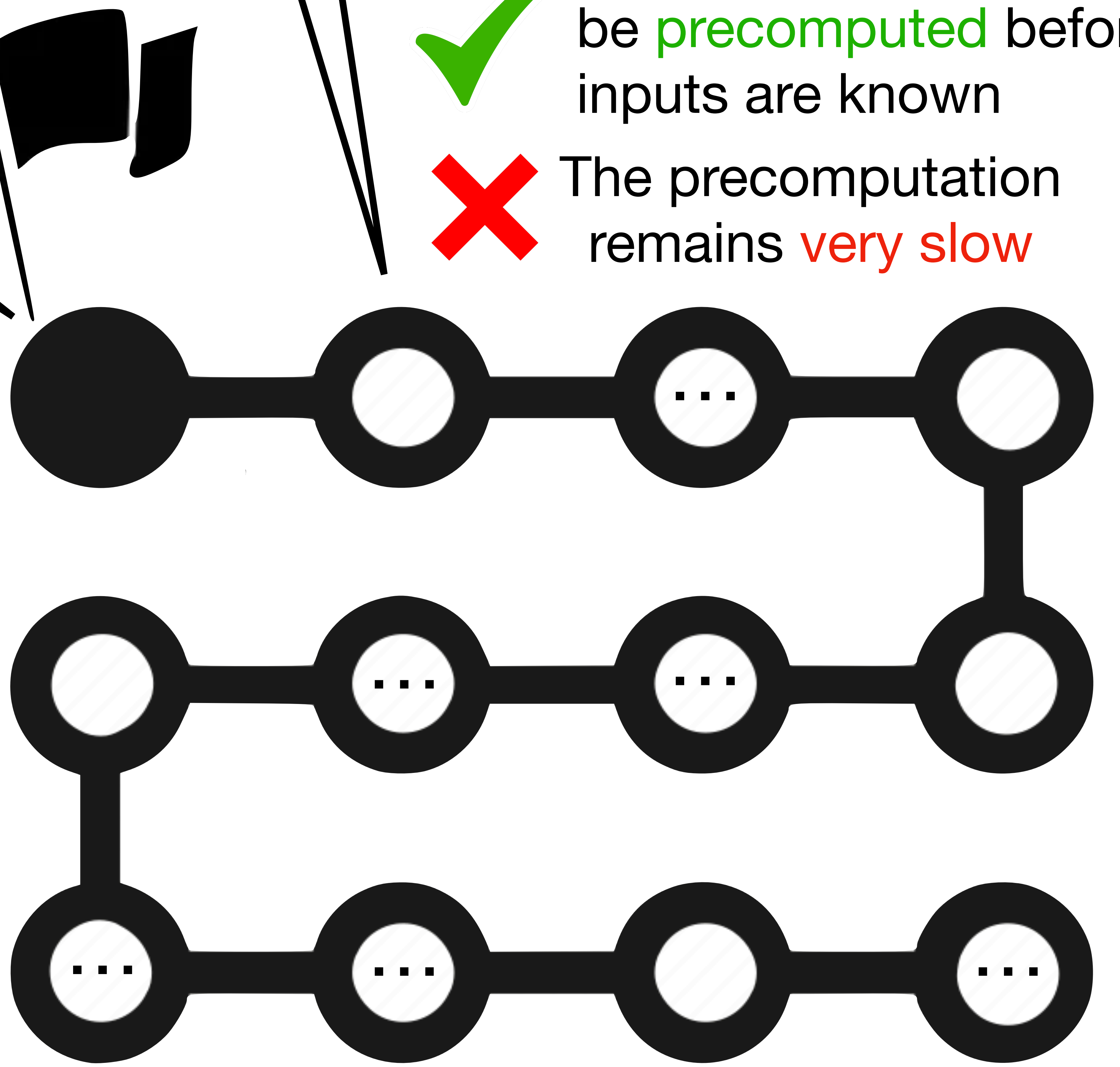

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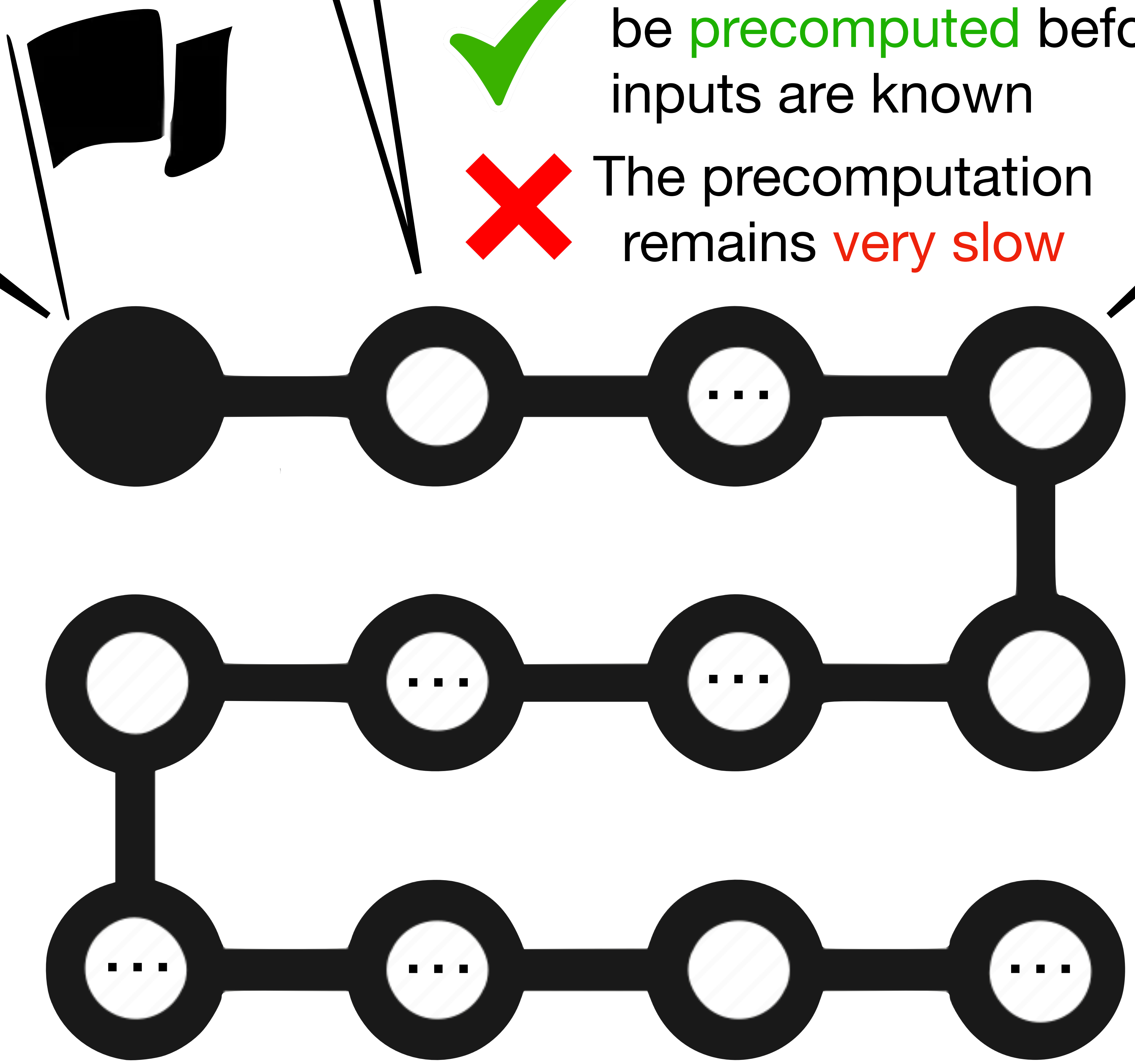
✗ Mostly theoretical result, and still requires heavy communication



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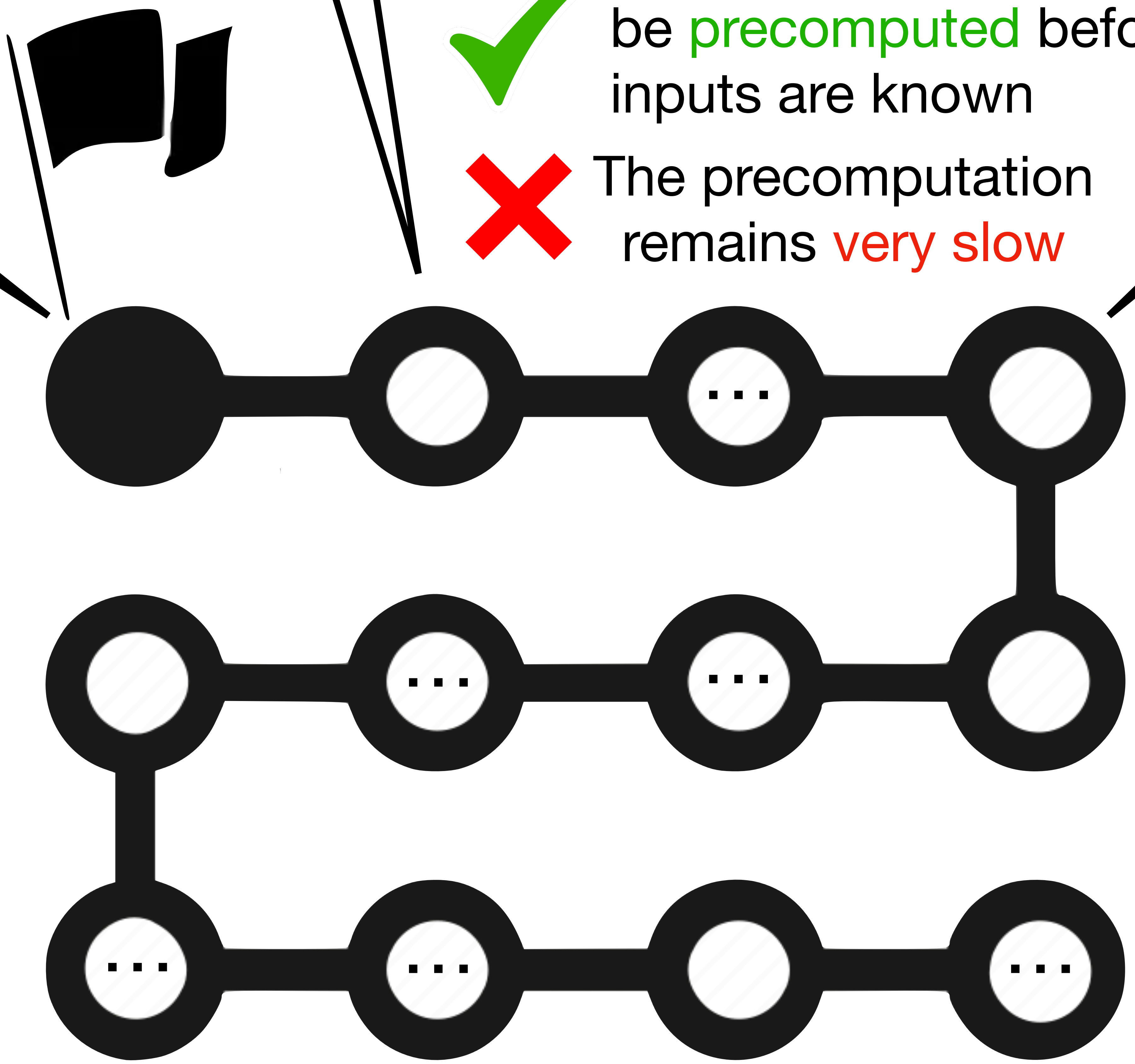

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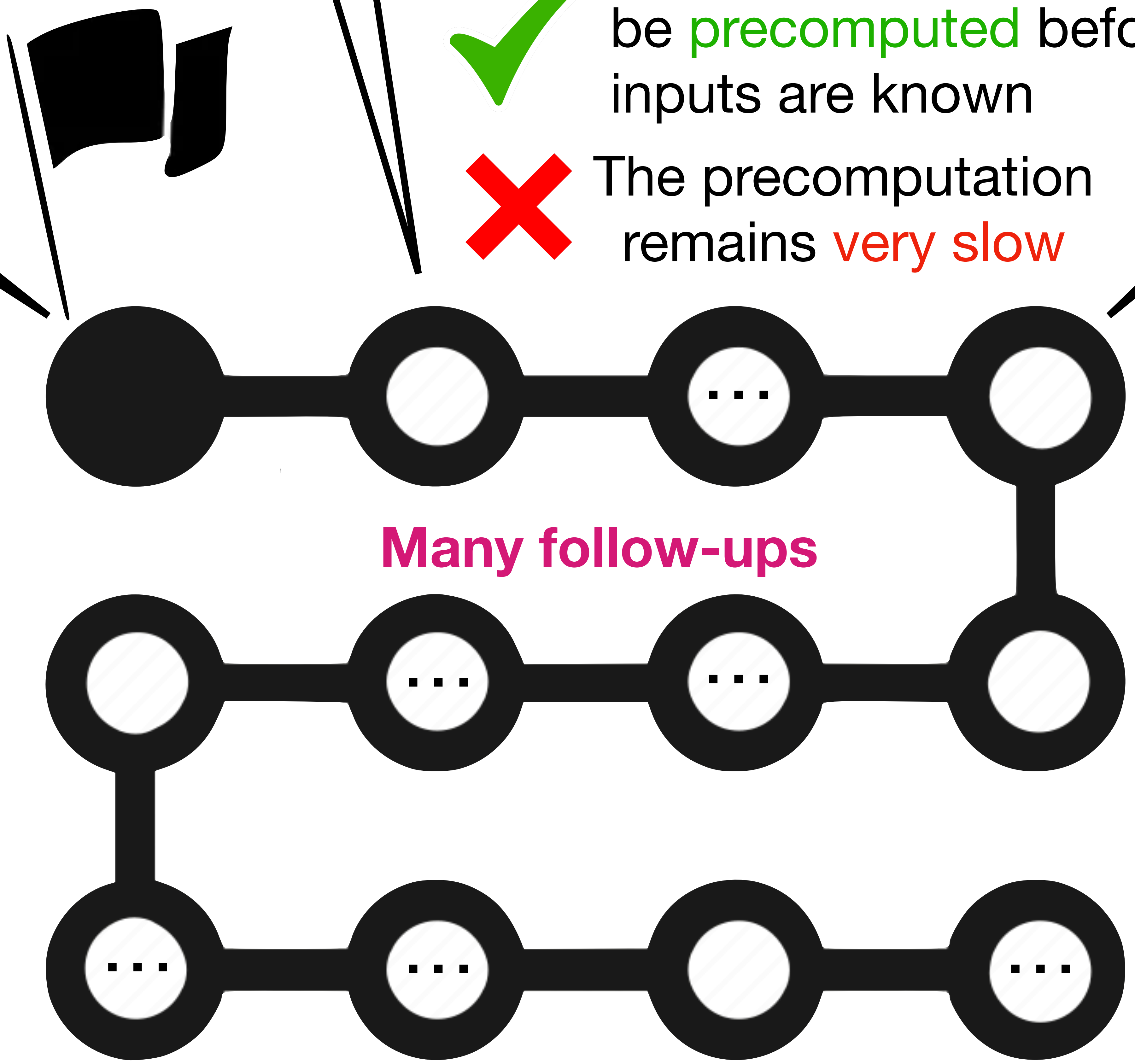

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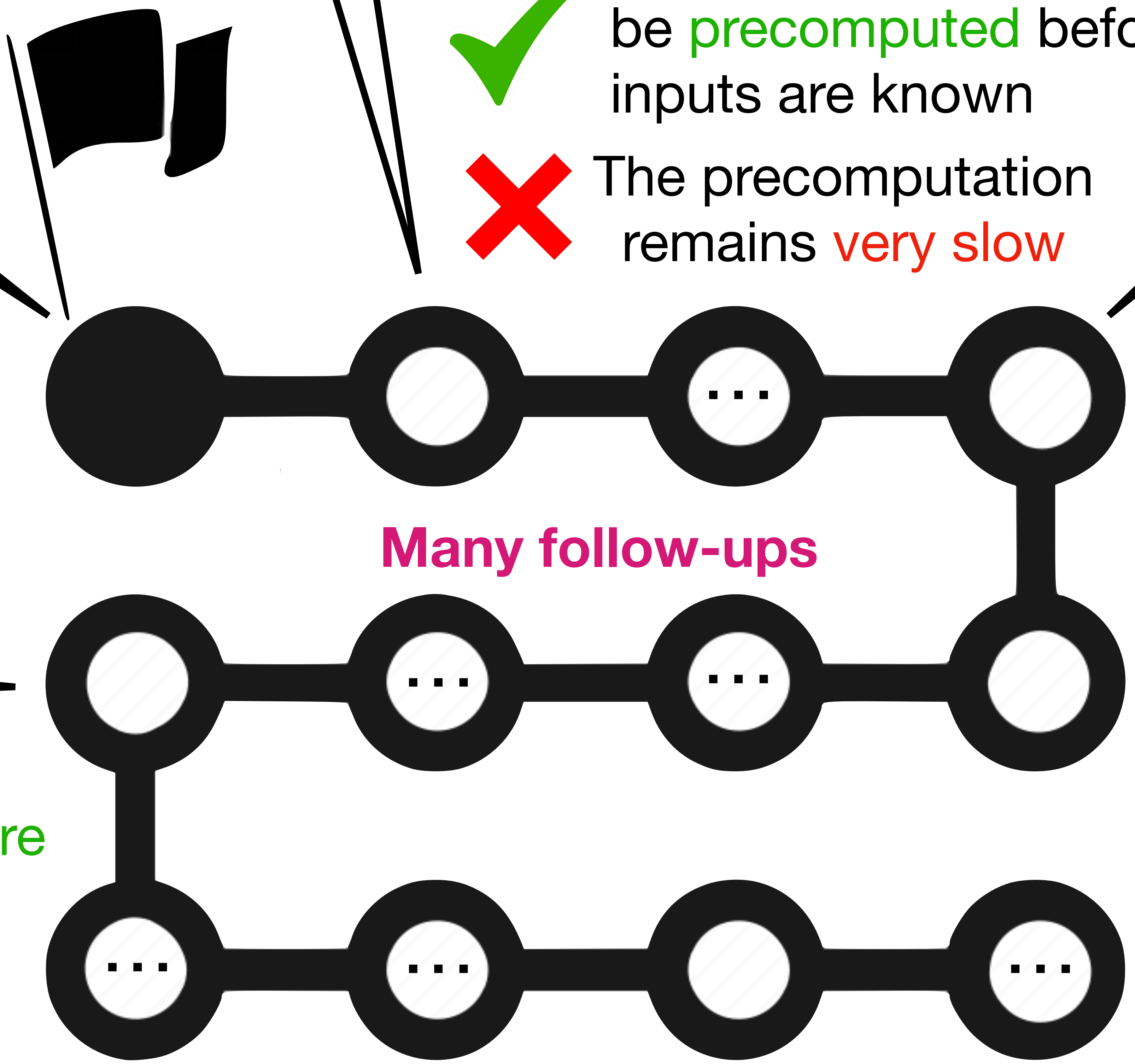

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The MPC explosion

Tons of follow-ups, improvements, first real-world deployments…

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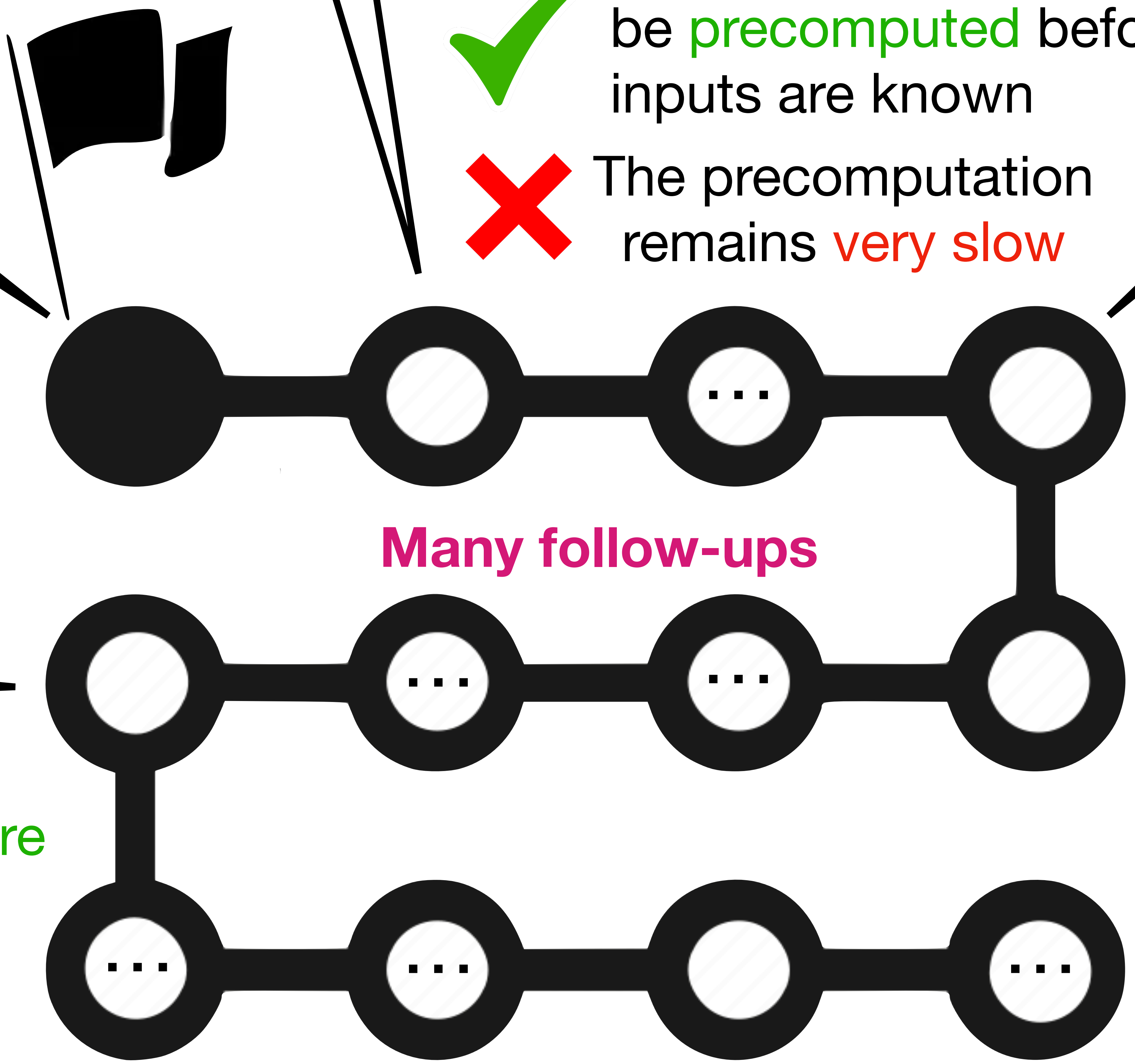

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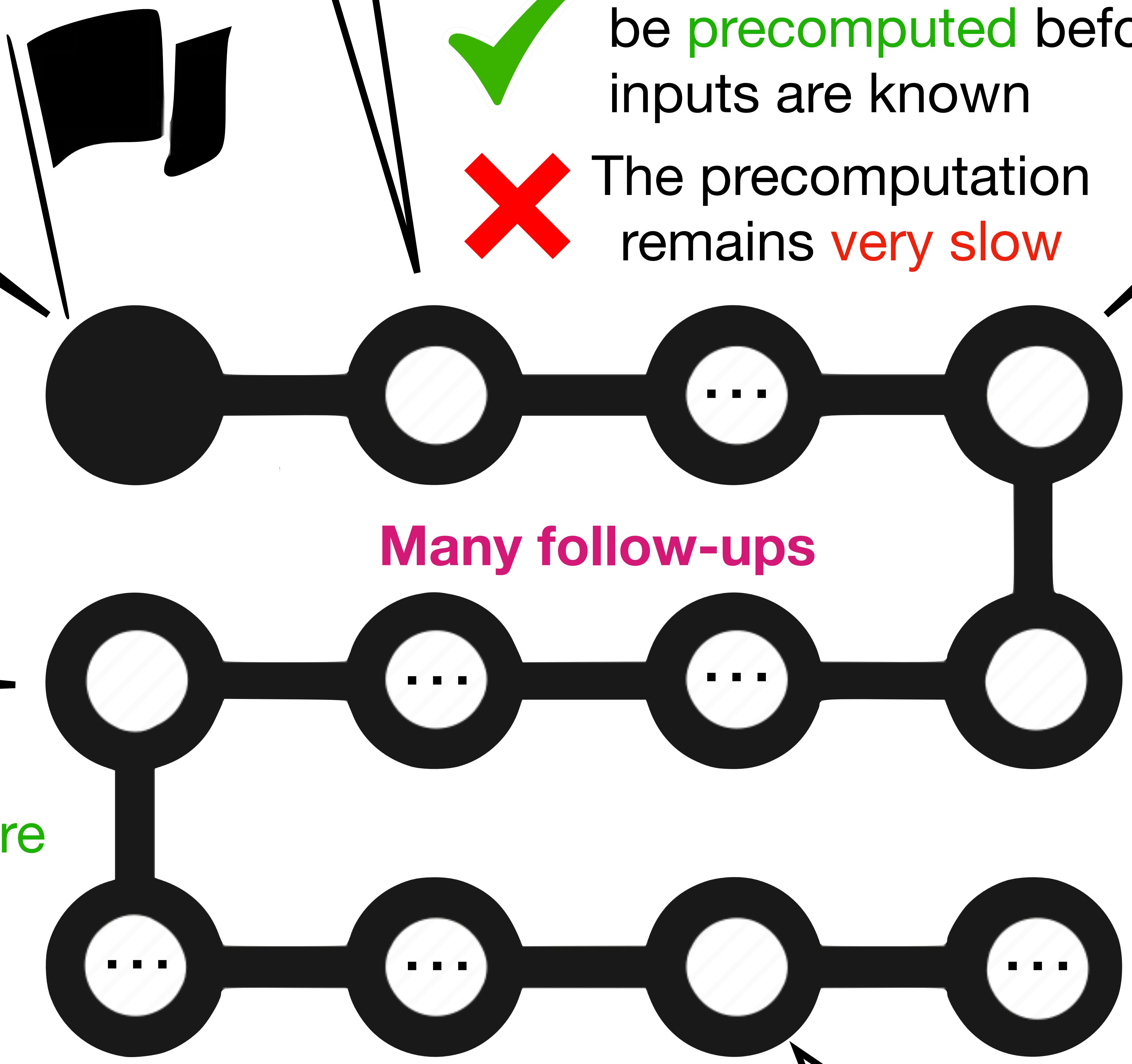

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The MPC explosion

Tons of follow-ups, improvements, first real-world deployments…

Many follow-ups

BCGI, 2018

Pseudorandom correlations

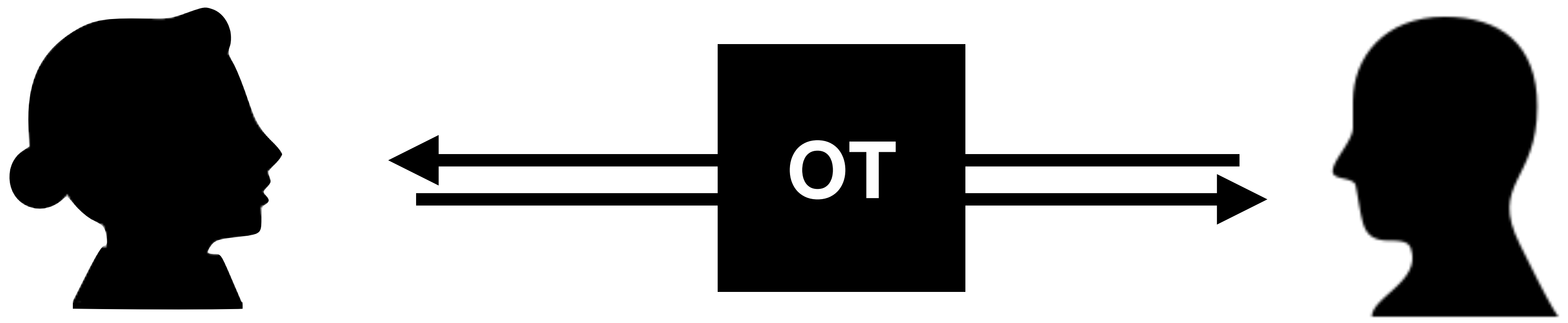
✓ Only lightweight communication

✗ Mostly for two parties

Secure Computation from Oblivious Transfer

Oblivious Transfer

A minimal example of secure computation…



(s_0, s_1) b

Output: Bob learns s_b

Security: Alice does not learn b, Bob does not learn s_{1-b}.

Secure Computation from Oblivious Transfer

Oblivious Transfer

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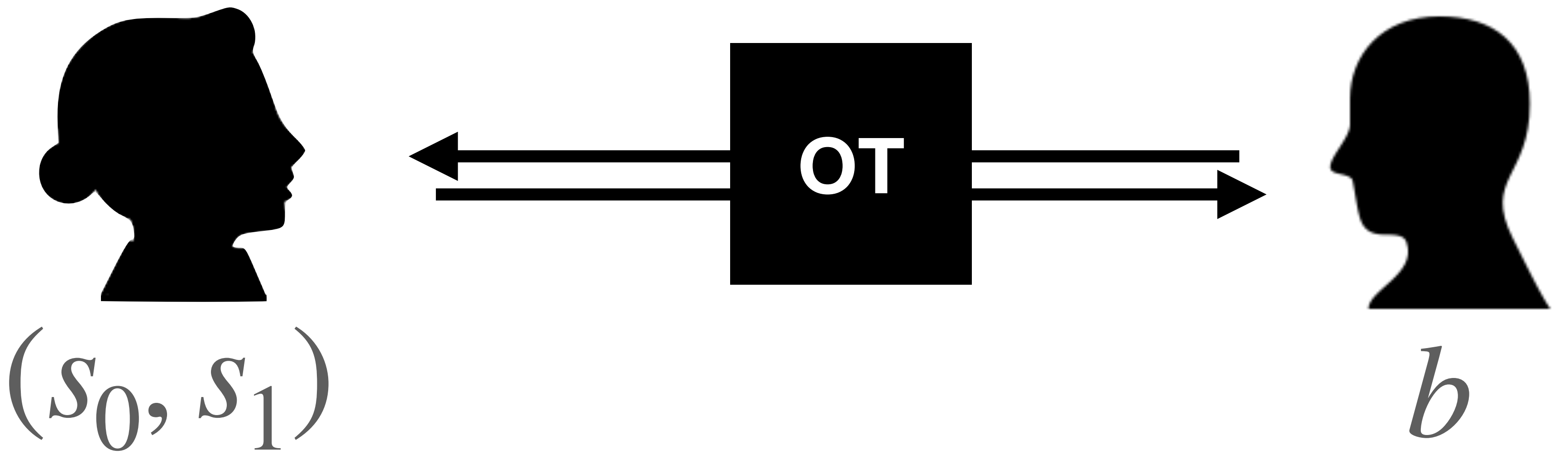

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GMW, 1987

Secure Computation for all functions

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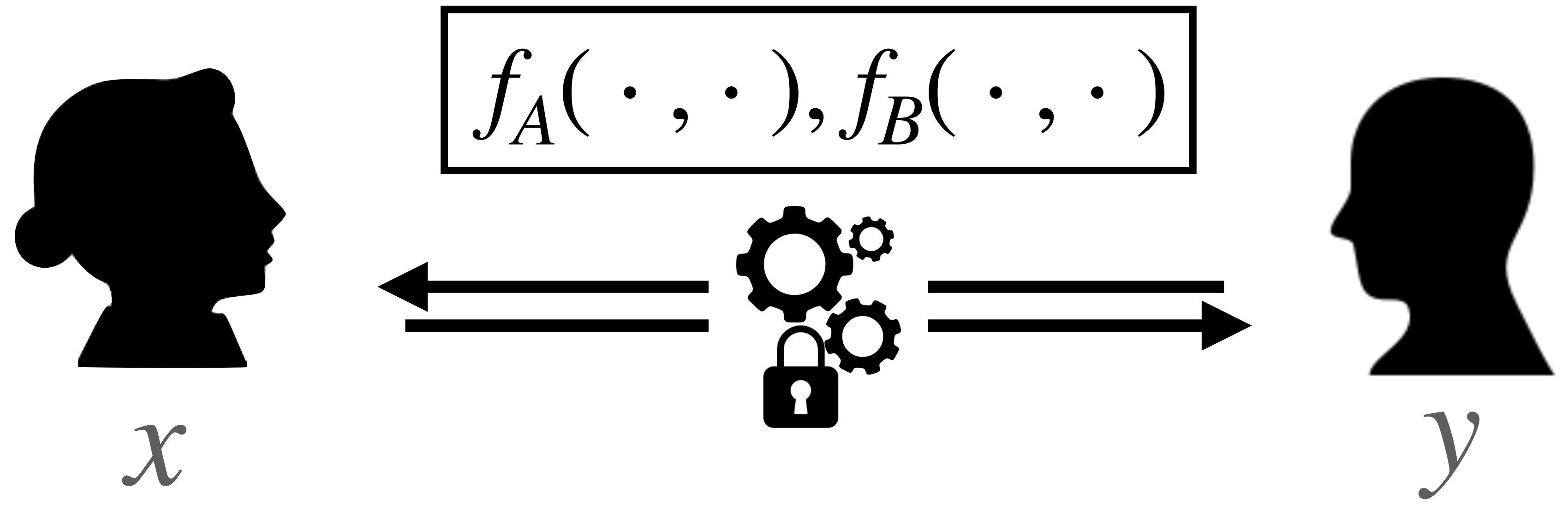

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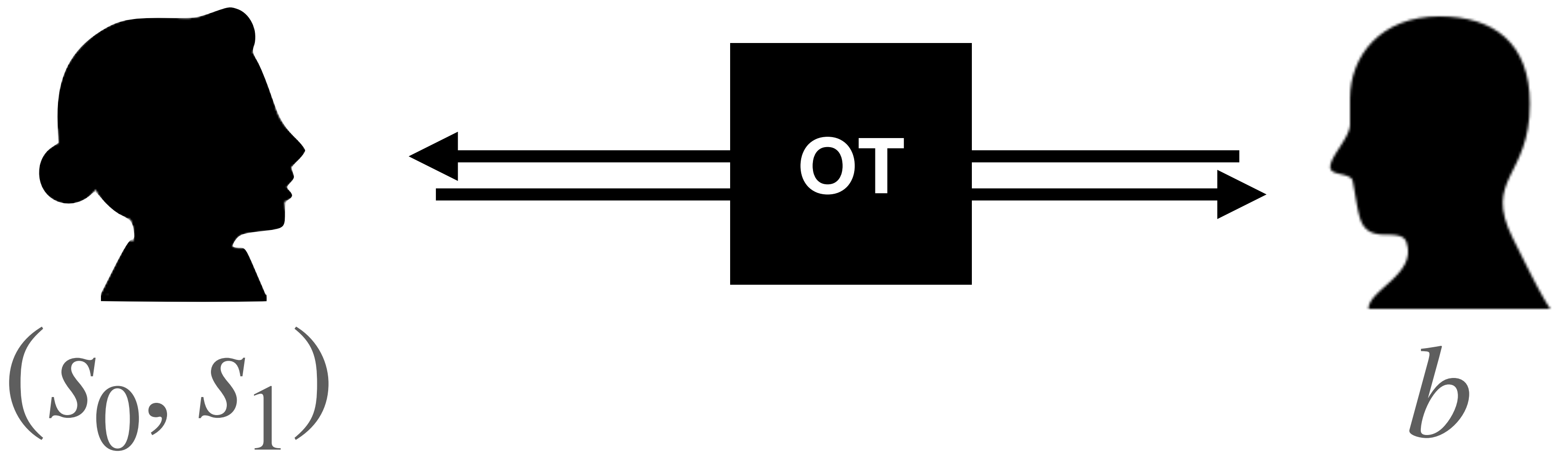

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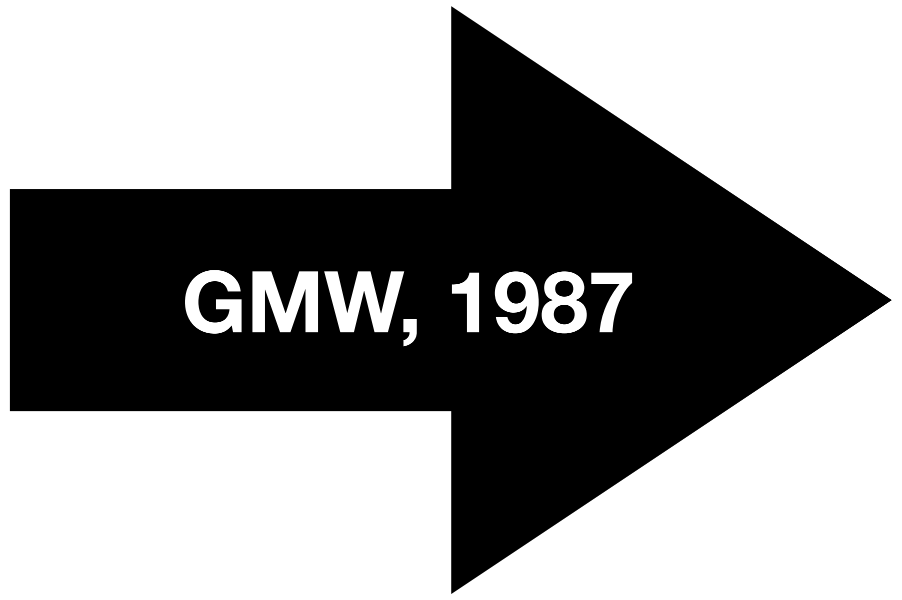

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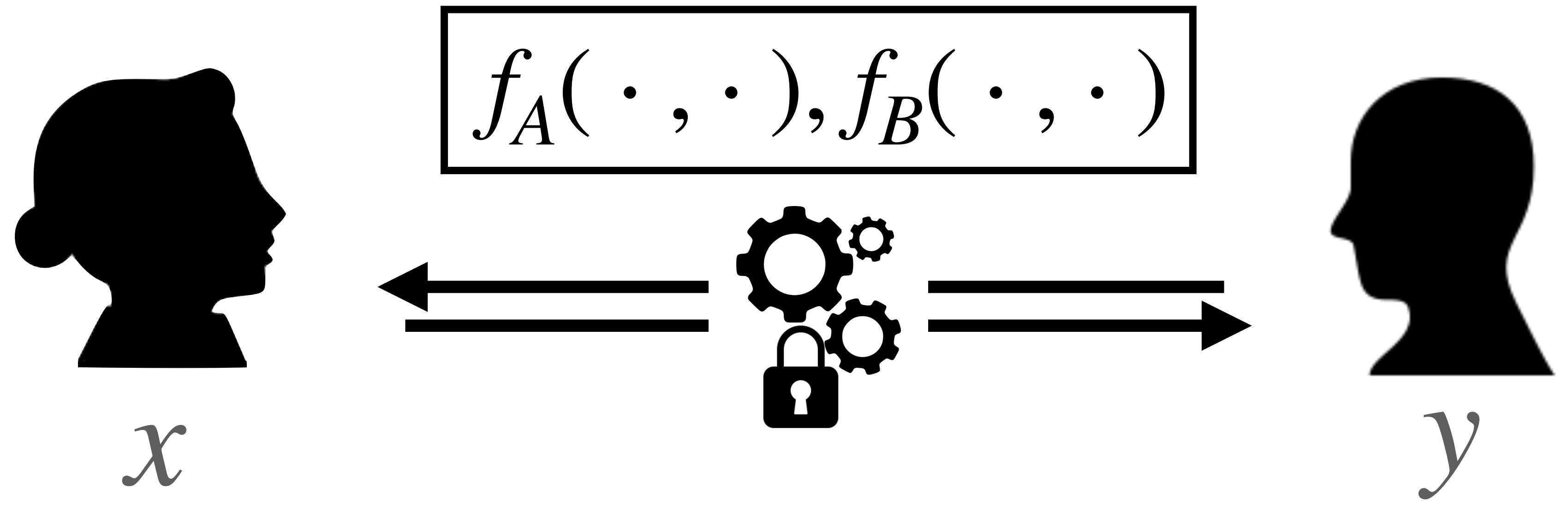

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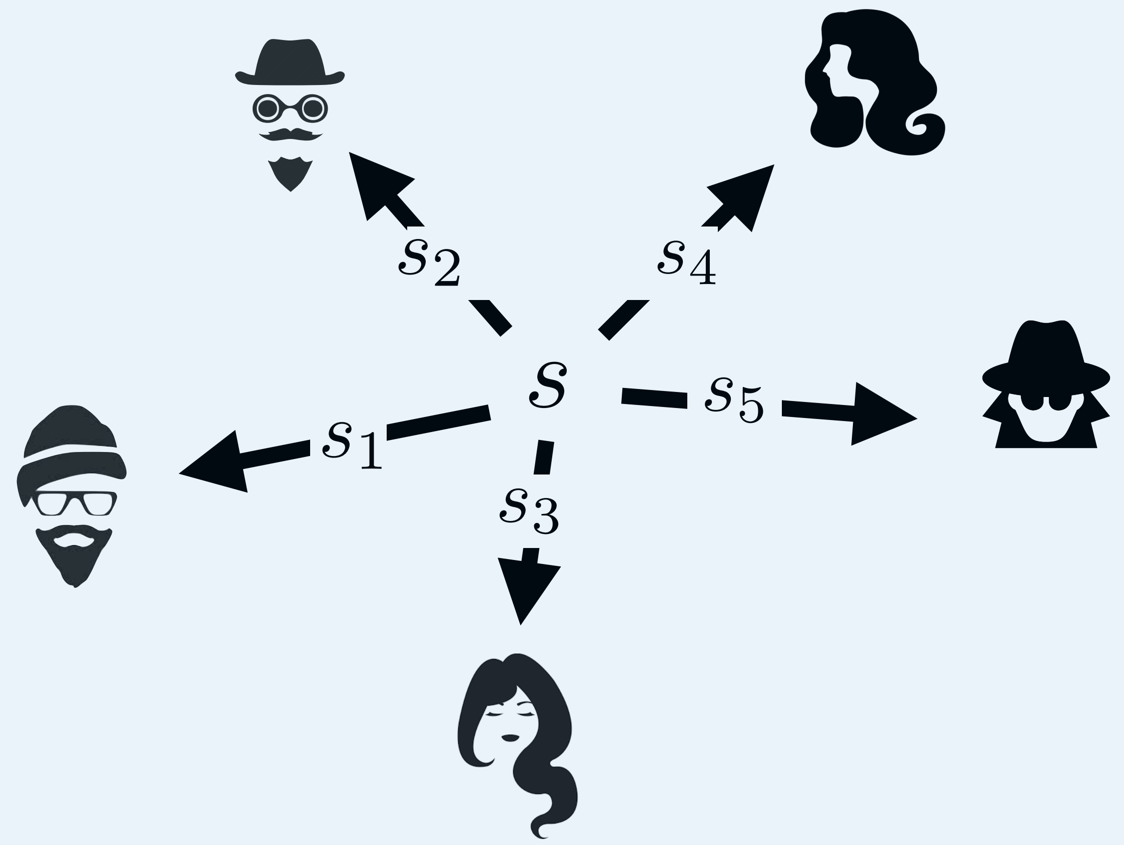
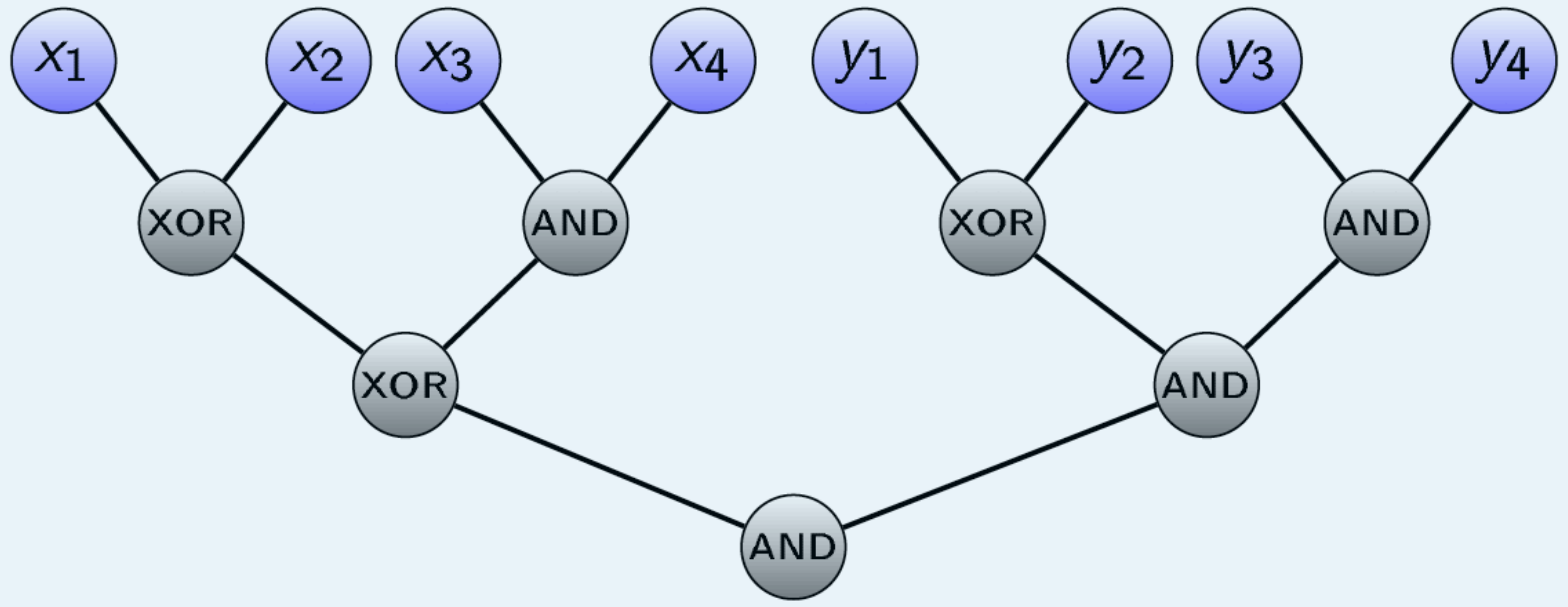
1. Use (additive) secret sharing



2. Write the function as a circuit



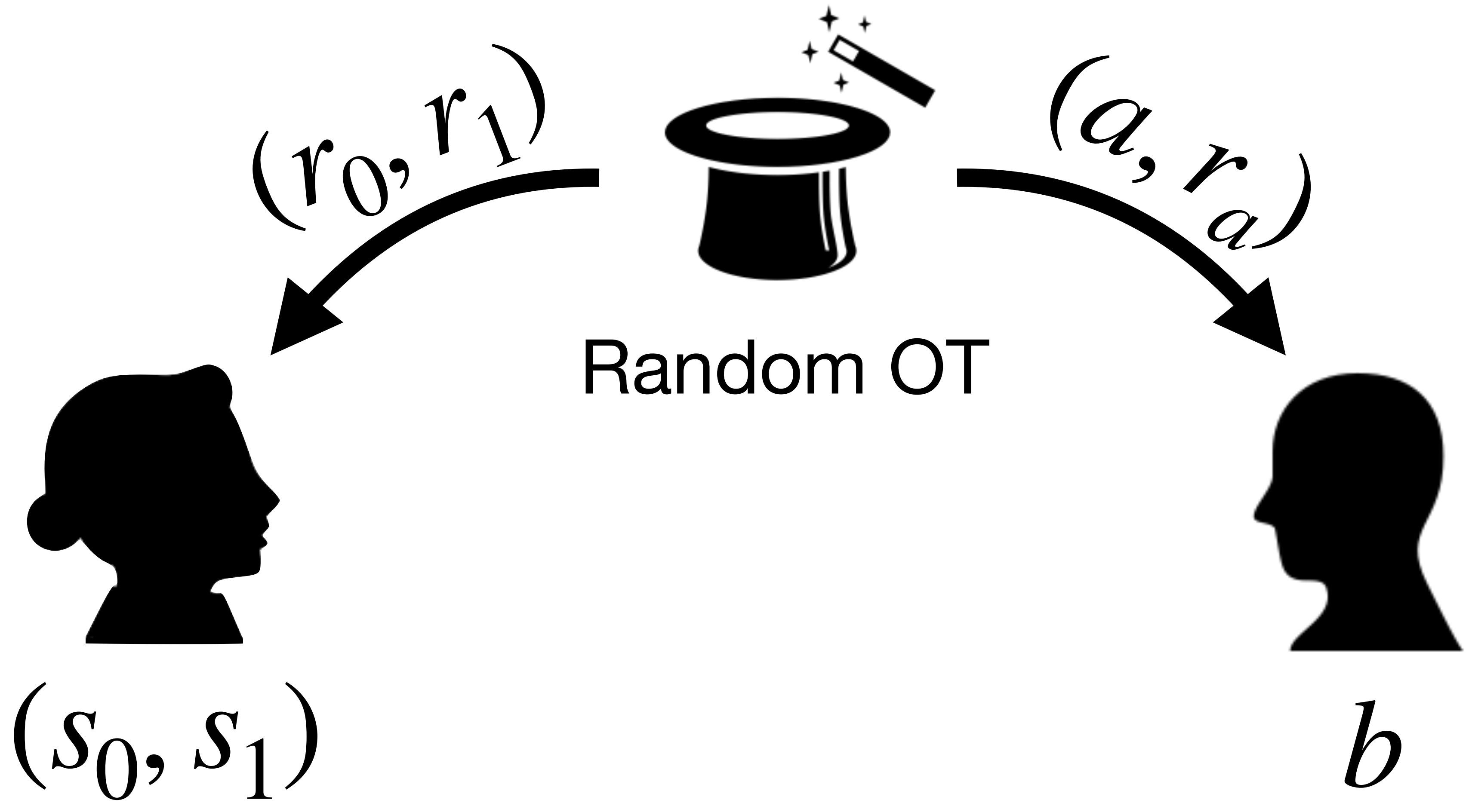
3. Use OT to compute the gates

share(x, y) $\Longrightarrow$ share(GATE(x, y))

I'll skip the details for now, but feel free to ask for them!

Precomputing Oblivious Transfers (Beaver, 1995)

Given a **random** oblivious transfer, two parties can construct a **standard** oblivious transfer



The (simple) protocol:

- If $a = b$ and Bob gets $(s_0 \oplus r_0, s_1 \oplus r_1)$, he can get $s_b = s_a$, since he knows only $r_b = r_a$.
- If $a = 1 - b$ and Bob gets $(s_0 \oplus r_1, s_1 \oplus r_0)$, he again gets s_b, since he knows only r_{1-b}.
- Bob simply tells Alice whether $a = b$ (leaks nothing since a is random!), and Alice sends the appropriate pair.

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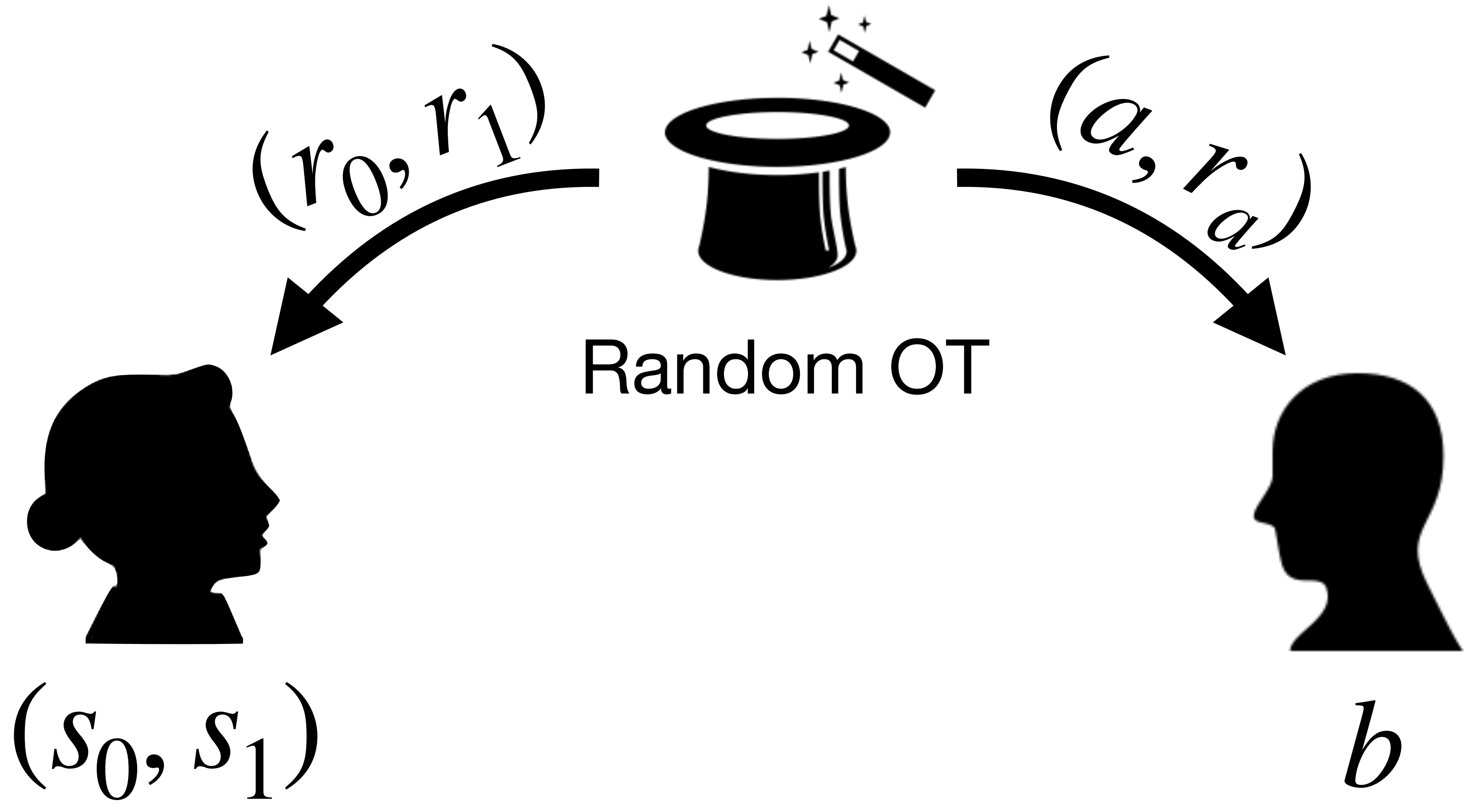

The protocol is:

- Perfectly secure (no assumption required)
- Very fast: only three bits exchanged per OT

$\Longrightarrow$ Almost all computations can be executed **ahead of time** to precompute many OTs

$\Longrightarrow$ Reduces *efficient secure computation* to the task of securely and efficiently **distributing long correlated strings** (here, random pairs (r_0, r_1) an (a, r_a))

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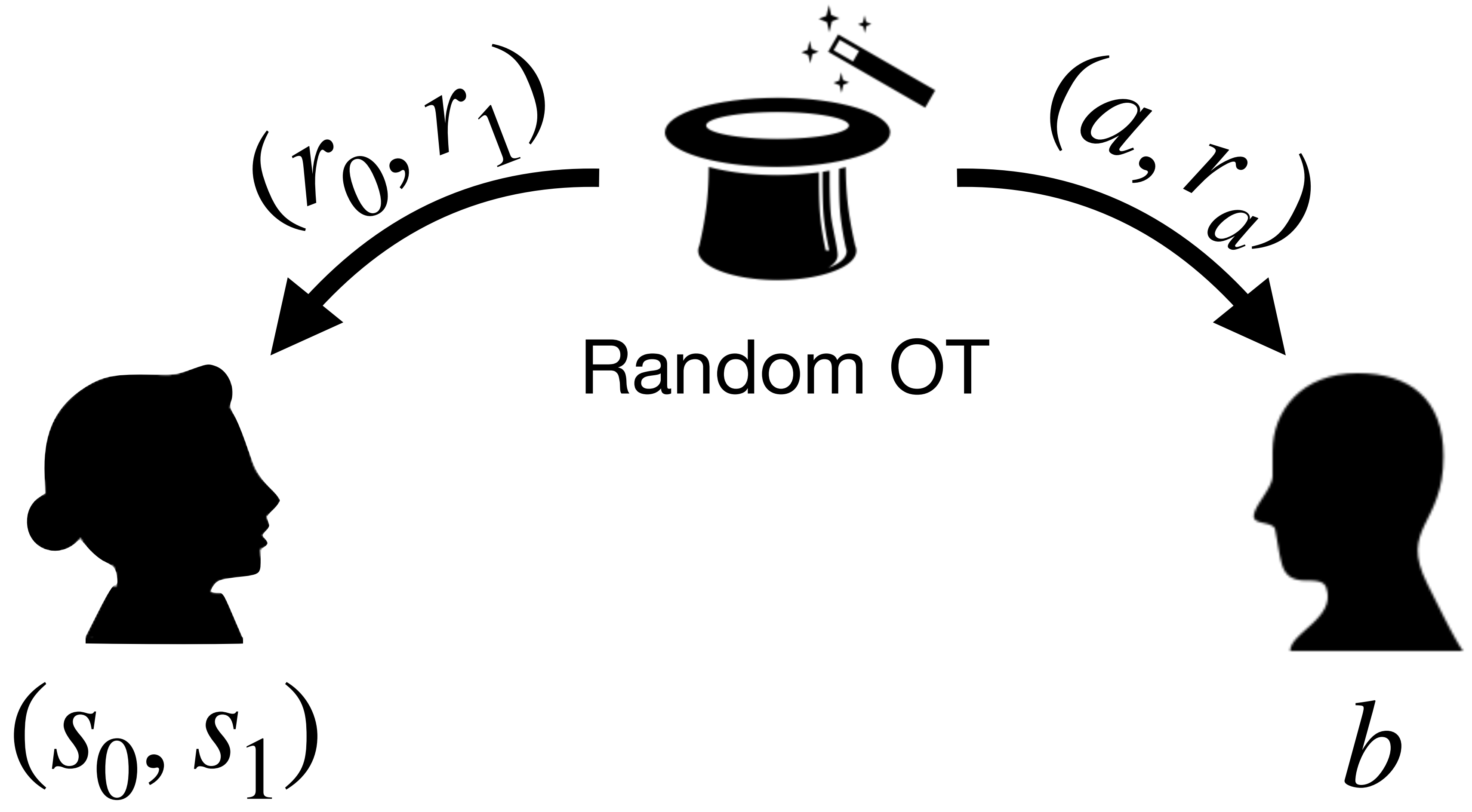

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Ishai-Killian-Nissim-Petrank 2003:

Computing n random OTs can be done using

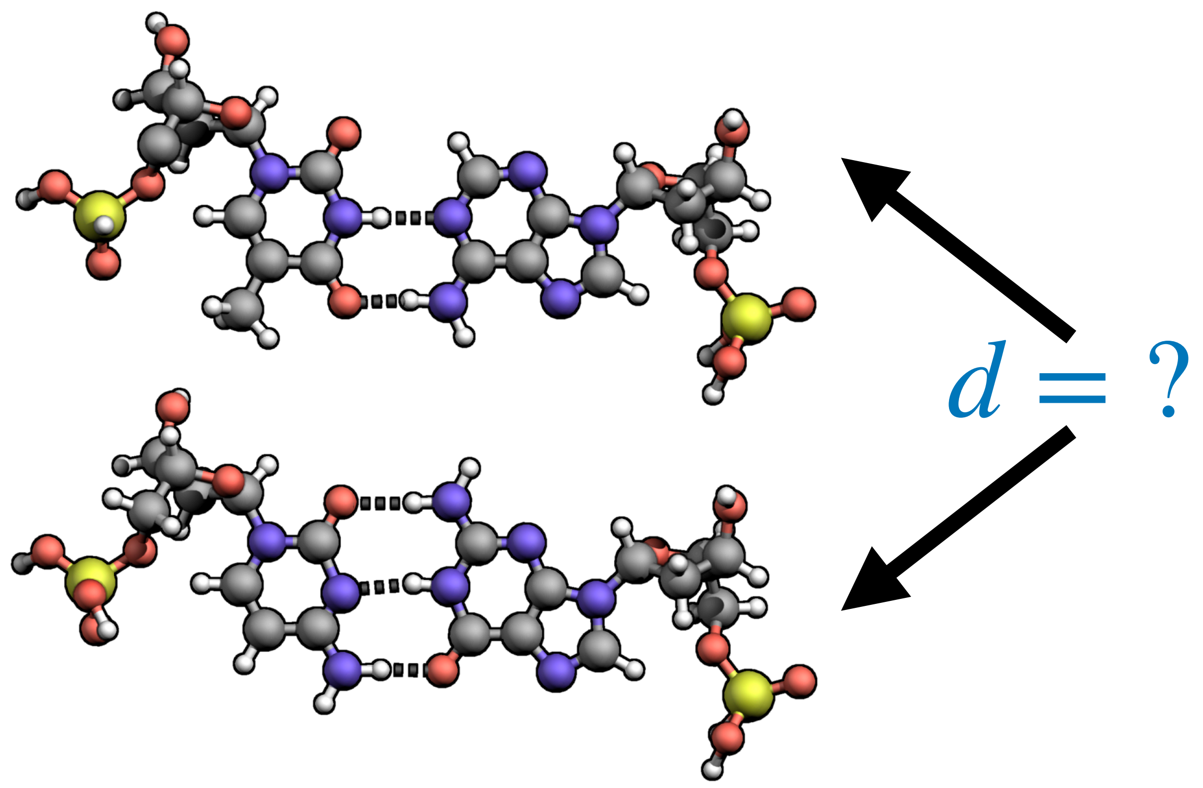
✔ 128 « base » oblivious transfers

✔ 3 **evaluations of a hash function** per OT

✘ $\sim$ 100 bits of communication per OT

Just to Get a Sense of Scales…

- **Edit distance:** number of insertions, deletions, and substitutions to convert one string into another
- Widely used to measure similarities, e.g. in genomics
- This is by all mean a relatively **simple function**



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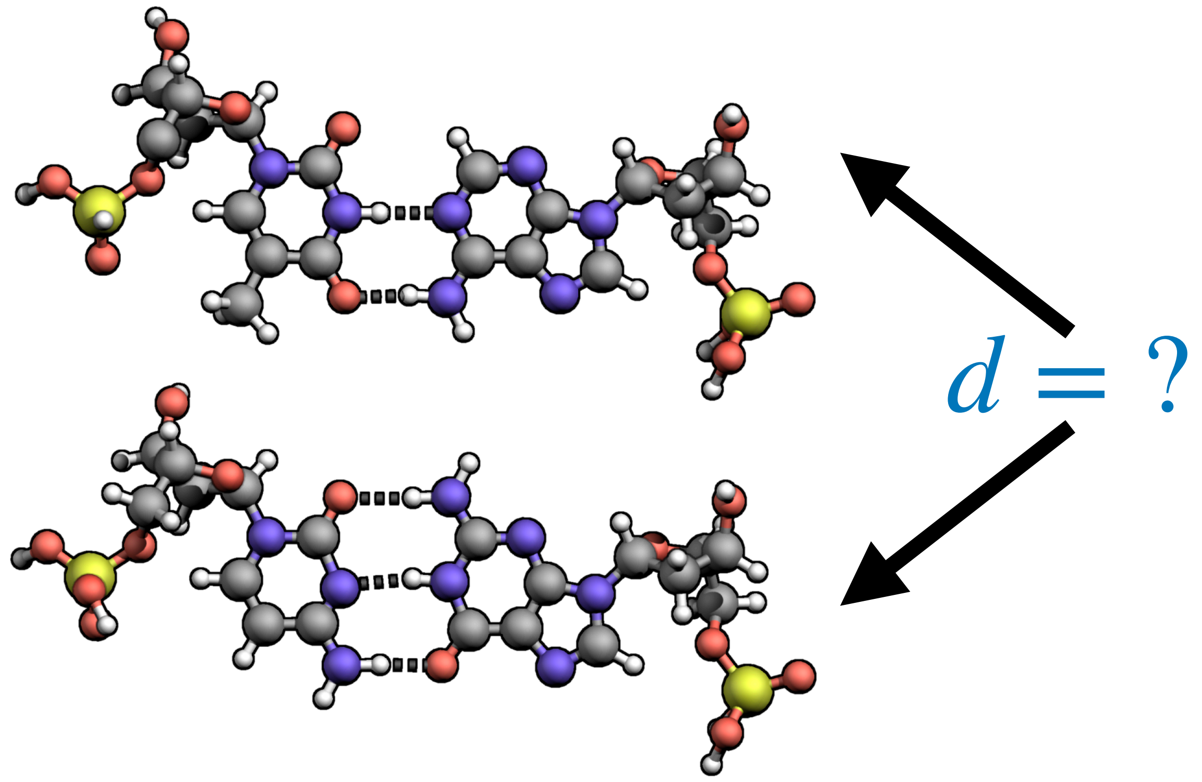

Assume Alice and Bob want to securely compute the edit distance between 512-byte inputs (that is, *small* inputs). This requires:

- Converting the function to a boolean circuit $\Longrightarrow$ 5,901,194,475 AND gates according to [1]
- Securely computing the circuit $\Longrightarrow$ 5,901,194,475 $\times$ 100 bits $\approx$ 70 Gigabytes of communication

This is **doable but expensive**, and communication is **typically the bottleneck** in secure computation protocols.

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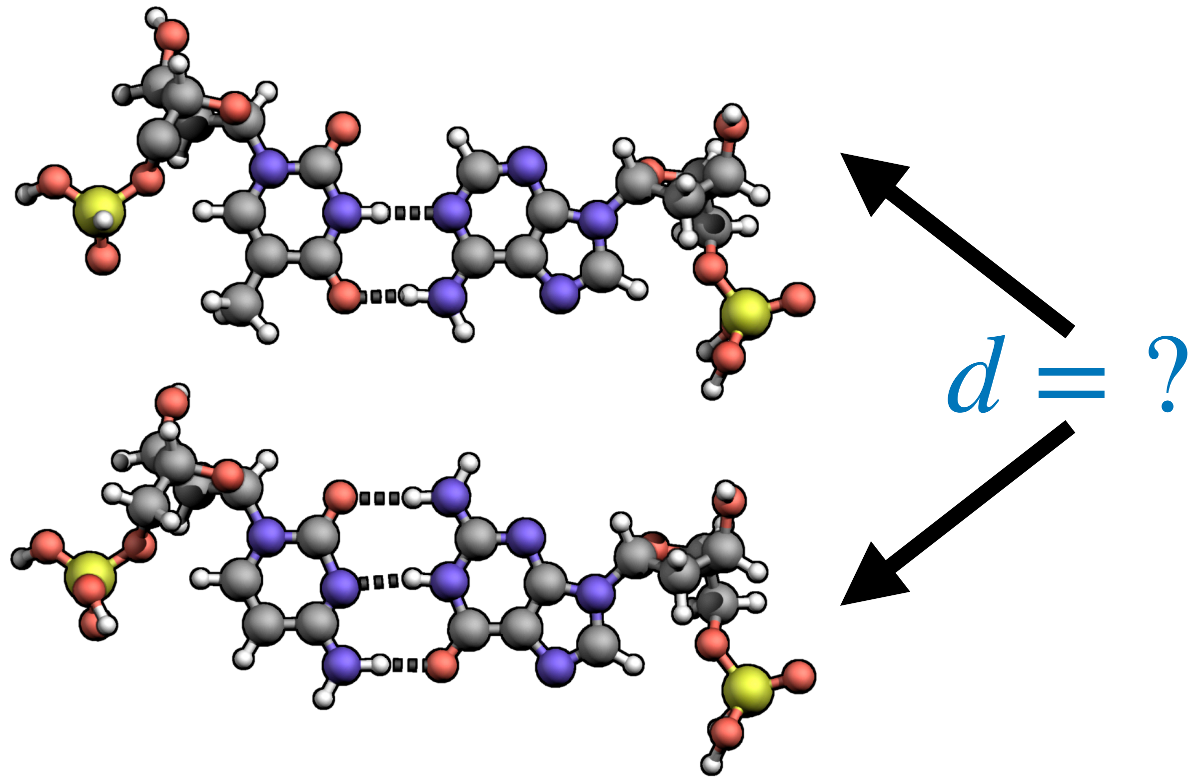

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$\Longrightarrow$ **Can we precompute random OTs using much less communication?**

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Back to Secure Computation

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Computing n random OTs can be done using	Computing n random OTs can be done using
✓ 128 « base » oblivious transfers	✓ A few hundred « base » oblivious transfers
✓ 3 **evaluations of a hash function** per OT	✓ 2 **evaluations of a hash function** per OT
✗ $\sim$ 100 bits of communication per OT	✓ $\sim$ **0 bits of communication per OT**
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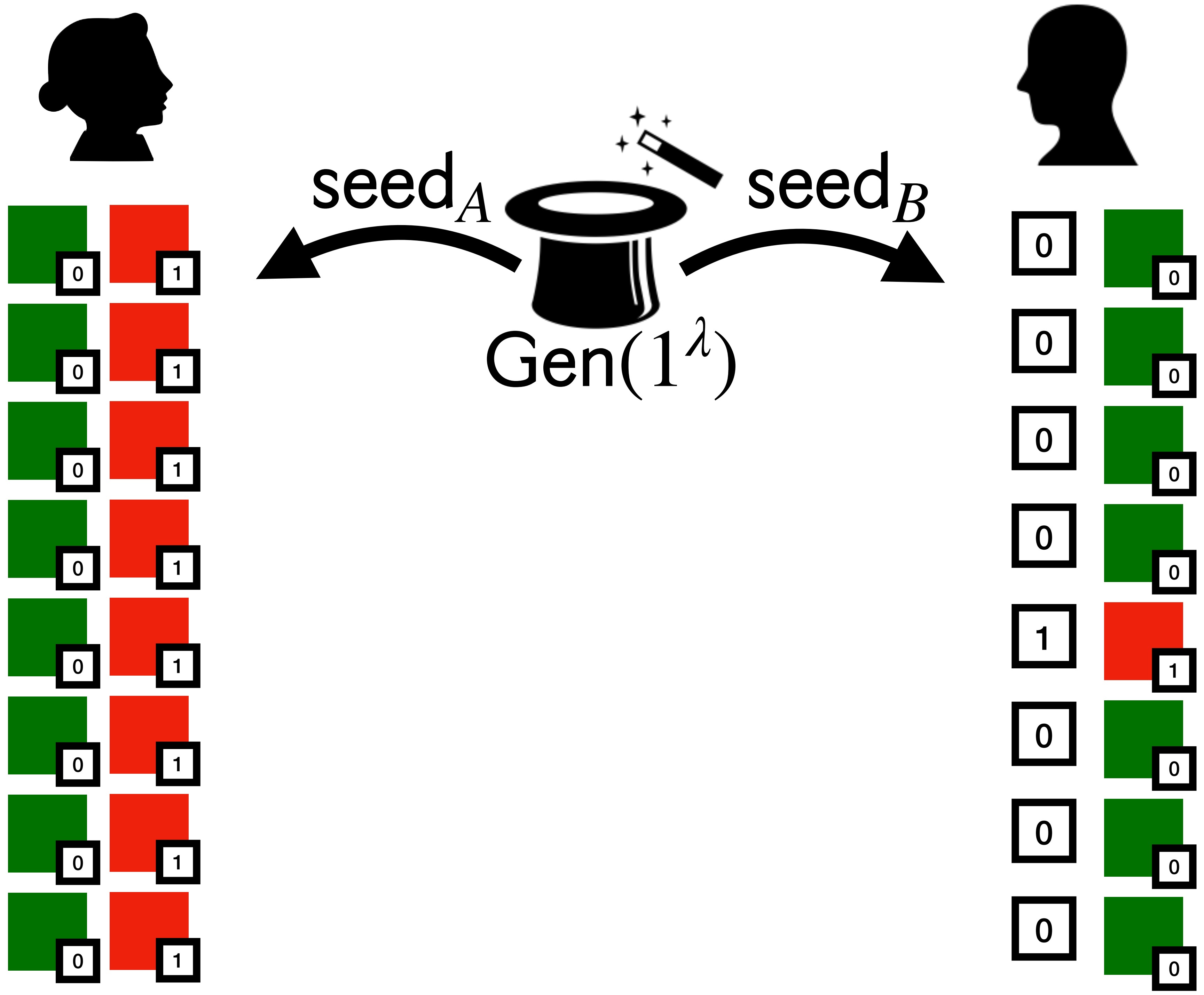
Computing n random OTs can be done using
- ✓ A few hundred « base » oblivious transfers
- ✓ 2 **evaluations of a hash function** per OT
- ✓ **$\sim$ 0 bits of communication per OT**
- ? Computing an n-by-$2n$ matrix-vector product

- Choosing the « right » matrix is related to deep questions in coding theory
- Latest exciting works (**C**RR'21, B**C**GIKS'22) provide **extremely efficient instantiations**
- Many fundamental questions remain partially open:
  - ➡ Achieving more powerful correlations (related to deep questions in algebraic coding theory)
  - ➡ Extending efficiently to n parties (currently works best for two parties)
  - ➡ …

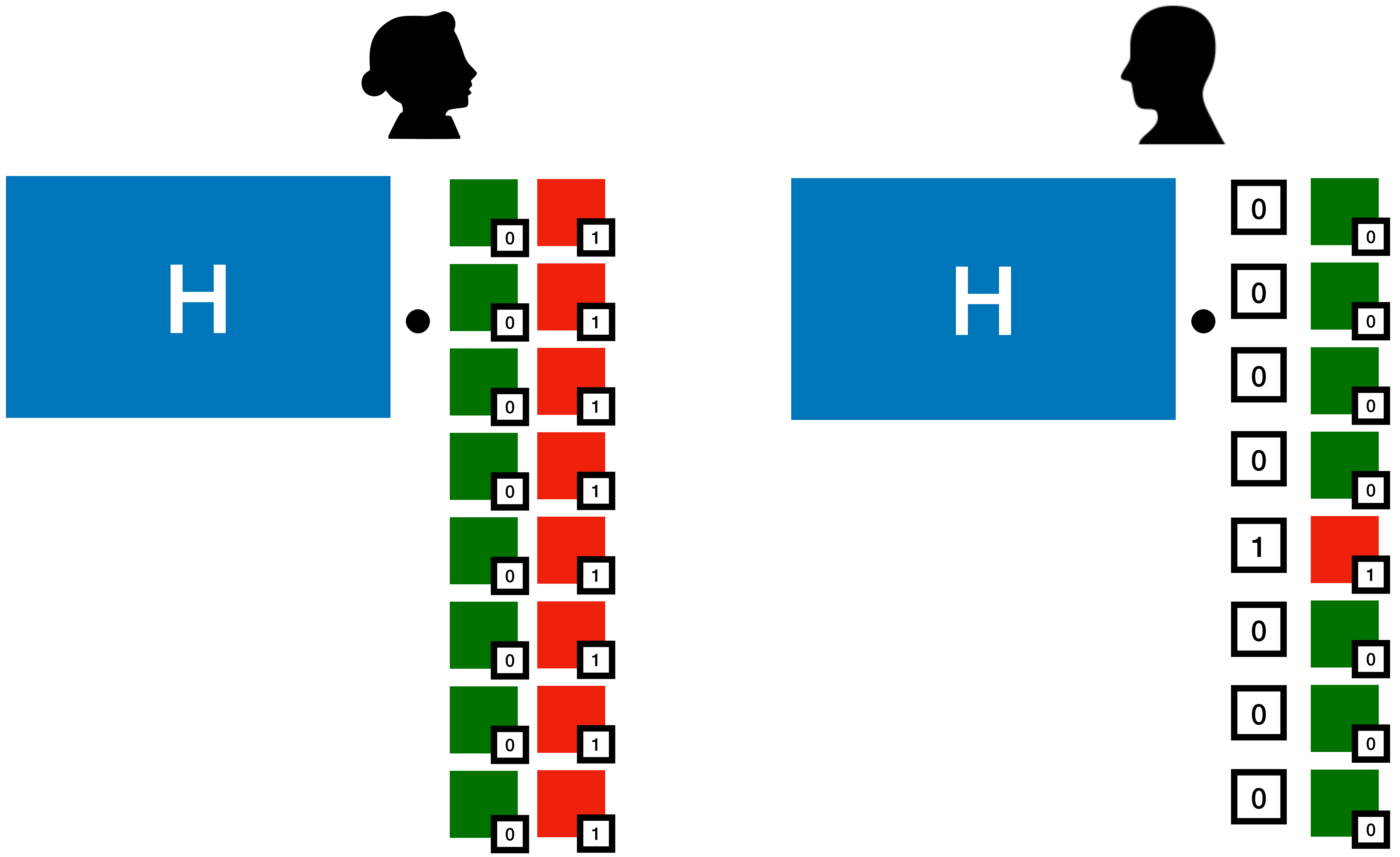
A 10s Walkthrough of the Core Ideas

Reminder: Alice and Bob want to get many (pseudorandom) oblivious transfers from *short* seeds.

Step 1. Design a strategy, using cryptographic techniques, to get a solution when Bob's selection bits are **all equal to 0** except t.



Step 2. **Scramble** the bits using a large, public, **structured**, compressive matrix multiplication



The natural way to attack is to distinguish from random by looking for a *bias* in $H \cdot \vec{b}$, i.e., finding $\vec{v}$ s.t. $\vec{v}^\intercal \cdot H \cdot \vec{b}$ is **biased**

$\iff \langle \vec{v} \cdot H, \vec{b} \rangle = 0$ with high probability

$\iff \vec{v}$ has low weight… Which is impossible when $H^\intercal$ generates a **good code**

$\implies$ **the goal is to find structured good codes where the computation of $x \to H^\intercal \cdot x$ is very fast**

Thank You for Your Attention!

Questions?

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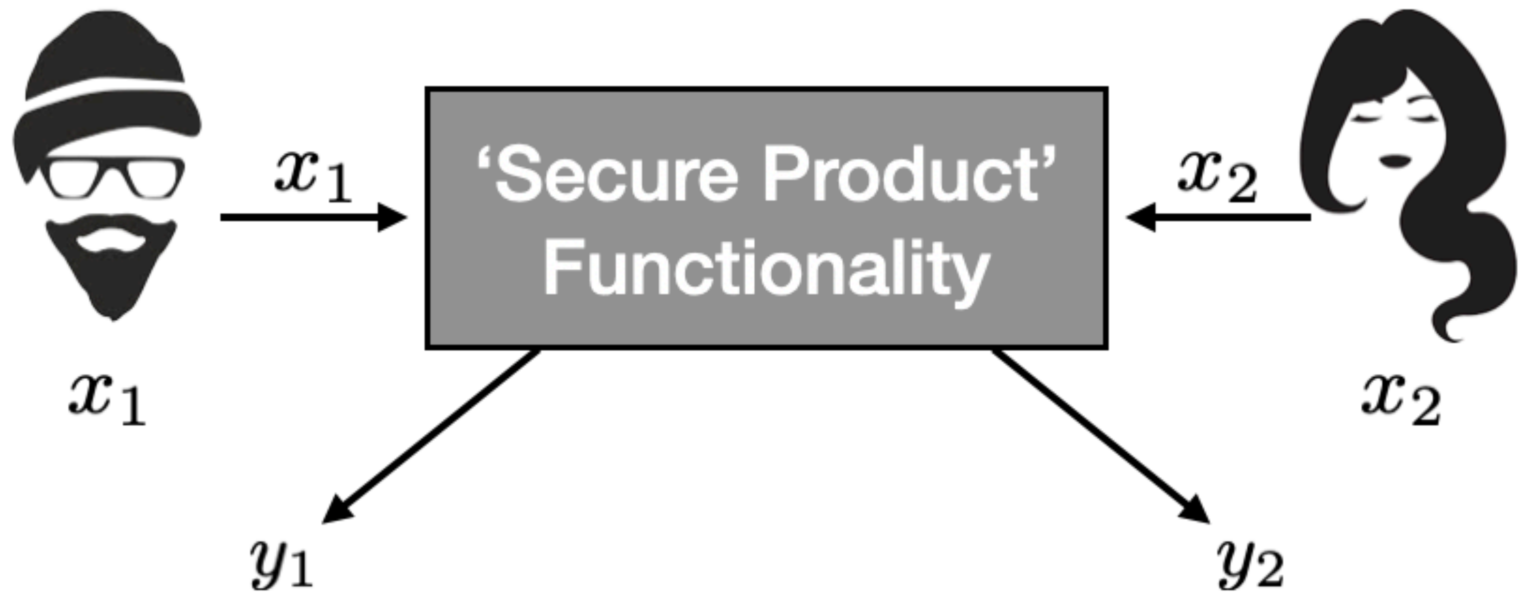
Backup Slides

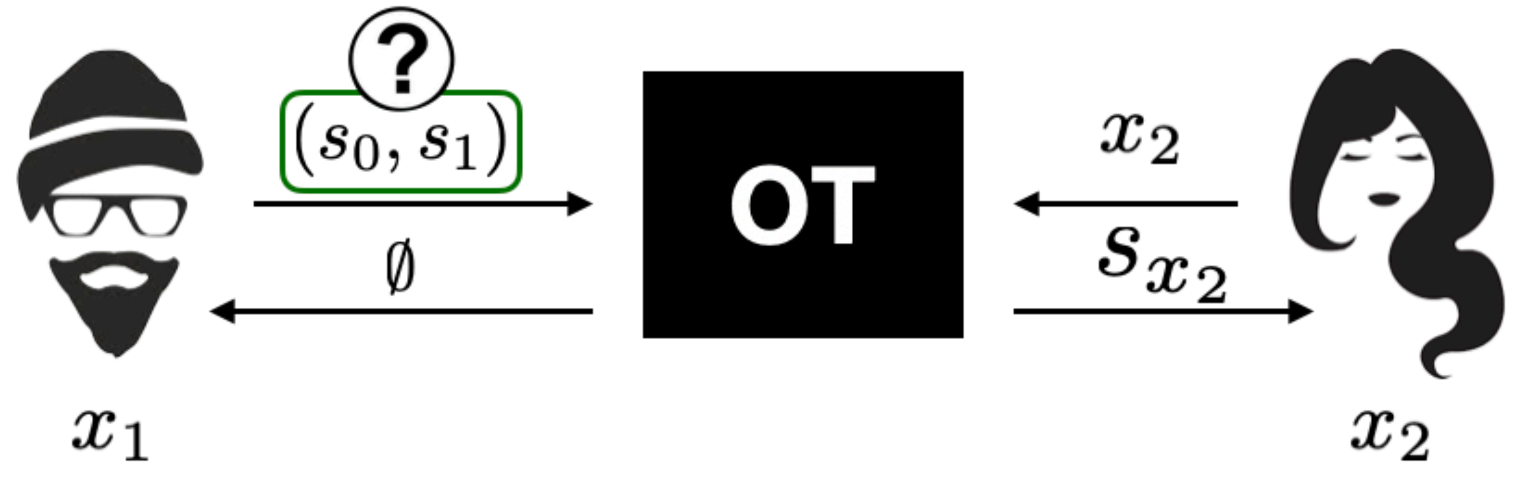
Secure Computation from Oblivious Transfer

Warm-up I: 2-Party Product Sharing



(y_1, y_2) random conditioned on $y_1 \oplus y_2 = x_1 x_2$

Step-by Step Solution



- We use an OT functionality where Alice is the receiver, and her *selection bit* is her input x_2
- What should be Bob's input? Let's work out the equation:

$$\begin{aligned} s_{x_2} &= x_2 \cdot s_1 + (1 - x_2) \cdot s_0 \\ &= x_2 \cdot s_1 \oplus (1 \oplus x_2) \cdot s_0 \\ &= s_0 \oplus (s_0 \oplus s_1) \cdot x_2 \end{aligned}$$

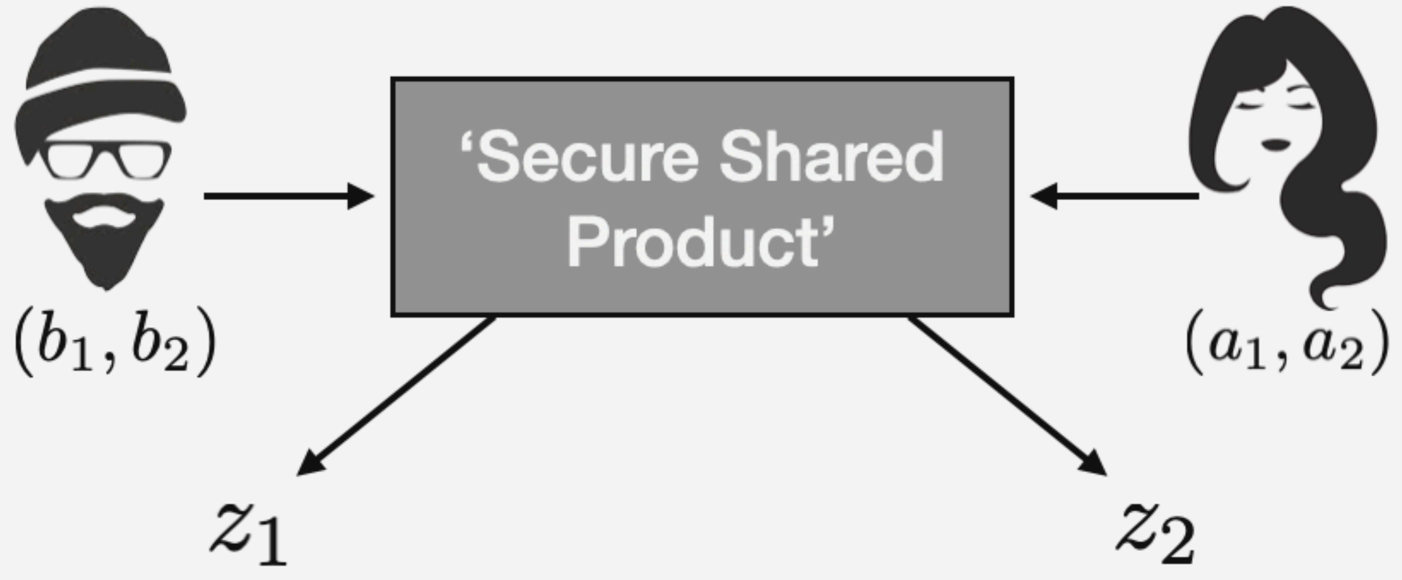
$\Longrightarrow \boxed{s_0} \oplus s_{x_2} = \boxed{(s_0 \oplus s_1)} \cdot x_2$

Share of Bob   This should be x_1

$\Longrightarrow (s_0, s_1)$ are (2,2)-shares of x_1.

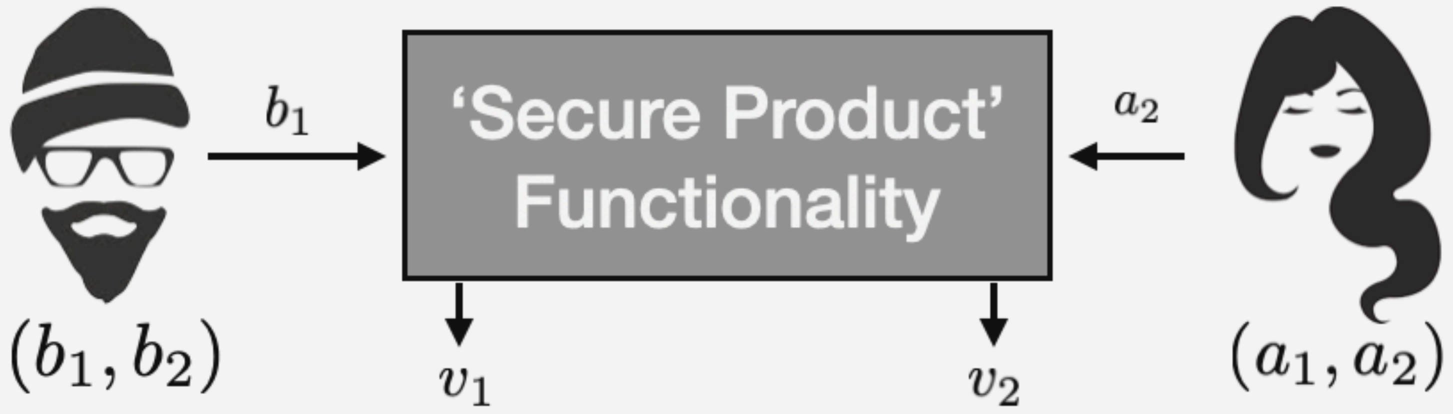
Warm-up II: Variant

This time, Alice and Bob start with *shares* of values (x,y), and want to compute shares of the product x.y



(a_1, b_1) are shares of x
(a_2, b_2) are shares of y
(z_1, z_2) are random shares of $z = x \cdot y$

Solution



$$x \cdot y = (a_1 + b_1) \cdot (a_2 + b_2)$$
$$= \boxed{a_1 \cdot a_2} + \boxed{a_1 \cdot b_2} + \boxed{a_2 \cdot b_1} + \boxed{b_1 \cdot b_2}$$

Value known to Alice   Value known to Bob

Each of these values is the product of a value known to Alice and a value known to Bob